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Intuitionistic fuzzy ideals in Regular duo ring

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Abstract

The aim of this paper is to characterise regular duo ring R by using the concept of intuitionistic fuzzy left(right,bi,quasi) ideals of R.

Key words : Regular duo ring, Intrinsic product,Intuitionistic fuzzy left(right,bi,quasi) ideals

1. Introduction

The theory of fuzzy set was initiateed by Zadeh[14] and so many researchers were conducted on the generalizations of the notion of fuzzy sets. The idea of *intuitionistic fuzzy set* was first published by Atanassov [2,3] as a generalization of the notion of fuzzy sets. In [5], Banerjee and Basnet applied the concept of intuitionistic fuzzy sets to the theory of rings, and introduced the notions of intuitionistic fuzzy subrings and intuitionistic fuzzy ideals of a ring. In [7], Hur et al. introduced the notions of intuitionistic fuzzy (completely) prime ideals and intuitionistic fuzzy weak completely prime ideals in a ring. Present aurhor [11] introduced the notions of intuitionistic product of intuitionistic fuzzy ideals ,and characterizations of regular rings are proved. In this paper we introduc the intrinsic product of intuitionistic fuzzy sets and intuitionistic fuzzy left(right,bi,quasi) ideals in a ring. The aim of this paper is to characterise regular duo ring R by using the concept of intuitionistic fuzzy left(right,bi,quasi) ideals of R.

2. Preliminaries

In this section, we introduce some definitions and lemmas which will be used in this paper. For more details we refer to [5],[10],[11] and [12]. Let R be a ring. Let A and B be subsets of R. Then the multiplication of A and B is defined as follows:

$$AB = \Big\{ \sum_{\text{finite}} a_i b_i \mid a_i \in A,\, b_i \in B \Big\}$$

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An additive subgroup Q of a ring R is called a *quasi-ideal* of R if $QR \cap RQ \subseteq Q$, and an additive subgroup B of a ring R is called a *bi-ideal* of R if $BB \subseteq B$ and $BRB \subseteq B$.

As an important generalization of the notion of fuzzy sets in M, Atanassov [2,3] introduced the concept of an *intuitionistic fuzzy set* defined on a non-empty set M as objects having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in M\},$$

where the functions $\mu_A : M \to [0,1]$ and $\gamma_A : M \to [0,1]$ denote the *degree of membership* (namely $\mu_A(x)$) and the *degree of nonmembership* (namely $\gamma_A(x)$) of each element $x \in M$ to A respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for all $x \in M$.

Such defined objects are studied by many authors (see for example two journals: 1. *Fuzzy Sets and Systems* and 2. *Notes on Intuitionistic Fuzzy Sets*) and have many interesting applications not only in mathematics (see Chapter 5 in the book [4].

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \gamma_A)$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in M\}$.

Definition 2.1. ([2])Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets in a set M.

(1). $A \subseteq B \Leftrightarrow (\forall x \in M)\,(\mu_A(x) \le \mu_B(x),\ \gamma_A(x) \ge \gamma_B(x))$.

(2). $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.

(3). $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.

(4). $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.

(5). $0_\sim = (0,1)$ and $1_\sim = (1,0)$.

Definition 2.2. ([5,10]) An intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in a ring R is called an *intuitionistic fuzzy subring* of R if it satisfies the following conditions:

(1). $(\forall x, y \in R)\ (\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\})$.

(2). $(\forall x, y \in R)\ (\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\})$.

(3). $(\forall x, y \in R)\ (\gamma_A(x-y) \leq \max\{\gamma_A(x), \gamma_A(y)\})$.

(4). $(\forall x, y \in R)\ (\gamma_A(xy) \leq \max\{\gamma_A(x), \gamma_A(y)\})$.

Definition 2.3. ([5,10]) An intuitionistic fuzzy set set $A = (\mu_A, \gamma_A)$ in a ring R is called an *intuitionistic fuzzy left* (resp. *right*) *ideal* of R if it satisfies the following conditions:

(1). $(\forall x, y \in R)\ (\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\})$.

(2). $(\forall x, y \in R)\ (\gamma_A(x-y) \leq \max\{\gamma_A(x), \gamma_A(y)\})$.

(3). $(\forall a, x \in R)\ \mu_A(ax) \geq \mu_A(x)(\gamma_A(xa) \leq \gamma_A(x))$

If $A = (\mu_A, \gamma_A)$ is both an intuitionistic fuzzy left and intuitionistic fuzzy right ideal of a ring R, then $A = (\mu_A, \gamma_A)$ is called an *intuitionistic fuzzy ideal* of R.

Definition 2.4. ([11]) Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets in a ring R. The *intrinsic product* of $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ is defined to be the intuitionistic fuzzy set $A * B = (\mu_{A*B}, \gamma_{A*B})$ in R given by

$$\mu_{A*B}(x) := \bigvee_{\substack{x=\sum a_i b_i \\ \text{finite}}} \min\left\{\begin{array}{l}\mu_A(a_1), \mu_A(a_2), \cdots, \mu_A(a_m), \\ \mu_B(b_1), \mu_B(b_2), \cdots, \mu_B(b_m)\end{array}\right\}$$

$$\gamma_{A*B}(x) := \bigwedge_{\substack{x=\sum a_i b_i \\ \text{finite}}} \max\left\{\begin{array}{l}\gamma_A(a_1), \gamma_A(a_2), \cdots, \gamma_A(a_m), \\ \gamma_B(b_1), \gamma_B(b_2), \cdots, \gamma_B(b_m)\end{array}\right\}$$

if we can express $x = a_1b_1 + a_2b_2 + \cdots + a_mb_m$ for some $a_i, b_i \in R$ and for some positive integer m where each $a_ib_i \neq 0$. Otherwise, we define $A * B = 0_\sim$, i.e., $\mu_{A*B}(x) = 0$ and $\gamma_{A*B}(x) = 1$.

Definition 2.5. ([11]) An intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in a ring R is called an *intuitionistic fuzzy quasi-ideal* of R if

(1). $(\forall x, y \in R)\ (\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\})$,

(2). $(\forall x, y \in R)\ (\gamma_A(x-y) \leq \max\{\gamma_A(x), \gamma_A(y)\})$,

(3). $(A * 1_\sim) \cap (1_\sim * A) \subseteq A$.

Definition 2.6. ([11]) An intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in a ring R is called an *intuitionistic fuzzy bi-ideal* of R if

(1). $(\forall x, y \in R)\ (\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\})$,

(2). $(\forall x, y \in R)\ (\gamma_A(x-y) \leq \max\{\gamma_A(x), \gamma_A(y)\})$,

(3). $A * A \subseteq A$ and $A * 1_\sim * A \subseteq A$.

Lemma 2.7. ([11]) For an intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in a ring R, the following assertions are equivalent:

(1). $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy subring of R.

(2). $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy subgroup of the additive group $(R, +)$ and $A * A \subseteq A$.

Lemma 2.8. ([11]) An intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in a ring R is an intuitionistic fuzzy left (resp. left) ideal of a ring R if and only if

(1). $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\gamma_A(x-y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$,

(2). $1_\sim * A \subseteq A$ (resp. $A * 1_\sim \subseteq A$).

Lemma 2.9. ([11]) Every intuitionistic fuzzy left (resp. right, two-sided) ideal of a ring R is an intuitionistic fuzzy quasi-ideal of R.

Lemma 2.10. ([11]) Any intuitionistic fuzzy quasi-ideal of a ring R is an intuitionistic fuzzy bi-ideal of R.

For a subset X of a ring R,we denote $\widetilde{X} = \{\langle x, \mu_{\widetilde{X}}(x), \gamma_{\widetilde{X}}(x)\rangle \mid x \in R\}$ defined by

$$\mu_{\widetilde{X}}(x) := \begin{cases} 1 & \text{if } x \in X \\ 0 & \text{otherwise} \end{cases}$$

and

$$\gamma_{\widetilde{X}}(x) := \begin{cases} 0 & \text{if } x \in X \\ 1 & \text{otherwise} \end{cases}$$

for all $x \in R$. For the sake of simplicity, we shall use the symbol $\widetilde{X} = (\mu_{\widetilde{X}}, \gamma_{\widetilde{X}})$ for the $\widetilde{X} = \{\langle x, \mu_{\widetilde{X}}(x), \gamma_{\widetilde{X}}(x)\rangle \mid x \in X\}$.

Lemma 2.11. ([11]) Let A and B be any subsets of a ring R. Then we have

(1). $\widetilde{A} * \widetilde{B} = \widetilde{AB}$.

(2). $\widetilde{A} \cap \widetilde{B} = \widetilde{A \cap B}$.

Lemma 2.12. ([11]) Let A be a nonempty subset of R, Then the following holds assertions.

(1). A is a subring of a ring R if and only if $\widetilde{A}$ is an intuitionistic fuzzy subring of R.

(2). A is a left(right) ideal of R if and only if $\widetilde{A}$ is an intuitionistic fuzzy left(right) ideal of R.

(3). A is a quasi ideal of R if and only if $\widetilde{A}$ is an intuitionistic fuzzy quasi ideal of R.

3. Regular duo ring

Recall that a ring R is said to be *duo* if every one-sided ideal of R is a two sided ideal. A ring R is called fuzzy *intuitionistic duo* if every one-sided intuitionistic fuzzy ideal of R is a two-sided intuitionistic fuzzy ideal. Recall that a ring R is said to be *regular ring* if every each element a of R ,there exist elements x of R such that $a = axa$

Lemma 3.1. ([13]) For a ring R, the following assertions are equivalent:

(1). R is a regular duo ring.

(2). $A \cap B = AB$ for every left ideal A and every right ideal B of R.

(3). $Q^2 = Q$ for every quasi-ideal of R.

(4). $EQE = E \cap Q$ for every two-sided ideal E and every quasi-ideal Q of R.

Proposition 3.2. Let R be a regular ring R. Then R is duo if and only if R is intuitionistic fuzzy duo.

Proof. Assume that R is a duo ring. Let $A = (\mu_A, \gamma_A)$ be any intuitionistic fuzzy left ideal of R, and a and b any elements of R. Then we have Ra is left ideal of R. By the assumption, Ra is a two-sided ideal of R. Since R is regular we have $a \in aRa$. It follows that

$$ab \in (aRa)b = \{(aR)a\}R \subseteq (Ra)R \subseteq Ra.$$

Thus there exists an $x \in R$ such that $ab = xa$. From this and A is a intuitionistic fuzzy left ideal of R, we have $\mu_A(ab) = \mu_A(xa) \geq \mu_A(a)$ and $\gamma_A(ab) = \gamma_A(xa) \leq \gamma_A(a)$. Hence A is a intuitionistic fuzzy right ideal of R. It can be seen in a similar way that any intuitionistic fuzzy right ideal of R. Therefore R is intuitionistic fuzzy duo ring. Conversely, suppose R is intuitionistic fuzzy duo ring. Let A be any right ideal of R. By Lemma 2.12 $\tilde{A}$ is a intuitionistic fuzzy fuzzy right ideal of R,and so $\tilde{A}$ is a intuitionistic fuzzy two-sided ideal of R by the assumption. Then it follws from Lemma 2.12 that A is a two sided ideal of R. Similarly, we can see that any left ideal of R is two-sided. Therefore R is duo ring. □

Lemma 3.3. Let R be a ring and let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy bi-ideal of a ring R. Then we have $\mu_A(axa) \geq \min\{\mu_A(a), \mu_A(b)\}$ and $\gamma_A(axa) \leq \max\{\gamma_A(a), \gamma_A(b)\}$ for all a, b and x of R.

Proof. Let a, b, and x be any element of R. Then since $A \supseteq A * 1_{\sim} * A$, we have

$$\begin{aligned}
&\mu_A(axb) \\
&\geq \mu_{A*1_\sim *A}(axb) \\
&= \bigvee_{axb=\sum_{\text{finite}} a_ib_i} \min\left\{ \begin{array}{l} \mu_A(a_1), \cdots, \mu_A(a_m), \\ \mu_{1_\sim *A}(b_1), \cdots, \mu_{1_\sim *A}(b_m) \end{array} \right\} \\
&\geq \min\{\mu_A(a), \mu_{1_\sim *A}(xb)\} \\
&= \min\{\mu_A(a), \bigvee_{xb=\sum_{\text{finite}} p_iq_i} \min\{1(p_1), \cdots, 1(p_m), \\
&\qquad\qquad \mu_A(q_1), \cdots, \mu_A(q_m)\}\} \\
&\geq \min\{\mu_A(a), \min\{1(x), \mu_A(b)\}\} \\
&= \min\{\mu_A(a), \min\{1, \mu_A(b)\}\} \\
&= \min\{\mu_A(a), \mu_A(b)\}\}
\end{aligned}$$

and

$$\begin{aligned}
&\gamma_A(axb) \\
&\leq \gamma_{A*1_\sim *A}(axb) \\
&= \bigwedge_{axb=\sum_{\text{finite}} a_ib_i} \max\left\{ \begin{array}{l} \gamma_A(a_1), \cdots, \gamma_A(a_m), \\ \gamma_{1_\sim *A}(b_1), \cdots, \gamma_{1_\sim *A}(b_m) \end{array} \right\} \\
&\leq \max\{\gamma_A(a), \gamma_{1_\sim *A}(xb)\} \\
&= \max\{\gamma_A(a), \bigwedge_{xb=\sum_{\text{finite}} p_iq_i} \max\{0(p_1), \cdots, 0(p_m), \\
&\qquad\qquad \gamma_A(q_1), \cdots, \gamma_A(q_m)\}\} \\
&\leq \max\{\gamma_A(a), \max\{0(x), \gamma_A(b)\}\} \\
&= \max\{\gamma_A(a), \max\{0, \gamma_A(b)\}\} \\
&= \max\{\gamma_A(a), \gamma_A(b)\}\}.
\end{aligned}$$

□

Proposition 3.4. Any intuitionistic fuzzy fuzzy bi-ideal of a regular intuitionistic fuzzy duo ring R is a intuitionistic fuzzy fuzzy two-sided ideal of R.

Proof. Let $A = (\mu_A, \gamma_A)$ be any intuitionistic fuzzy fuzzy bi-ideal of R, and a and b any elements of R. Then Ra is a left ideal of R. Then,since R is duo by Lemma 3.2, Ra is a right ideal of R. since R is regular we have $a \in aRa$. Hence

$$ab \in (aRa)b \subseteq a\{(Ra)R\} \subseteq aRa.$$

This implies that there exists an element x in R such that $ab = axa$. Then,since A is a intuitionistic fuzzy bi-ideal of R, by Lemma 3.3, we have $\mu_A(ab) = \mu_A(axa) \geq \min\{\mu_A(a), \mu_A(a)\} = \mu_A(a)$, and $\gamma_A(ab) = \gamma_A(axa) \leq \max\{\gamma_A(a), \gamma_A(a)\} = \gamma_A(a)$, and so $A = (\mu_A, \gamma_A)$ is a intuitionistic fuzzy fuzzy right ideal of R. It can be seen in a similar way that A is a intuitionistic fuzzy fuzzy left ideal of R. Therefore A is a intuitionistic fuzzy fuzzy two-sided ideal of R. This completes the proof. □

Lemma 3.5. ([11]) A ring R is regular if and only if $A * B = A \cap B$ for every intuitionistic fuzzy right ideal $A = (\mu_A, \gamma_A)$ of R and every intuitionistic fuzzy left ideal $B = (\mu_B, \gamma_B)$ of R.

Proposition 3.6. For a ring R, the following assertions are equivalent:

(1). R is a regular duo ring.

(2). R is a regular intuitionistic fuzzy duo ring.

(3). $A * B = A \cap B$ for all intuitionistic fuzzy bi-ideals A and B of R.

(4). $A * B = A \cap B$ for every intuitionistic fuzzy bi-ideal A and every intuitionistic fuzzy quasi-ideal of R.

(5). $A * B = A \cap B$ for every intuitionistic fuzzy bi-ideal A and every intuitionistic fuzzy right-ideal of R.

(6). $A * B = A \cap B$ for every intuitionistic fuzzy quasi-ideal A and every intuitionistic fuzzy right-ideal of R.

(7). $A * B = A \cap B$ for every intuitionistic fuzzy left-ideal A and every intuitionistic fuzzy right-ideal of R.

Proof. (1) $\Rightarrow$ (2). From Lemma 3.2 ,it follows that (1) and (2) are equivalent.

(2) $\Rightarrow$ (3). Assume that R is a regular intuitionistic fuzzy duo ring. Let A and B be any intuitionistic fuzzy bi-ideal of R. Then it follows from Proposition 3.4 that A is a intuitionistic fuzzy right ideal of R and B is a intuitionistic fuzzy left ideal of R. Since R is regular, it follows from Lemma 3.5 that $A * B = A \cap B$.

(3) $\Rightarrow$ (4) $\Rightarrow$ (5) $\Rightarrow$ (6) $\Rightarrow$ (7). Straightforward.

(7) $\Rightarrow$ (1). Assume that (7) holds. Let A and B be any left ideal and any right ideal of R, respectively. Then it follows from Lemma2.12 that $\widetilde{A} = (\mu_{\widetilde{A}}, \gamma_{\widetilde{A}})$ and $\widetilde{B} = (\mu_{\widetilde{B}}, \gamma_{\widetilde{B}})$ are a intuitionistic fuzzy left ideal and a intuitionistic fuzzy right ideal of R. Let $a \in A \cap B$. Then, we have

$$\begin{aligned} \mu_{\widetilde{AB}}(a) &= \mu_{\widetilde{A}*\widetilde{B}}(a) \\ &= (\mu_{\widetilde{A}} \wedge \mu_{\widetilde{B}})(a) \\ &= \min\{\mu_{\widetilde{A}}(a), \mu_{\widetilde{B}}(a)\} \\ &= \min\{1, 1\} \\ &= 1 \end{aligned}$$

and

$$\begin{aligned} \mu_{\widetilde{AB}}(a) &= \gamma_{\widetilde{A}*\widetilde{B}}(a) \\ &= (\gamma_{\widetilde{A}} \vee \gamma_{\widetilde{B}})(a) \\ &= \max\{\gamma_{\widetilde{A}}(a), \gamma_{\widetilde{B}}(a)\} \\ &= \max\{0, 0\} \\ &= 0 \end{aligned}$$

and so $a \in AB$. Thus $A \cap B \subseteq AB$.

In order to see that the converse inclusion hold, le a be any element of AB. Thus

$$\begin{aligned} \min\{\mu_{\widetilde{A}}(a), \mu_{\widetilde{B}}(a)\} &= (\mu_{\widetilde{A}} \wedge \mu_{\widetilde{B}})(a) \\ &= (\mu_{\widetilde{A}} * \mu_{\widetilde{B}})(a) \\ &= \mu_{\widetilde{AB}}(a) \\ &= 1 \end{aligned}$$

and

$$\begin{aligned} \max\{\gamma_{\widetilde{A}}(a), \gamma_{\widetilde{B}}(a)\} &= (\gamma_{\widetilde{A}} \vee \gamma_{\widetilde{B}})(a) \\ &= (\gamma_{\widetilde{A}} * \gamma_{\widetilde{B}})(a) \\ &= \gamma_{\widetilde{AB}}(a) \\ &= 0 \end{aligned}$$

This implies that $a \in A$ and $a \in B$. Thus we have $a \in A \cap B$, and $AB \subseteq A \cap B$. Therefore, we have $AB = A \cap B$. Then it follows form Lemma3.1 that R is regular duo ring. □

Proposition 3.7. For a ring R, the following assertions are equivalent:

(1). R is a regular duo ring.

(2). R is a regular intuitionistic fuzzy duo ring.

(3). $A * B = A \cap B$ for all intuitionistic fuzzy quasi-ideals A and B of R.

(4). Every intuitionistic fuzzy quasi-ideal of R is idempotent.

Proof. $(1) \Rightarrow (2)$. From Proposition 3.2 ,it follows that (1) and (2) are equivalent.
$(2) \Rightarrow (3) \Rightarrow (4)$. Straightforward.
$(4) \Rightarrow (1)$. Let Q be any quasi-ideal of R, and a any element of Q. By Lemma 2.12 , $\widetilde{Q}$ is an intuitionistic fuzzy quasi-ideal of R. Then we have $\mu_{\widetilde{Q^2}}(a) = \mu_{\widetilde{Q}*\widetilde{Q}}(a) = \mu_{\widetilde{Q}}(a) = 1$ and $\gamma_{\widetilde{Q^2}}(a) = \gamma_{\widetilde{Q}*\widetilde{Q}}(a) = \gamma_{\widetilde{Q}}(a) = 0$ and so $a \in Q^2$, that is, $Q \subseteq Q^2$. Since the reverse inclusion always holds, we obtain $Q^2 = Q$. It follows from Lemma 3.1 , R is a regular duo ring. □

Recall that a ring R is said to be *intra-regular ring* if every each element of R ,there exist elements x_i and y_i of R such that $a = \sum_{i=1}^{n} x_i a^2 y_i$.

Lemma 3.8. ([11]) For a ring R, the following conditions are equivalent:

(1). R is both regular and intra-regular ring.

(2). $A * A = A$ for every intuitionistic fuzzy bi-ideal A of R.

(3). $A * A = A$ for every intuitionistic fuzzy quasi-ideal A of R.

Corollary 3.9. For a regular ring R, the following conditions are equivalent:

(1). R is a intra-regular ring.

(2). R is a duo ring.

Proof. It follows from Lemma 3.8 and Propostion 3.6 □

Proposition 3.10. For a ring R, the following conditions are equivalent:

(1). R is regular duo ring

(2). $A*B*A = B \cap A$ for every intuitionistic fuzzy two-sided ideal A and every intuitionistic fuzzy bi-ideal B of R.

(3). $A*B*A = B \cap A$ for every intuitionistic fuzzy two-sided ideal A and every intuitionistic fuzzy quasi-ideal B of R

Proof. $(1) \Rightarrow (2)$. Assume that (1) holds. Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be any intuitionistic fuzzy two-sided ideal and any intuitionistic fuzzy bi-ideal of R, respectively. Then we have

$$A * B * A \subseteq (A * 1_\sim) * 1_\sim \subseteq A * 1_\sim \subseteq A$$

and so $A * B * A \subseteq B \cap A$

Let a be any element of R. Since R is regular, there exists an element x in R such that $a = axa (= axaxa)$. Then we have

$$\begin{aligned}
&\mu_{A*B*A}(a) \\
&= \bigvee_{a=\sum_{\text{finite}} a_i b_i} \min\left\{ \begin{array}{l} \mu_A(a_1), \cdots, \mu_A(a_m), \\ \mu_{B*A}(b_1), \cdots, \mu_{B*A}(b_m) \end{array} \right\} \\
&\geq \min\{\mu_A(ax), \mu_{B*A}(axa)\} \\
&= \min\{\mu_A(ax), \bigvee_{axa=\sum_{\text{finite}} p_i q_i} \min\{\mu_B(p_1), \cdots, \\
&\qquad \mu_B(p_m), \mu_A(q_1), \cdots, \mu_A(q_m)\}\} \\
&\geq \min\{\mu_A(ax), \min\{\mu_B(a), \mu_A(xa)\}\} \\
&= \min\{\mu_A(a), \min\{\mu_B(a), \mu_A(a)\}\} \\
&= \min\{\mu_A(a), \mu_B(a)\}\} \\
&= (\mu_A) \wedge \mu_B)(a)
\end{aligned}$$

and

$$\begin{aligned}
&\gamma_{A*B*A}(a) \\
&= \bigwedge_{a=\sum_{\text{finite}} a_i b_i} \max\left\{ \begin{array}{l} \gamma_A(a_1), \cdots, \gamma_A(a_m), \\ \gamma_{B*A}(b_1), \cdots, \gamma_{B*A}(b_m) \end{array} \right\} \\
&\geq \max\{\gamma_A(ax), \gamma_{B*A}(axa)\} \\
&= \max\{\gamma_A(ax), \bigvee_{axa=\sum_{\text{finite}} p_i q_i} \max\{\gamma_B(p_1), \cdots, \\
&\qquad \gamma_B(p_m), \gamma_A(q_1), \cdots, \gamma_A(q_m)\}\} \\
&\geq \max\{\gamma_A(ax), \max\{\gamma_B(a), \gamma_A(xa)\}\} \\
&= \max\{\gamma_A(a), \max\{\gamma_B(a), \gamma_A(a)\}\} \\
&= \max\{\gamma_A(a), \gamma_B(a)\}\} \\
&= (\gamma_A \wedge \gamma_B)(a)
\end{aligned}$$

and so we have $A \cap B \subseteq A * B * A$. Thus we obtain that $A * B * A = A \cap B$.

$(2) \Rightarrow (3)$. Straightforward.

$(3) \Rightarrow (1)$. Let U and V be any twosided ideal and quasi-ideal of R. By Lemma 2.12, $\widetilde{U}$ is an intuitionistic fuzzy two-sided ideal and $\widetilde{V}$ is an intuitionistic fuzzy quasi ideal of R. Let a be any element of $U \cap V$.

$$\begin{aligned}
&\mu_{\widetilde{UVU}}(a) \\
&= \mu_{\widetilde{U}*\widetilde{V}*\widetilde{U}}(a) \\
&= (\mu_{\widetilde{U}} \wedge \mu_{\widetilde{V}})(a) \\
&= \min\{\mu_{\widetilde{U}}(a), \mu_{\widetilde{V}}(a)\} \\
&= \min\{1, 1\} \\
&= 1
\end{aligned}$$

and

$$\begin{aligned}&\gamma_{\widetilde{UVU}}(a)\\&=\gamma_{\widetilde{U}*\widetilde{V}*\widetilde{U}}(a)\\&=(\gamma_{\widetilde{U}}\vee\gamma_{\widetilde{V}})(a)\\&=\max\{\mu_{\widetilde{U}}(a),\mu_{\widetilde{V}}(a)\}\\&=\max\{0,0\}\\&=0\end{aligned}$$

and so $a\in UVU$. Thus $U\cap V\subseteq UVU$. To see that the converse inclusion holds, let a be any element of UVU. Then we have

$$\begin{aligned}&\mu_{\widetilde{U\wedge V}}(a)\\&=(\mu_{\widetilde{A}}\wedge\mu_{\widetilde{B}})(a)\\&=\mu_{\widetilde{U}*\widetilde{V}*\widetilde{U}}(a)\\&=(\mu_{\widetilde{U}}\wedge\mu_{\widetilde{V}})(a)\\&=\mu_{\widetilde{UVU}}(a)\}\\&=1\end{aligned}$$

$$\begin{aligned}&\gamma_{\widetilde{U\vee V}}(a)\\&=(\gamma_{\widetilde{A}}\vee\gamma_{\widetilde{B}})(a)\\&=\gamma_{\widetilde{U}*\widetilde{V}*\widetilde{U}}(a)\\&=(\gamma_{\widetilde{U}}\vee\gamma_{\widetilde{V}})(a)\\&=\gamma_{\widetilde{UVU}}(a)\}\\&=0\end{aligned}$$

and so $a\in U\cap V$. Thus we obtain that $UVU\subseteq U\cap V$, and so $UVU=U\cap V$. Then it follows from Lemma 3.1 that R is regular duo ring.

□

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