International Journal of CAD/CAM Vol. 7, No. 1, pp. 61~69 (2007)

INTERNATIONAL JOURNAL OF CAD/CAM
www.ijcc.org

Reliability-Based Topology Optimization for Different Engineering Applications

G. Kharmanda*, S. Lambert[1], N. Kourdi, A. Daboul and A. Elhami[1]
Faculty of Mechanical Engineering, Aleppo University, Aleppo, Syria
[1]Laboratoire de Mécanique de Rouen, INSA de Rouen, France

Abstract – The objective of this work is to integrate reliability analysis into topology optimization problems. We introduce the reliability constraint in the topology optimization formulation, and the new model is called Reliability-Based Topology Optimization (RBTO). The application of the RBTO model gives a different topology relative to the classical topology optimization that should be deterministic. When comparing the structures resulting from the deterministic topology optimization and from the RBTO model, the RBTO model yields structures that are more reliable than the deterministic ones (for the same weight). Several applications show the importance of this integration.

Keywords: Reliability-Based Topology Optimization, Reliability-Based Design Optimization, Reliability Analysis, Sensitivity Analysis, Finite Element Analysis

1. Introduction

Significant research over the last four decades has focused on the search for the "optimum" structural system. Two basic design philosophies are used in structural optimization: Deterministic Structural Optimization philosophy and Reliability-Based Structural Optimization philosophy (Frangopol 1995). Deterministic structures are treated with deterministic (or fixed) data concerning geometry, loading, materials …, and the designer aims to obtain the solution without caring about the effects of uncertainties (or randomness) concerning geometry and loading data. However, structural problems are generally non-deterministic that necessitates concepts and methods of probability. The integration of reliability (or probability) analysis can be carried out into the three structural optimization families: Sizing, Shape and Topology optimization. Therefore, the reliability-based optimization aims to define the best compromise between cost and safety. When integrating the reliability concept into the sizing and shape optimization, the model is called Reliability-Based Design Optimization (RBDO), which allows us to design structures, which satisfy economy and safety requirements (see Kharmanda *et al.* 2001-2007).

But when coupling the reliability analysis with the topology optimization being considered non-quantitative of nature, the new model is called Reliability-Based Topology Optimization (RBTO). The purpose of the Reliability-Based Topology Optimization (RBTO) is to consider some uncertainties of the geometry or the loading of the structure, by introducing the reliability criteria in the optimization procedure. This integration takes into account the randomness of the applied loads and the geometry description.

*Corresponding author:
Tel: +963-(0)21-5112-319
Fax: +963-(0)21-3313-910
Email: mgk@aloola.sy or mgk@scs-net.org

2. RBTO models

Topology Optimization is used to increase the performance of structures. The deterministic topology optimization has great impact on the performance of structures, and the last decades have seen an enormous interest in this important sub-area of structural optimization (Bendsøe and Kikuchi 1988; Bendsøe 1995; Olhoff *et al.* 1998; Beckers 1999; Olhoff 2000; Eschenauer and Olhoff 2001). Recently, a new model called, Reliability-Based Topology Optimization (RBTO), has been proposed from two points of views:

From point of view **topology optimization**, Kharmanda and Olhoff (2001) have elaborated un RBTO model with object of providing the designer with several reliability-based structures however in the classical topology optimization, the designer produces only one deterministic topology. It has been shown the importance of the RBTO model yields structures that are more reliable than those produced by deterministic topology optimization (for the same weight, see Kharmanda and Olhoff 2002; Kharmanda *et al.* 2004). In the new model reliability constraints have been introduced into deterministic topology optimization problem (reliability-based constraints with SIMP approach for continuum structures). The used limit state function is a linear combination of the random variables. Therefore, the proposed approach is a heuristic strategy that aims to reduce mass while improving the reliability level of the structure without greatly increasing its weight. But the limit state function used by them was not based on failure criteria for the structure. This formulation considered uncertainty with respect to geometrical dimension and applied load only.

Also their reliability analysis seems to be independent of the boundary and loading condition, so their results showed similar values for the uncertain variables for different structures (Mozumder and Renaud 2006).

From a point of view **reliability analysis**, the classical topology optimization is formulated as finding the stiffest structural layout with a volume constraint. However, the feasibility of volume constraint is not critical in structural design problems. It is more important to consider the variations of the stiffness under uncertainties. To maintain the robustness of stiffness in the topology design, Bae and Wang (2002) formulated the topology design optimization as volume minimization problem with a displacement constraint and applied the RBDO technique. They minimize the structural volume subject to linear limit state function. In the research of Jung *et al.* (2003), the extension of the work of Bae and Wang for the geometrically non linear problems is studied. They minimize the structural volume subject to a nonlinear limit state function. Next, Tovar *et al.* (2004) have developed the Hybrid Cellular Automaton (HCA) method for structural synthesis of continuum material where the state of each cell is defined by both density and strain energy. In Agarwal (2004), a decoupled RBDO approach is employed such that the topology optimization is separate from the reliability analysis. Patel *et al.* (2005) showed the use of RBTO using the gradient free Hybrid Cellular Automata (HCA) method. Their formulation incorporates uncertainty with respect to material property also. They considered limit state function based on failure modes on the output displacements.

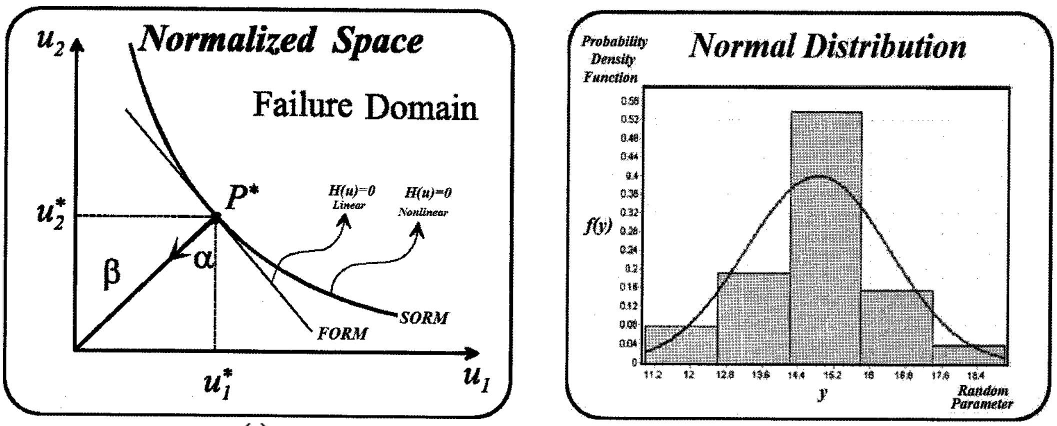
So the topology point of view philosophy seems to be interesting for topology designers because it provides several reliability-based structures relative the reliability index changes. Some works have been carried RBTO from point of view philosophy. It led to different topology structure with a high percentage of the grey elements which has no sense for the following optimization stages.

2.1. Formulation

The main difference between the deterministic topology optimization procedure and the proposed RBTO model is to take into account the randomness (variability) of the most important variables that exhibit strong influence on the resulting optimal topology. The deterministic topology problem allows for the prediction of the gross shape of the body and it is possible to predict placement and shape of holes in the structure. However, the RBTO model leads to a different set of optimal topologies with respect to that produced by the deterministic topology optimization procedure. In order to control the produced topologies, a reliability index β (see Hasofer and Lind 1974) is introduced with a normalized vector $\mathbf{u}$ that defines the relation between the random variables $\mathbf{y}$ and the design variables $\mathbf{x}$. The general calculation of the reliability index (RIA: Reliability Index Approach; Tu *et al.* 1999) can be realized by the following form:

$$\beta = \min(\sqrt{\mathbf{u}^T\mathbf{u}}) \text{ subject to } H(\mathbf{u}) = 0 \qquad (1)$$

The optimum value of β corresponds to the best distribution of the vector $\mathbf{u}$ relative to the studied random variables. The solution of this problem is called the *design point* P^*, as illustrated in Fig. 1a. The evaluation of reliability index is carried out by FORM (First Order Reliability Method) for linear limit state function and by SORM (Second Order Reliability Method) for nonlinear limit state function. When the mechanical model is defined by numerical methods, such as the finite element method, the evaluation of the reliability implies a special coupling procedure between both reliability and mechanical models. According to a statistical study of failure probability, several distributions can be approximated (normal, lognormal, uniform...). For both theoretical and practical reasons, the normal distribution is probably the most important distribution in statistics. For example, many classical statistical tests are based on the assumption that the data follow a normal distribution. Figure 1b shows an example of a statistical study that can be approximated to normal distribution case, the target (or allowed) reliability index β_t (Haldar and Mahadevan 2000; Ditlevsen and Madsen 1996)



(a) (b)

Fig. 1. (a) Normalized space and (b) Statistical study.

is calculated using the failure probability as follows:

$$P_f = \int f(\mathbf{y}) dy_1 ... dy_n \quad (2a)$$

The target reliability index that correspond to the resulting failure of probability of equation 2 (see Figure 1b), in linear limit state function is numerically computed as follows

$$P_f \approx \Phi(-\beta_t) \text{ or } \beta_t \approx -\Phi^{-1}(P_f) \quad (2b)$$

where $\Phi(.)$ is the standard Gaussian cumulated function given as follows.

$$\Phi(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{z^2}{2}} dz, \quad (2c)$$

The normalized variable for normal distribution case of a random parameter y_i can be written as:

$$u_i = \frac{y_i - m_i}{\sigma_i}, \quad i = 1, ..., n \quad (3a)$$

where n is the number of random variables, m_i and σ_i are respectively standard-deviations and mean value of i^{th} random parameter y_i. Equation 2b numerically gives the target reliability index β_t to be respected and the reliability index $\beta(\mathbf{u})$ is then presented by the following formulation:

$$\beta(\mathbf{u}) = \min \sqrt{u_1^2 + ... + u_i^2 + ... + u_n^2} \quad (3b)$$

In general the nuclear and spatial studies necessitate very small failure probability, the failure probability should be: $P_f \in [10^{-6} - 10^{-8}]$ that corresponds to a reliability index $\beta \in [4.75 - 5.6]$ when using equations 2b and 2c, while in structural studies, the failure probability should be: $P_f \in [10^{-3} - 10^{-5}]$ that corresponds to a reliability index $\beta \in [3 - 4.25]$. In this paper, we consider the target reliability index which is used in structural engineering as $\beta_t = 3.8$. Before presenting the RBDO model, we start writing the formulation of the classical topology problem using the SIMP approach (Bendsoe 1989), we then have:

$$\text{min: } \mathit{Compliance}(\mathbf{x}) = \mathbf{q}^T \mathbf{K} \mathbf{q} = \sum_{e=1}^{N} p(x_e)^{p-1} q_e^T k_0 q_0$$

$$\text{subject to: } \frac{\mathit{Volume}(\mathbf{x})}{V_0} = f(\mathbf{x}) \quad (4a)$$

where $\mathbf{q}$ and $\mathbf{K}$ are the global displacement vector and the global stiffness matrix, respectively. q_e and k_0 are the element displacement vector and stiffness matrix, respectively. N is the number of elements to discretize the design domain, p is the penalization power, *Volume* and V_0 are the material volume and design domain volume, respectively, and f is the prescribed volume fraction. The design variables $\mathbf{x}$ are the dimensions of the discretization elements. In the DTO, the input vector $\mathbf{m}$ regroups the geometry and loading parameters being deterministic (fixed). However, the input vector $\mathbf{y}$ of the RBDO is computed according to a reliability analysis (see figure 2) and the problem can be then written as:

$$\text{min: } \mathit{Compliance}(\mathbf{x}) = \mathbf{q}^T \mathbf{K} \mathbf{q} = \sum_{e=1}^{N} -p(x_e)^{p-1} q_e^T k_0 q_0$$

$$\text{subject to: } \beta(\mathbf{u}) \geq \beta_t \text{ and } \frac{\mathit{Volume}(\mathbf{x})}{V_0} = f(\mathbf{x}) \quad (4b)$$

$\beta(\mathrm{u})$ and β_t are the reliability index of the system and the target (or allowable) reliability index, respectively. In equation (4b) the evaluation of the reliability constraint $\beta(\mathbf{u}) \geq \beta_t$ is carried out by an optimization process to find the minimum distance between the origin of the normalized space and the limit state function using FORM or SORM (see Fig. 1). For simplicity, we use in this work the FORM to solve equation (1) for linear limit state function.

2.2 RBTO algorithm

We define our strategy, which implies a coupling between the reliability analysis and the topology design problem. After having proposed a set of parameter assembled in the vector $\mathbf{m}$ which will be called the deterministic input parameter vector, this vector concerns the applied loads and geometry of the structure. The selection of these active parameters depends on the role of each one in the structure (for example: geometry, loading, materials …). If these parameters are not given as requirements, the designer can analytically, semi-analytically or numerically study the sensitivity analysis to identify random parameters which have significant effect on the objective function. This selection is considered a facultative step. However, our RBDO algorithm consists of two main steps:

Step 1: Reliability index evaluation

The evaluation of the reliability index can be carried out by a particular optimization procedure. This index is the minimum distance in the normalized space (Fig. 1a). For simplicity, the *limit state is considered as linear function* (Fig. 1a), the reliability problem is then given by:

$$\beta = \min d(\mathbf{u}) \sqrt{u_1^2 + ... + u_i^2 + ... + u_n^2} \text{ subject to } \beta(\mathbf{u}) \geq \beta_t \quad (5a)$$

During the optimization procedure, we can *analytically* provide the derivative of the distance d with respect to u_i by the following form:

$$\frac{\partial d}{\partial u_i} = \frac{u_i}{d(\mathbf{u})} \quad (5b)$$

The resulting vector $\mathbf{u}$ of the problem in equation (5a) will be used to evaluate the random vector $\mathbf{y}$. The selected parameters will be assembled in the random parameter vector **y** as input values. The used optimization process is simply the *gradient-based method* to update **u**.

Step 2: Topology optimization procedure

After satisfying the reliability constraints and determining the vector of the random parameters **y**, we call the topology optimization procedure with the resulting input random vector **y**. The resulting optimal topology principally depends on the target reliability index values β. The topology problem (4b) seeks to minimize the compliance using the new values of the random variables **y** as inputs. The used optimization method to update **x** is the *standard optimality criteria method*. For more details on the derivation and implementation of the optimality criteria method, the reader is referred to the literature (e.g. Bendsøe 1995). During the topology optimization process, the sensitivity analysis of the objective function is *analytically* carried out with respect to the design variables **x** (Sigmund 2001) as follows:[2]

$$\frac{\partial C}{\partial x_e} = -p(x_e)^{p-1} q_e^T k_0 q_0 \qquad (6)$$

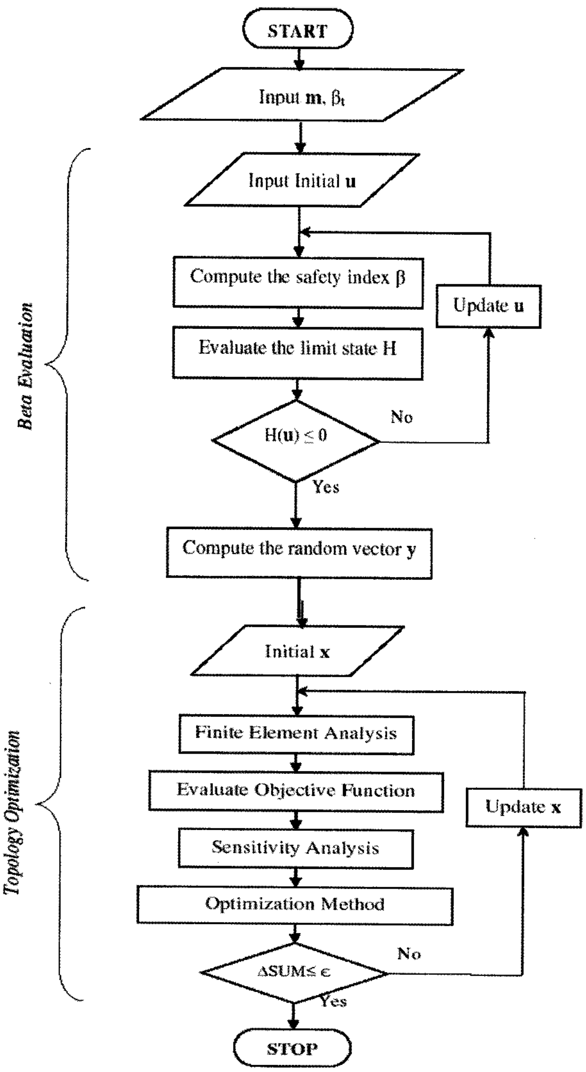


Fig. 2. RBTO procedure

Fig. 2 shows both steps of the RBTO algorithm. The construction of this algorithm is first to regroup the most influent parameters in an input vector **m**. Next, the reliability index β will be evaluated in satisfying the associated linear constraint and the resulting normalized vector **u** will be used to formulate the random parameter vector **y** that will be considered as input vector for the following topology optimization process. Finally, we apply the SIMP approach to obtain the new reliable and optimal topology (Fig. 2).

3. Numerical Applications

The reliability-based design optimization (RBDO) tries to find a highly reliable design by ensuring the satisfaction of probabilistic (or reliability) constraints. In the conventional RBDO method using the reliability index evaluation, the probabilistic constraints are stated in terms of the reliability indices obtained from the first-order reliability method (FORM). In the FORM, the reliability indices are calculated by determining the most probable point (MPP) in the standard normal space of random variables (**u**-space). At each iteration in the RBDO, the MPP search is necessary to obtain the reliability indices. The computational requirements are costly because numerous reliability analyses should be performed. Furthermore, the MPP search is very expensive for highly nonlinear constraints in the **u**-space (Jung and Cho 2004). This way often leads to very high computing time and weak convergence stability. The topology optimization process is already expensive. We have to note that when coupling with the reliability constraints, the new RBTO problem becomes extremely complex. Thus, we need to simplify the RBDO process by using the topology point of view that allows the designer to generate different topologies according to the variability of reliability index. A truss modeling in Kharmanda *et al.* (2004) for resulting topologies shows that the reliability-based topology contains a number of bars more than the deterministic topology which generally leads to more reliable structure. In order to validate the topology point of view, we apply the deterministic topology optimization procedure and the RBTO model to several cases as follows:

3.1 RBTO for static analysis

The design domain considered and the boundary and loading conditions are illustrated Fig. 3a. It concerns a MBB-beam submitted to a single static load. When using SIMP approach, the DTO problem seeks to minimize the compliance subject to a given volume fraction. When considering the input parameter vector **m** and the design vector **x** as optimization variables, we can write:

$$\text{min: } \textit{Compliance}\,(\mathbf{x}) \text{ subject to: } \frac{Volume(\mathbf{x})}{V_0} = f(\mathbf{x}) \qquad (7)$$

where *Volume* and V_0 are the actual volume and design domain volume, respectively, and f is the prescribed volume fraction. Here, the number of elements in the horizontal and vertical directions: $nelx = 120$ and $nely = 20$, respectively,

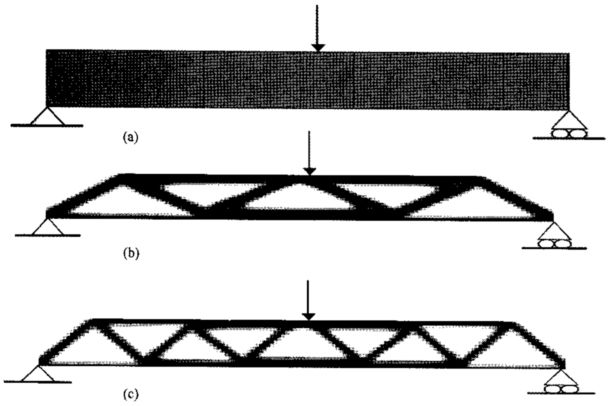


Fig. 3. Resulting DTO and RBDO structure for the MBB-beam under a static load.

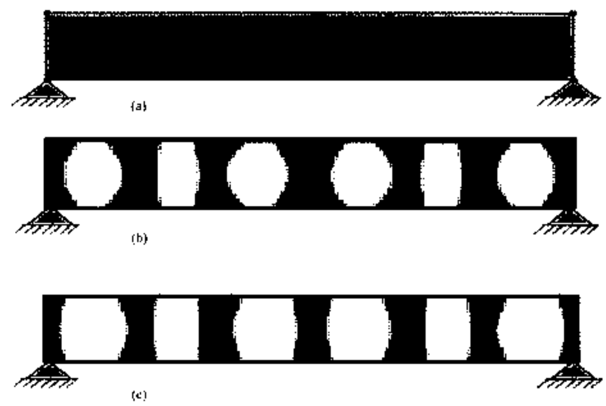


Fig. 4. Resulting DTO and RBDO structure for the MBB-beam under free vibrations.

$f = 0.5$ is the volume fraction, $F = 1$ is the applied force. For RBTO, these parameters are regrouped to be the mean values $m_i = \{120,20,0.5,-1\}$ of the selected four random variables. The standard-deviations are considered as given proportional values of the mean vector: $\sigma_i = 0.1m_i$. Now we find the random variable values y_i using equations (1) and (2). When the reliability constraint is satisfied, the optimum normalized vector $\mathbf{u}^*$ leads to the random vector **y** that contains new values of the horizontal number *nelx*, the vertical number *nely*, the volume fraction f and the applied load F. For the RBTO procedure, we consider the random vector **y** as geometry and loading inputs and add the structure reliability index as a constraint to satisfy the target (required) reliability index $\beta_t = 3.8 \in [3 - 4.25]$ as presented in Section 2.

So the RBTO problem is to minimize compliance subject to a given volume fraction and reliability constrains. When considering the input random vector **y** as geometry and loading parameter from problem (1) and the design variables **x**, we can write:

$$\text{min: } Compliance\,(\mathbf{x})$$
$$\text{subject to: } \beta(\mathbf{u}) \geq \beta_t \text{ and } \frac{Volume(\mathbf{x})}{V_0} = f(\mathbf{x}) \qquad (8)$$

When applying the DTO procedure, we get the resulting topology illustrated in Fig. 3b while for the application of the RBTO procedure, the resulting topology is illustrated in Fig. 3c. During the optimization process of this example, the RBTO needs more time to converge relative to DTO because the resulting RBTO topology has a smaller volume fraction value (see Table 1).

3.2. RBTO for modal analysis

The initial design domain considered and the boundary conditions are illustrated in Fig. 4a. We apply free vibrations on the studied beam and get several shape modes. Figs. 4b and 4c show the optimized topologies for DTO and RBTO, respectively. The classical topology optimization problem is to minimize compliance subject to a given volume fraction. When considering the input parameter vector **m** and the design vector **x** as optimization variables, we can write:

$$\text{min: } Compliance\,(\mathbf{x}) \text{ subject to: } \frac{Volume(\mathbf{x})}{V_0} = f \qquad (9)$$

The number of elements in the horizontal and vertical directions: $nelx = 120$ and $nely = 20$, respectively, and $f = 0.5$ is the volume fraction. For RBTO, these parameters are regrouped to be the mean values $m_i = \{120,20,0.5\}$ of the selected three random parameters. The standard-deviations are also considered as given proportional values of the mean vector: $\sigma_i = 0.1m_i$. Now we find the random values y_i using equations (1) and (2). When the reliability constraint is satisfied, the optimum normalized vector $\mathbf{u}^*$ leads to the random vector **y** that contains new values of the horizontal number *nelx*, the vertical number *nely*, and the volume fraction f. For the RBTO procedure, we consider the random vector **y** as geometry and loading inputs and add the structure reliability index as a constraint to satisfy the target (required) reliability index $\beta_t = 3.8$. So the RBTO problem is to minimize compliance subject to a given volume fraction and reliability constrains. When considering the random vector **y** resulting from problem (1) and the design variables x, we can write:

$$\text{min: } Compliance\,(\mathbf{x})$$
$$\text{subject to: } \beta(\mathbf{u}) \geq \beta_t \text{ and } \frac{Volume(\mathbf{x})}{V_0} = f \qquad (10)$$

When applying the DTO procedure, the DTO and RBTO algorithms do not provide us with significant topologies for

Table 1. Input parameters and results of DTO & RBTO model for static analysis.

Model	Input vector	Input parameters					Output results		
		nelx	nely	volfrac	F	β_t	Objective	Iteration	CPU-Time
DTO	m	120	20	0.5000	-1.000	---	204.43	134	01m50s
RBTO	y	142	16	0.4057	-1.189	3.8	980.21	155	16m18s

Table 2. Input parameters and results of DTO & RBTO model for modal analysis

Model	Input vector	Input parameters				Output results		
		nelx	nely	volfrac	β_t	Objective	Iteration	CPU-Time
DTO	m	120	20	0.5000	---	868.90	35	38m21s
RBTO	y	142	16	0.3797	3.8	1262.82	39	36m09s

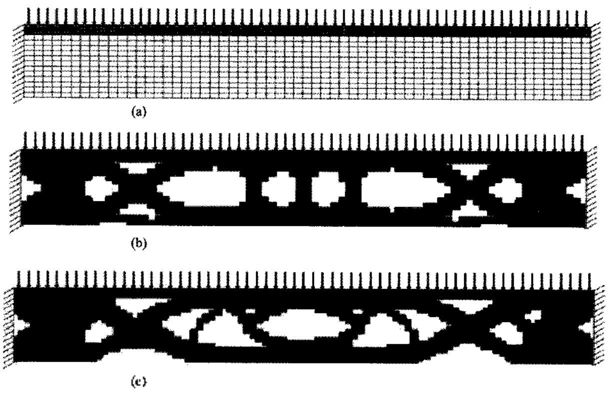


Fig. 5. Resulting DTO and RBDO structure for the two sides fixed beam under fatigue distributed loads.

the first four shape modes but at the fifth shape mode we get the resulting topology illustrated in Fig. 4b and c. We found that both DTO and RBTO algorithms need to more conditions to converge. Table 2 shows that the computing time consummation of the RBTO process is smaller than the DTO one that leads to not consider the computational time as a drawback of the RBTO model but the main importance is to provide the designer with several generated topologies and to control their reliability levels.

3.3. RBTO for fatigue analysis

The initial design domain considered and the boundary conditions are illustrated in Fig. 5a. The MBB-beam is submitted to a distributed fatigue load. Figs. 5b and 5c show the optimized topologies for DTO and RBTO, respectively. When considering the input parameter vector **m**, the optimization problem is to minimize the maximum damage subject to can be written as:

$$\text{min: max } Damage\,(\mathbf{x}) \text{ subject to: } \frac{Volume(\mathbf{x})}{V_0}=f(\mathbf{x}) \quad (11)$$

where **x** is the vector of design variables corresponds to the number of element to be optimized. The number of elements in the horizontal and vertical directions: $nelx=80$ and $nely=16$, respectively and the force $F=-4.10^{12}$, For RBTO, these parameters are regrouped to be the mean values of the selected three random variables $m_i=\{80,16, -4.10^{12}\}$. The standard-deviations are considered as given proportional values of the mean vector: $\sigma_i=0.1m_i$. Now we find the random values y_i using equations (1) and (2). When the reliability constraint is satisfied, the optimum normalized vector $\mathbf{u}^*$ leads to the random vector **y** that contains new values of the horizontal number *nelx*, the vertical number *nely*, and the applied force *F*. For the RBTO procedure, we consider the random vector **y** as geometry and loading inputs and add the structure reliability index as a constraint to satisfy the target (required) reliability index $\beta_t=3.8$. So the RBTO problem is to minimize the maximum damage subject to a given volume fraction ($f=0.7$) and reliability constrains. When considering the random vector **y** resulting from problem (1) and the design variables **x**, we can write:

$$\text{min: max } Danage\,(\mathbf{x})$$

$$\text{subject to: } \beta(\mathbf{u})\geq\beta_t \text{ and } \frac{Volume(\mathbf{x})}{V_0}=f(\mathbf{x}) \quad (12)$$

When applying the DTO procedure, we get the resulting topology illustrated in Fig. 5b but the application of the RBTO procedure, the reliability-based topology at the same mode leads to the resulting topology illustrated in Fig. 5c. The beam structure is subjected to a multiaxial stress state due to the action of the distributed load. Among the great number of multiaxial fatigue criteria, a frequency formulation of the Crossland's damage criterion is chosen as fatigue damage assessment method. The formulation is well suited to random vibration problems and gives a fast and accurate estimation of the structural fatigue damage from the stress power spectral densities (PSD). The classical time domain approach of this criterion (Crossland, 1956) based on a global approach has been validated for multiaxial periodic loads and appears to be one of the most widely used in high cycle fatigue. The criterion assumes the structure reliability after a period T if the following inequality is satisfied at every point of the structure:

$$\frac{\sqrt{J_{2,a}}+\alpha_m \max p(t)}{\beta_m}\leq 1, \quad t\in[0,T] \quad (13)$$

$\sqrt{J_{2,a}}$ is the maximum amplitude of the second invariant of the stress deviator, this expression is related to the Von Mises stress $\sigma_c(t)$. $p(t)$ is the hydrostatic pressure defined as a function of the first invariant of the stress tensor, α_m and β_m are material parameters function of endurance limits. The frequency formulation proposed by Pitoiset (2000), partly relies on the peak factor theory and can be applied directly after a spectral analysis as classically performed in random vibration. Over an observation period T the peak factor in our case allows to estimate the extreme value reach by a process based on the Von Mises stress $\sigma_c(t)$ and the hydrostatic pressure $p(t)$, thus:

$$\sqrt{J_{2,a}}\approx\frac{1}{\sqrt{3}}\int_{-\infty}^{+\infty}\Phi_c(\omega)\mathrm{d}\omega\,\eta_c \quad (14)$$

$$\max p(t)\approx\int_{-\infty}^{+\infty}\Phi_p(\omega)\mathrm{d}\omega\,\eta_p \quad (15)$$

The equivalent Von Mises stress PSD $\Phi_c(\omega)$ and the PSD of the hydrostatic pressure $\Phi_p(\omega)$ can be calculated from the PSD matrix $\Phi_\sigma(\omega)$ of the stress vector $\sigma(t)$, the procedure for the evaluation of these terms is detailed by Segalman *et al.* (2000). η_c and η_p stand for the peak factors of the PSD $\Phi_c(\omega)$ and $\Phi_p(\omega)$. The mean of the process peak factor could be approximated as a function of the process spectral moments following a Davenport's expression (Davenport, 1964). The obtained value for the criterion can be considered as the resulting fatigue damage after a period T. Moreover the frequency domain formulation appears to be computationally by far more efficient than the time domain formulation allowing great computer saving in an optimization procedure. The design variables of the optimization procedure are binary design variables {0,1} stating the absence (0) or presence (1) of each finite element. So the obtained results are presented in black and white but when considering the element material density as design variables, the resulting topology can be presented in different colors according to the interval [0-1]. For the presented case, the CPU-time of the RBTO process generally (not always) needs more computing time than the DTO procedure in order to generate several topologies starting from the same input values and to control certain parameters (see Table 3). So the application of the RBTO model gives a different topology relative to the deterministic topology optimization.

4. Importance and Validation

4.1 Analytical demonstration: Truss modeling

In order to demonstrate the importance of the integration of reliability constraints into the classical topology optimization, we consider a cantilever beam (Fig. 6a) submitted to a single load F. The structure is loaded by a vertical force F. The mean value of this force is $m_F = 8$ KN, the safety factor is $S_C = 1.25$ and the allowable stress is $\sigma_w = 235$ MPa. We now calculate analytically the bar areas of the structures obtained by Deterministic Design Optimization and Reliability-Based Design Optimization, respectively. The optimization problem is to minimize the structural volume subject to mechanical stress constraints. The truss structures are illustrated in Fig. 6d,e. Fig. 6b shows the optimal topology when using the deterministic procedure and Fig. 6c presents the topology when integrating the reliability in the topology procedure (RBTO). The dimensions and the angles are: $L = 1000$ mm, $H = 875$ mm, $\alpha = 45°$ and $\beta = 30°$, respectively. For the deterministic design, the structural volume of the deterministic topology is $V_c = 132367$ mm^3, but when introducing the reliability, the structure volume will be $V = 114325$ mm^3. The weight reduction is thus: 13.6%. This example shows that the new topology reduces the structural weight by 13.6% for the same conditions. It means that the introduction of the reliability analysis during the topology optimization reduces the structural weight when using a shape optimization module (for deterministic design). Now if we consider that the force is the only random variable, according to the normal distribution law, we can evaluate the reliability of the structure by calculating the normalized variable in considering that the standard-deviation $\sigma_F = 0.1 \times m_F$, we then obtain $\beta = |u| = 2.5$. However, for the Reliability-Based Design Optimization model, when considering a simple case of one random variable (the applied force F) and the target reliability level $\beta_t = 3.0$, we have $u = 3.0$ and then $F = 10.4$ KN. For the same example, if we replace $F = 10$ KN by $F = 10.4$ KN and seeing that the relation of the stress $\sigma = N/A$ is linear, the structural volume of the deterministic topology procedure is $V_c^{RBDO} = 137662$ mm^3

Table 3. Input parameters and results of DTO & RBTO model for fatigue analysis.

Model	Input vector	Input parameters				Output results		
		nelx	nely	force	β_t	Objective	Iteration	CPU-Time
DTO	m	80	16	-4.10^{12}	—	0.40	74	01h37m47s
RBTO	y	98	20	$-4.7.10^{12}$	3.8	0.51	71	03h32m42s

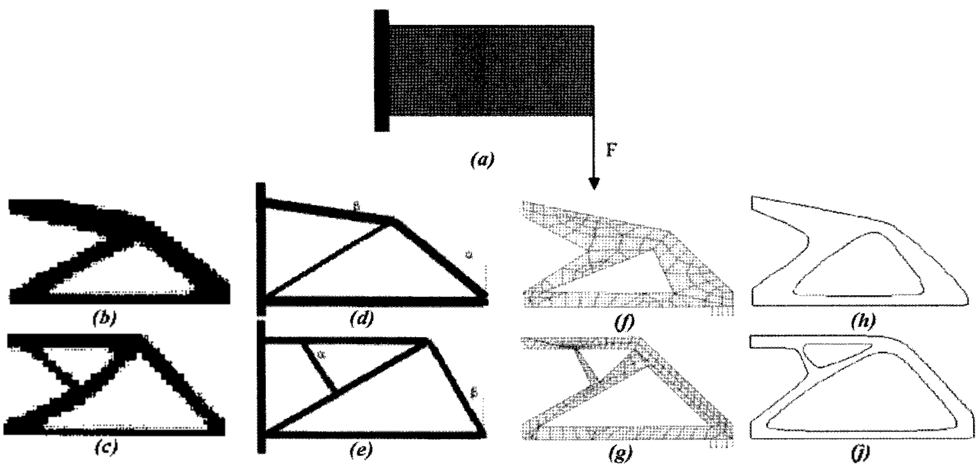


Fig. 6. Topology, truss and CAD/CAE modeling of a cantilever beam.

but when introducing the reliability the structure volume will be $V^{RBDO} = 118898\,mm^3$. The weight reduction is 13.6%, the same as of the deterministic one. This reduction demonstrates the importance of the reliability in the topology optimization. This importance can also be verified when considering shape and sizing optimization for deterministic structural optimization as well as for reliability-based structural optimization. The truss modeling for both resulting topologies for the three cases (static, modal and fatigue) shows that the reliability-based topology contains a number of bars more than the deterministic topology which generally leads to more reliable structure. Here, we modeled the structures by truss for same boundary conditions and same geometrical dimensions. The truss modeling of the static and fatigue cases shows that the RBTO truss modeling has is bigger number of bars relative to the DTO, except the DTO result for the modal case in Figure 3b that cannot be approximated to a truss model. Both resulting topologies for the three cases (static, modal and fatigue) show that the reliability-based topology contains a number of bars more than the deterministic topology which may generally leads to more reliable structure.

4.2 Numerical demonstration: CAD/CAE modeling

The RBTO model contains the principal successive processes: reliability index evaluation, and a topology optimization process. The cantilever beam illustrated in Fig. 6a, is subjected to a single external load. The objective is to show the difference between the resulting deterministic topology and the reliability-based one. In Fig. 6b and Fig. 6c show the resulting deterministic topology and the reliability-based one with reliability index $\beta_t = 3.8$. Now we apply a shape optimization algorithm to the meshed models for both cases, illustrated in Fig. 6f and Fig. 6g. The shape optimization problem is to minimize the structural volume subject to mechanical stress, displacement constraints and parameter limitations. The structure is loaded by a vertical force $F = 3$ kN as indicated in Fig. 6a. The safety factor is $S_f = 1.5$, the allowable stress is $\sigma_w = 90$ Mpa and the allowable displacement is $d_w = 1$ mm. The beam length and height are: $L = 1000$ mm and $H = 700$ mm, respectively. For the resulting deterministic topology, the structural volume of the optimal configuration, illustrated in Fig. 6h, is $Vc = 268938\,mm^3$. However, when introducing the reliability, the structural volume of the optimal configuration, illustrated in Fig. 6j, is only $V = 216747\,mm^3$. This example shows that the new topology reduces the structural weight by 19.4% for the same conditions. It means that the introduction of the reliability analysis during the topology optimization reduces the structural weight when using a shape optimization module (for deterministic design optimization). This importance can also be verified when considering reliability-based design optimization. The interested reader can see different topologies for different reliability levels ($\beta \in [1-6]$) for only static cases in Kharmanda et al. 2004. This strategy allows us to generate different topologies because the resulting optimal topologies principally depend on the reliability index value.

5. Conclusion

The proposed RBTO model aims to consider randomness (variability) of the most important quantities of a structure such as the geometry and the applied loads. This model can provide designers with different topologies. Another advantage is the reduction of structural weight for the same conditions. This weight reduction will manifest itself in deterministic design optimization as well as in reliability-based design optimization.

References

[1] Agarwal. H. (2004), *Reliability Based Design Optimization: Formulations and Methodologies*. PhD thesis, University of Notre Dame.

[2] Bae K. and Wang S. (2002), Reliability-based topology optimization. *Proceedings of 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization* 2002, AIAA 2002-5542.

[3] Beckers, M. (1999), Topology optimization using a dual method with discrete variables. *Structural Optimization*. **17**, 14-24.

[4] Bendsøe, M. P. and Kikuchi, N. (1988), Generating optimal topologies in optimal design using a homogenization method. *Comp. Meth. Appl. Mech. Engrg*. **71**, 197-224.

[5] Bendsøe, M. P. (1995), *Optimization of structural topology, shape and material*. Berlin, Heidelberg, New York: Springer.

[6] Crossland, B. (1956), Effects of large hydrostatic pressures on the torsional fatigue strength of an alloy steel, International conference on fatigue of metals, London.

[7] Davenport, A. G. (1964), Note on the distribution of the largest value of a random function with application to gust loading, *Proc. Inst. Civ. Eng*., **28**, pp.187-196.

[8] Ditlevsen, O. and Madsen, H. (1996), *Structural reliability methods*, In: John Wiley & Sons (eds).

[9] Eschenauer, H. A. and Olhoff, N. (2001), Topology optimization of continuum structures: A review, American Society of Mechanical Engineers, *Appl. Mech. Rev*. **54**(4), 331-390.

[10] Frangopol, D. M. (1995), Reliability-based optimum structural design. In: Sundarajan, C. (ed.) *Probabilistic structural mechanics handbook*, pp. 352-387. New York: Chapman & Hall.

[11] Jung H-S. and Cho S. (2004), Reliability-based topology optimization of geometrically nonlinear structures with loading and material uncertainties, *Finite Elements in Analysis and Design* **43**, 311-331.

[12] Haldar A. and Mahadevan S. (2000), *Probability, reliability and statistical methods in engineering design*. Wiley, New York.

[13] Hasofer, A. M. and Lind, N. C. (1974), An exact and invariant first order reliability format. *J. Eng. Mech*., ASCE, EM1 **100**, 111-121.

[14] Jung H-S., Cho S. and Yang Y-S. (2003), Reliability-based Robust Topology Design Optimization of Nonlinear Structures, *WCSMO5*, Italy.

[15] Kharmanda, G., Mohamed, A. and Lemaire, M. (2002), Efficient reliability-based design optimization using a hybrid space with application to finite element analysis, *Structural and Multidisciplinary Optimization*, **24**, pp 233-245.

[16] Kharmanda, G. and Olhoff, N. (2001), *Reliability-Based*

Topology Optimization, Report N°: 110, Institute of Mechanical Engineering, Aalborg University, Denmark, December 2001.

[17] Kharmanda, G. and Olhoff, N. (2002), Reliability-Based Topology Optimization as a New Strategy to Generate Different Topologies, In: *the Nordic Seminar in Computational Methods*, (eds. E. Lund, N. Olhoff and J. Stegmsen), Aalborg University, Denmark, October, 18-19 2002.

[18] Kharmnda, G, Olhoff, N., Mohamed, A. and Lemaire, M. (2004), Reliability-based topology optimization, *Structural and Multidisciplinary Optimization*, vol 26.

[19] Kharmanda, G, Olhoff, N. and El-Hami, A. (2004), Optimum values of structural safety factors for a predefined reliability level with extension to multiple limit states, *Structural and Multidisciplinary Optimization*, **27**, pp 421-434.

[20] Kharmanda, G. and Olhoff, N. (2007), An extension of optimum safety factor approach to nonlinear reliability-based design optimization, *Structural and Multidisciplinary Optimization*, **34**, 367-380.

[21] Mozumder C. K. and Renaud J. E. (2006), An Investigation of reliability-based topology optimization *20th Aerospace and Mechanical Engineering Graduate Student Conference*, Notre Dame, Indiana, USA, 19 October 2006

[22] Olhoff, N. (2000), Comparative study of optimizing the topology of plate-like structures via plate theory and 3-D theory of elasticity, In: Rozvany, G.I.N. and Olhoff, N. (eds), *Topology optimization of structures and composite continua*, 37-48.

[23] Patel, N. M., Agarwal, H., Tovar, A. and Renaud, J. (2005), Reliability based topology optimization using the hybrid cellular automaton method. In *1st AIAA Multidisciplinary Design Optimization Specialist Conference*, Austin, Texas, April 18-21, 2005.

[24] Pitoiset, X. and Preumont, A., (2000), Spectral methods for multiaxial random fatigue analysis of metallic structures, *International Journal of Fatigue*, **22**, pp. 541-550

[25] Tovar, A. Patel, N. M., Letona G. A., Kaushik, A. K. and Renaud J. E. (2004), Hybrid cellular automata: A biologically-inspired structural optimization technique. In *Proceedings of the 10th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, number AIAA 2004-4558, Albany, New York, Aug 30 - Sep. 1 2004

[26] Tu, J., Choi, K. K. and Park, Y. H. (1999), A new study on reliability-based design optimization, *Journal of Mechanical Design*, ASME, **121**(4), 557-564.

[27] Segalman, D., Reese, G., Field, R. and Fulcher, C., (2000) Estimating the probability distribution of Von Mises stress for structures undergoing random excitation, *ASME Journal of Vibrations and Acoustics*, **122**, pp. 42-48

[28] Sigmund, O. (2001) A 99 line topology optimization code written in Matlab, *Structural and Multidisciplinary Optimization*, **21**, pp 120-127