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Optimal Disposal Policy in a Hybrid Production System with Manufacturing and Remanufacturing

Eun Gab Kim†

Associate Professor, College of Business Administration, Ewha Womans University, Seoul 120-750, Korea

신제품 생산과 회수제품 재가공이 이루어지는 생산시스템에서 최적 처분 정책에 대한 연구

김 은 갑

이화여자대학교 경영대학

We address a disposal issue of returned products in a product recovery system where a single product is stocked in order to meet a demand from customers who may return products after usage. Product returns occur randomly and can be accepted for remanufacturing or disposed of depending on the state of the system. We examine the structure of the optimal disposal policy for returned product that utilizes the information of the inventory of both serviceable and remanufacturable products. Numerical study indicates that it can be characterized by a monotonic threshold type of the curve. A disposal is allowed only when the remanufacturable inventory level exceeds a threshold which is the function of the inventory level of serviceable product and it is decreasing as the serviceable inventory level increases. Sensitivity analysis also indicates that the optimal disposal policy and the optimal profit have monotonic properties with respect to system parameters.

Keywords: Reverse Logistics, Disposal, Remanufacturing, Product recovery, Inventory Management

1. Introduction

There has been considerable interest in inventory control for joint manufacturing and remanufacturing systems, since remanufacturing is less costly than producing originally new product, recovery of used products may prove beneficial due to savings in material and manufacturing costs. It differs from traditional inventory management situations in essentially two aspects. First, product returns represent an exogenous inbound material flow causing a loss of monotonicity of stock levels of serviceable products which serve customer demands. Second, two alternative supply sources are available for replenishing the serviceable inventory. One source is to procure externally or produce internally while the other comes from remanufacturing activity. While both aspects as such are not new to inventory theory it is their combination that leads to novel situations.

In this paper, we address a problem of product recovery management where a single product is stocked in order to meet a demand from customers who may return products after usage. Demands occur randomly and each demand satisfied from on-hand inventory of serviceable product generates a revenue. If on-hand serviceable inventory is not

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† Corresponding author : Eun Gab Kim, College of Business Administration, Ewha Womans University 11-1 Daehyeon-dong, Seodaemun-gu, Seoul 120-750, Korea, Tel : +82-3277-3970, Fax : +82-3277-2835, E-mail : evanston@ewha.ac.kr
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available, arriving demands are lost. Product returns occur randomly and each of returned product can be accepted for remanufacturing or disposed of depending on the state of the system. It makes sense to consider the possibility to dispose of returned product into the model, since an increase in the recovery of used products may not result in higher profits for the firm. The reduction in the profits is due to increasing inventory costs of both manufacturable and remanufacturable products. If accepted returns undergo a remanufacturing process, they are assumed to be as good as new after recovering. Besides remanufacturing, new products are produced by a manufacturing facility on the continuous production basis. Serviceable products obtained from both manufacturing and remanufacturing processes are held in inventory to meet customer demands. The primary goal of this paper is to examine an optimal disposal policy for product return. To this end, we provide a simple model, discussed in the next section, to gain insights into the nature of these problem.

Detailed literature surveys for product recovery models are found in Fleischmann *et al*. (1997), Mahadevan *et al*. (2003), and Savaskan *et al*. (2004). Product recovery models with disposal options under periodic inventory review are found in Simpson (1978), Inderfurth (1997), Teunter and Vlachos (2002). Simpson (2004) considers the tradeoff between material savings due to reuse of old products versus additional inventory carrying costs and shows optimality of a three parameter policy to control replenishment order, recovery, and disposal when neither fixed costs nor lead times are considered. Inderfurth (1997) considers the effects of fixed and deterministic lead times for replenishment order and recovery without fixed costs. For identical lead times, he shows that the structure of the optimal policy is the same as Simpson (2004). Inderfurth (1997) also considers the case of recovery not allowing storage of recoverable products. For identical lead times, a two parameter order upto and dispose down to policy is shown to be optimal. Teunter and Vlachos (2002) consider a single item production system with manufacturing and remanufacturing and deal with the issue of what the cost reduction associated with having a disposal option for returned item is. Using simulation, they show that there is only a considerable cost reduction if an item is very slow moving, the recovery rate is high, and remanufacturing is almost as expensive as manufacturing.

A parallel stream of research has evolved for continuous inventory review models. Heyman (1997) studies disposal policies for a model where demands and returns are independent stochastic process, remanufacturing and procurement are instantaneous, and no fixed costs of remanufacturing and procurement are taken into account. When demands and returns follow Poisson process, he shows the optimality of single parameter disposal policy and derives an explicit expression for the optimal disposal level. Van der Lann *et al*. (1996a) present an explicit modeling of non-zero remanufacturing process with a disposal option under which the number of remanufacturable products is limited to a certain maximum level. Van der Lann *et al*. (1996b) present a numerical comparison of several disposal policies and show that it is advantageous to base disposal decisions on both the inventory level of remanufacturable products and an adequately defined total inventory.

Our model differs from previous product recovery models with a disposal option in the following aspect. First, we examine the optimal disposal policy which reflects both probabilistic lead times for manufacturing and remanufacturing and unit costs of manufacturing and remanufacturing. Second, our disposal policy is based upon future demand and product return processes and serviceable and remanufacturable inventory processes. In contrast, previous research papers present static single parameter disposal policies based on the remanufacturable inventory level only or the (possibly weighted) sum of both inventory levels. It is the most distinct feature that a disposal decision in this paper is dynamically made according to the function of both serviceable and remanufacturable inventory levels.

The paper is organized as follows. In the next section, we provide a formulation of our model. The structure of the optimal disposal policy is studied in section 3. Section 4 presents a numerical example which graphically illustrates the optimal policy. In section 5, we numerically implement a sensitivity analysis of the optimal policy with respect to system parameters. Finally we state our conclusions in the last section.

2. Problem Formulation

When a demand for product arises, it is satisfied immediately from on-hand inventory of serviceable products. If they are not available, the demand is lost. Demands for product occur randomly with rate λ_1 and the sales price of a product is R. The production time for a new product is a random variable with mean μ_1^{-1} and the unit cost of manufacturing a new product is c_M. Product returns occur randomly with rate λ_2 and each returned product can be disposed of or accepted for remanufacturing. The time required for remanufacturing a returned product into a new one is a random variable with mean μ_2^{-1} and the unit cost of remanufacturing is c_R. We further assume that the unit cost of remanufacturing is the same for all remanufactured products and the quality of originally new product and remanufactured one are the same. This assumption is usually made in most of product recovery models. Holding costs are assessed at rate h_1 and h_2 for each product in serviceable inventory and each returned product in remanufacturable inventory, respectively. Disposal of a returned product accrues a positive salvage value of S (the model can be converted to the case that a disposal costs).

Let $x_1(t)$ and $x_2(t)$ respectively represent the number of products in serviceable inventory and the number of returned products in remanufacturable inventory at time t. Then, a state is described by the vector $(x_1(t), x_2(t))$ and the state space is denoted by z^{+^2}. At each epoch of a product return, the firm can take one of the following two actions: *Admit a returned product for remanufacturing* or *Dispose of it*. A control policy, π, specifies the action taken at any epoch of a product return given the current state of the system. Denote the initial state by (x_1, x_2) and the interest rate by α. Then, the expected discounted profit given (x_1, x_2) over an infinite horizon under π can be written as

$$J^{\pi}(x_1, x_2) = E^{\pi}\left[\int_0^{\infty} e^{-\alpha t}\left(-\sum_{i=1}^{2} h_i x_i(t)\,dt + \sum_{t\in B_1} R + \sum_{t\in B_2} S - \sum_{t\in B_3} c_M - \sum_{t\in B_4} c_R\right)\Big|(x_1, x_2)\right] \quad (1)$$

where B_1, B_2, B_3, and B_4 respectively denote the set of random instances of demand satisfaction, disposal of returned product, new product manufacturing completion, and remanufacturing completion under policy π. Then, the goal of this paper is to find the disposal policy which maximize the following expected discounted profit over an infinite horizon:

$$J(x_1, x_2) = \sup_{\pi} J^{\pi}(x_1, x_2) \quad (2)$$

where *sup* implies the supreme.

3. Structure of the Optimal Policy

In this section, we investigate the structure of an optimal disposal policy. Since it is not possible to identify the optimal disposal policy under general probability distribution, we assume that demand and product return follow a Poisson process and manufacturing and remanufacturing processes follow an exponential distribution.

Denote $D(x_1, x_2) = (x_1 - 1, x_2)$ if $x_1 > 0$; (x_1, x_2) otherwise, and $I(x_1, x_2) = (x_1 + 1, x_2 - 1)$ if $x_2 > 0$; (x_1, x_2) otherwise. Operator D and I respectively correspond to a demand arrival and a remanufacturing completion. From the theory of Markov decision processes(see chapter 6 in Puterman, 2005), we know that the optimal profit function $J(x_1, x_2)$ in (2) satisfies the following optimality equation:

$$\begin{aligned} J(x_1, x_2) = \frac{\beta}{\gamma}[&-\sum_{i=1}^{2} h_i x_1 + \lambda_1\{R1(x_1 > 0) \\ &+ J(D(x_1, x_2))\} + \mu_1\{J(x_1 + 1, x_2) - c_M\} \\ &+ \mu_2\{J(I(x_1, x_2)) - c_R 1(x_2 > 0)\} \\ &+ \lambda_2 \max\{J(x_1, x_2 + 1), J(x_1, x_2) + S\}] \end{aligned} \quad (3)$$

where $\gamma = \lambda_1 + \lambda_2 + \mu_1 + \mu_2$, $\beta = \gamma/(\alpha + \gamma)$, and $1(a) = 1$ if a is true, otherwise, 0.

Since γ is the sum of all transition rates, $1/\gamma$ in (3) is the expected transition time. β can be interpreted as the discount factor for this discrete-time Markov decision process. The terms multiplied by λ_1 represent sales revenues and transitions generated with the arrival of demand, the terms multiplied by μ_1 and μ_2 imply penalties and transitions associated with a production completion of a new product and a remanufacturing completion, respectively, and the terms multiplied by λ_2 represent transitions and disposal revenues generated with the arrival of a product return.

Define the value iteration operator T on J by

$$TJ(x_1,\, x_2) = \frac{\beta}{\gamma}[-\sum_{i=1}^{2} h_i x_1 + \lambda_1\{\mathrm{R1}(\mathrm{x}_1 > 0) \qquad (4)$$

$$+ J(D(x_1,\, x_2))\} + \mu_1\{\mathrm{J}(\mathrm{x}_1+1,\mathrm{x}_2) - \mathrm{c_M}\}$$
$$+ \mu_2\{\mathrm{J}(\mathrm{I}(\mathrm{x}_1,\, \mathrm{x}_2)) - \mathrm{c_R}1(\mathrm{x}_2 > 0)\}$$
$$+ \lambda_2 \max\{\mathrm{J}(\mathrm{x}_1,\, \mathrm{x}_2+1), \mathrm{J}(\mathrm{x}_1,\, \mathrm{x}_2) + \mathrm{S}\}]$$

Then, (3) can be rewritten as (see Equation (6.2.8) in Puterman (2005))

$$J(x_1,\, x_2) = TJ(x_1,\, x_2). \qquad (5)$$

Next, consider the following value iteration algorithm (see Bertsekas (1987) for the detail) to solve for equation(5) :

$$J_{k+1}(x_1,\, x_2) = TJ_k(x_1,\, x_2), k = 0,1,\cdots \qquad (6)$$

where $J_0(x_1,\, x_2)$ for every state $(x_1,\, x_2)$. Here $J_{k+1}(x_1,\, x_2)$ can be viewed as the optimal profit in state $(x_1,\, x_2)$ when the problem is terminated after k iterations. Since T is a contraction mapping, it is well known that $J_{k+1}(x_1,\, x_2)$ converges to $J(x_1,\, x_2)$ as k goes to the infinite.

Define $D_1 J(x_1,\, x_2) = J(x_1+1,\, x_2) - J(x_1,\, x_2)$ and $D_2 J(x_1,\, x_2) = J(x_1,\, x_2+1) - J(x_1,\, x_2)$. The quantity $D_1 J(x_1,\, x_2)$ and $D_2 J(x_1,\, x_2)$ respectively imply the marginal profit obtained when there is one more unit of product in serviceable inventory and one more unit of product in remanufacurable inventory.

Numerical investigation indicates that the optimal profit function J has the following set of the properties:

(i) *Submodularity*:
$D_1 J(x_1,\, x_2) \geq D_1 J(x_1,\, x_2+1)$ and
$D_2 J(x_1,\, x_2) \geq D_2 J(x_1+1,\, x_2)$

(ii) *-Diagonal dominance*:
$D_1 J(x_1,\, x_2+1) \geq D_1 J(x_1+1,\, x_2)$ and
$D_2 J(x_1+1,\, x_2) \geq D_2 J(x_1,\, x_2+1)$

(iii) *Concavity*:
$D_1 J(x_1,\, x_2) \geq D_1 J(x_1+1,\, x_2)$ and
$D_2 J(x_1,\, x_2) \geq D_2 J(x_1,\, x_2+1)$

Hence, *Submoularity* of J says that the marginal profit of holding one more product in serviceable inventory decreases as the remanufacturable inventory increases, and the incremental profit of holding one more returned product in remanufacturable inventory decreases as the serviceable inventory increases. If J is concave, then the incremental profit when there is one more unit of serviceable inventory is decreasing in its inventory, and the incremental profit when there is one more unit of remanufacturable inventory is decreasing in its inventory. *-Diagonal dominance* implies that the benefit of having one more unit of serviceable inventory decreases faster in its inventory level than in remanufacturable inventory level, and the benefit of having one more unit of remanufacturable inventory decreases faster in its inventory level than in serviceable inventory level. Note that *Submodularity* and-*Diagonal dominance* together imply *Concavity*.

From the *Submodularity*, *-Diagonal dominance*, and *Concavity* of the optimal profit function J, the following properties of the optimal disposal policy can be deduced. It can be easily verified that properties (i)~(iii) characterize the optimal disposal policy as a monotonic threshold type of the curve.

(i) *If the firm accepts a returned product for remanufacturing in state* $(x_1,\, x_2+1)$, *then it also accepts a returned product for remanufacturing in state* $(x_1,\, x_2)$.

(ii) *If the firm accepts a returned product for remanufacturing in state* $(x_1+1,\, x_2)$, *then it also accepts a returned product for remanufacturing in state* $(x_1,\, x_2)$.

(iii) *If the firm accepts a returned product for remanufacturing in state* $(x_1,\, x_2+1)$, *then it also accepts a returned product for remanufacturing in state* $(x_1+1,\, x_2)$.

The following theorem provides upper bounds on $D_1 J(x_1,\, x_2)$ and $D_2 J(x_1,\, x_2)$, which states that the marginal profit obtained with holding one more unit of serviceable (remanufacurable) product cannot exceed the one stage discounted unit sales price. These bounds are used in truncating the state space when the value iteration method computes the function J under the optimal policy.

Theorem 1

(i) $D_1 J(x_1,\, x_2) \leq \beta R$

(ii) $D_2 J(x_1, x_2) \le \beta R$

Proof: See the Appendix.

The arguments made for the discounted profit problem can be also applied to the average profit problem. We can define the optimal average profit problem in a similar way to the optimal discounted one as follows:

$$v(x_1,x_2) = \frac{1}{\gamma}[-\sum_{i=1}^{2} h_i x_1 + \lambda_1\{\mathrm{R}1(\mathrm{x}_1 > 0) + \mathrm{J}(\mathrm{D}(\mathrm{x}_1, \mathrm{x}_2))\} + \mu_1\{\mathrm{J}(\mathrm{x}_1+1, \mathrm{x}_2) - \mathrm{c_M}\} + \mu_2\{\mathrm{J}(\mathrm{I}(\mathrm{x}_1, \mathrm{x}_2)) - \mathrm{c_R}1(\mathrm{x}_2 > 0)\} + \lambda_2 \max\{\mathrm{J}(\mathrm{x}_1, \mathrm{x}_2+1), \mathrm{J}(\mathrm{x}_1, \mathrm{x}_2) + \mathrm{S}\}] \tag{7}$$

where v is the optimal value function. Then, from the theory of Markov decision processes,

$$v(x_1, x_2) + g = Tv(x_1, x_2) \tag{8}$$

where g is the optimal average profit during the expected transition time.

Again we consider a value iteration algorithm to solve for equation (8) in which $v_0(x_1, x_2) = 0$ for every state (x_1, x_2) and

$$v_{k+1}(x_1, x_2) = Tv_k(x_1, x_2),\ k = 0, 1, \cdots. \tag{9}$$

Here $v_k(x_1, x_2)$ can be viewed as the optimal value function when the problem is terminated after k iterations. The following result shows that equation (8) has a well-defined solution, and $v_k(x_1, x_2)$ in equation (9) converges to $v(x_1, x_2)$ in equation (8).

Theorem 2 *There exists an integer N, a constant g and a function v such that $v_{kN+r}(x_1, x_2) - (kN+r)g$ converges to $v(x_1, x_2)$ for all $r = 0, \cdots, N-1$ as $k \to \infty$.*

Proof : This lemma can be proven using the same argument as in Carr and Duenyas (2000). The essence of the proof is to convert the original problem to one with finite state space and apply Theorem 8.4.5 of Puterman (2005). Without loss of optimality, we can add the constraint to the original problem that we can not produce a new product when $h_1 x_1/\gamma > R$. If $h_1 x_1/\gamma > R$, this indicates that the expected amount of holding costs incurred by holding x_1 units of serviceable inventory until the next transition is greater than the opportunity cost (sales revenue) that would be incurred due to a serviceable product not being available if the next event were the demand arrival. Similarly, we can add the constraint to the original problem that we can not stock returned products when $h_2 x_2/\gamma > R - S$. Since we have finite action spaces and the model with finite state space is unichain, the result follows from Theorem 8.4.5 of Puterman.

4. A Numerical Example

We present an example to illustrate the results obtained in the previous section. The optimal policy can be found easily using value iteration. Because of the magnitude of the state space, however, it is necessary to truncate the inventory level of serviceable and remanufacturable products. We apply the linear to approximation for the value function J along and beyond the truncated state space (see Ha (1997) for linear approximation).

Arrivals for demand and product return are Poisson distributions with rates $\lambda_1 = 0.7$ and $\lambda_2 = 0.3$, respectively. Times for manufacturing a new product and remanufacturing a returned product are exponential with rates $\mu_1 = 0.4$ and $\mu_2 = 2$, respectively. Holding cost rates are $h_1 = 0.4$ and $h_2 = 0.2$. The sales price of a product is $R = 200$. The disposal revenue of a returned product is $S = 5$. The unit cost of manufacturing a new product is $c_M = 100$. The unit cost of remanufacturing is $c_R = 5$. The discount factor is $\beta = 0.99$.

The optimal disposal policy is graphically represents in <Figure 1> in which it is characterized by a monotonic threshold type of function, $\theta(x_1)$, which is decreasing in x_1 as discussed in the pre-

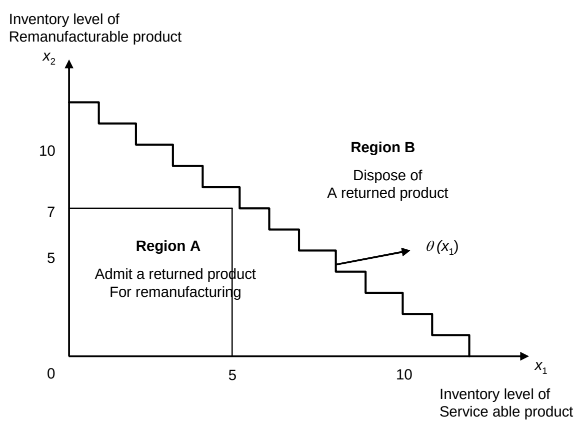

Figure 1. Graphical representation of optimal disposal policy

vious section. It separates the state space into two regions: 1) Dispose of a returned product and 2) Admit a returned product for remanufacturing. If $x_2 \geq \theta(x_1)$, it is optimal to dispose of a returned product. Otherwise, it is optimal to accept a returned product. In this example, if a product return occurs in state (0, 5), the firm should admit it for remanufacturing. If a product return occurs in state (5, 7), the firm should dispose of it.

If the system starts within Region A, we note that the remanufacturable inventory level cannot move up across the boundary of the curve. To see this, suppose that the system starts in state (5, 7) where there are following three possible transitions: the remanufacturable inventory level decreases by one and the inventory level of serviceable product increases by one, the inventory level of serviceable product increases by one, and the serviceable inventory level decreases by one. All the transition makes the system not move up across the boundary $x_2 = 7$.

5. Computational study

Having examined the structure of the optimal policy in the previous section, we next proceed to numerically investigate how the optimal disposal decision changes as a function of the problem parameters. To this end, we compute the expected discounted infinite horizon profits when there are no initial inventories of serviceable and remanufacturable products, $J(0, 0)$ and found the optimal disposal control. We refer the example presented in the previous section and test it with varying system parameters. For the detail of implementing the value iteration method, we refer to Section 5.2 of Bertsekas (1987).

Table 1. Reference example used in the numerical test

λ_1	λ_2	μ_1	μ_2	h_1	h_2	R	S	c_M	c_R	β
0.7	0.3	0.4	2	0.4	0.2	200	5	100	5	0.99

Based on the computational experiments, we observe the following monotonicity of the optimal disposal policy, $\theta(x_1)$, with respect to the time parameters:

- $\theta(x_1)$ is increasing as demand rate, λ_1, increases (see <Figure 2>).
- $\theta(x_1)$ is decreasing as product return rate, λ_2, increases (see <Figure 3>).
- $\theta(x_1)$ is decreasing as manufacturing rate, μ_1, increases (see <Figure 4>).

If demand rate increases while other conditions remain the same, the chance of enhancing sales revenue also increases. Hence, the firm will increase the disposal curve to accept more returned products rather than to dispose of them. To explain the second observation, suppose that product return rate increases with other conditions remaining the same. Then, it is reasonable to expect that the firm will delay a disposal decision because the firm will try to balance the inventory level between serviceable and remanufacturable products. The intuition behind the third observation as follows. The increase

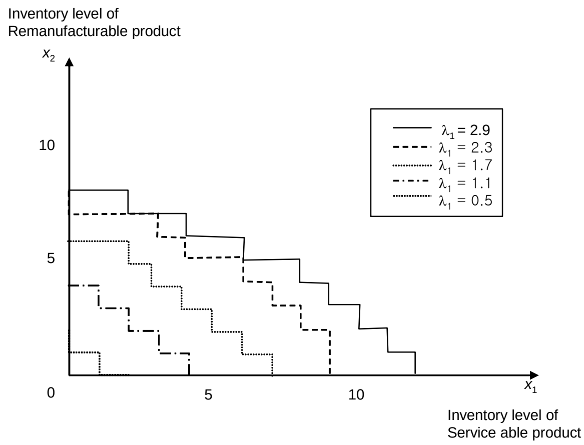

Figure 2. The effect of demand rate on the optimal disposal policy

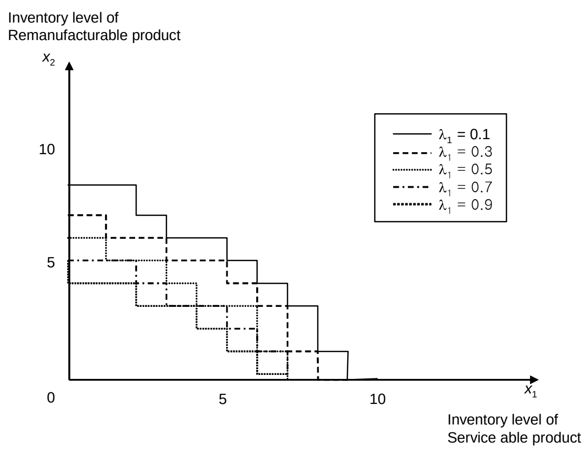

Figure 3. The effect of product return rate on the optimal disposal policy

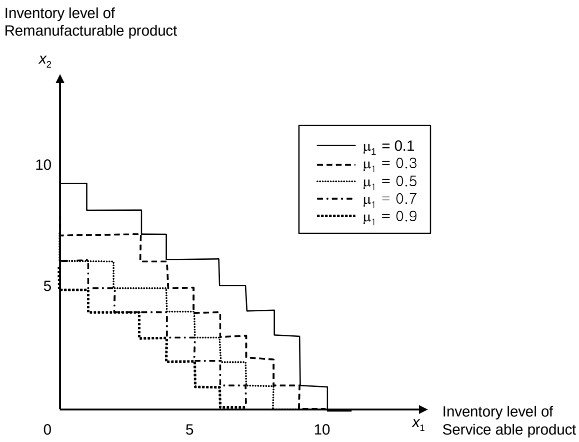

Figure 4. The effect of manufacturing rate on the optimal disposal policy

in the manufacturing rate will increase inventory of serviceable product more rapidly and thus the firm is willing to keep less inventory of remanufacturable product. Hence, the firm's strategy will prefer to disposing of a returned product.

Next are the results for the marginal analysis of the optimal disposal policy with respect to revenue and cost parameters.

- $\theta(x_1)$ is increasing as R increases (see <Figure 5>).
- $\theta(x_1)$ is decreasing as S increases (see <Figure 6>).
- $\theta(x_1)$ is decreasing as c_R increases (see <Figure 7>).
- $\theta(x_1)$ is decreasing as either h_1 or h_2 increases (see <Figure 8> and <Figure 9>).

The first observation states that as R gets larger, it may become optimal to switch from disposing of returned products to accepting them for remanufacturing in any given state. When R is increased, the policy tends to give more importance to the inventories of remanufacturable product. In contrast, if S is increased, disposing of returned product is more profitable and the policy works towards building a less inventory of remanufacturable product. If c_R is increased, transforming remanufacturable product into serviceable product becomes expensive and the policy tends to dispose of returned product than build an inventory of remanufacturable product. If inventory holding cost of either serviceable or remanufacturable products is increased with other conditions remaining the same, the policy works towards building a less inventory of remanufacturable product. Therefore, the disposal curve shifts down so that the firm can dispose of more returned products.

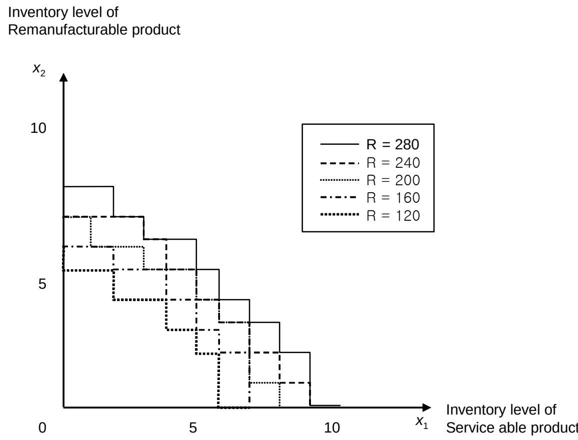

Figure 5. The effect of revenue on the optimal disposal policy

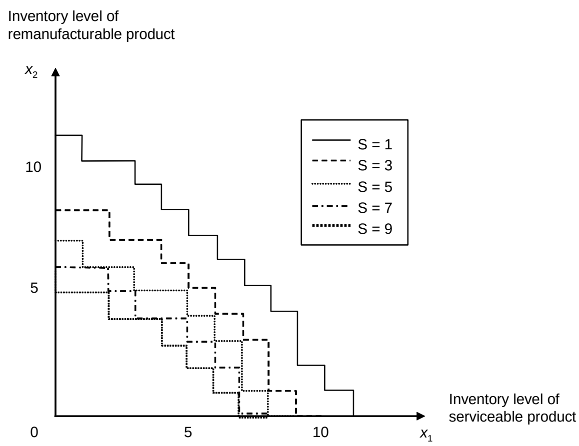

Figure 6. The effect of disposal revenue on the optimal disposal policy

Figure 7. The effect of remanufacturing cost on the optimal disposal policy

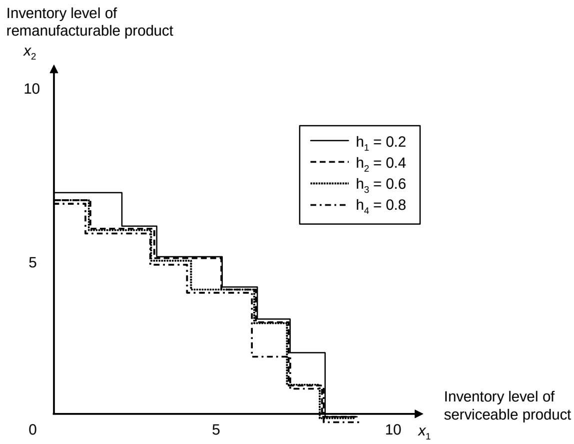

Figure 8. The effect of holding cost of serviceable products on the optimal disposal policy

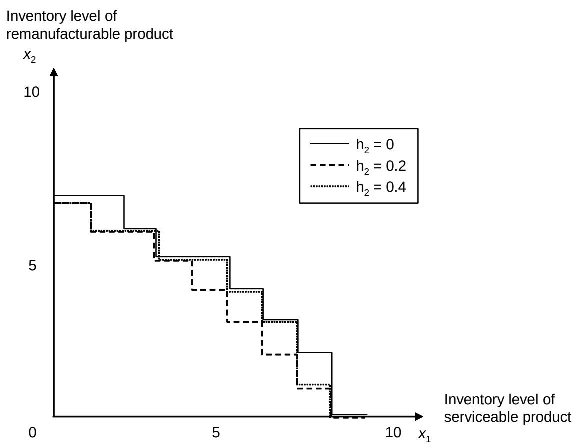

Figure 9. The effect of holding cost of remanufacturable products on the optimal disposal policy

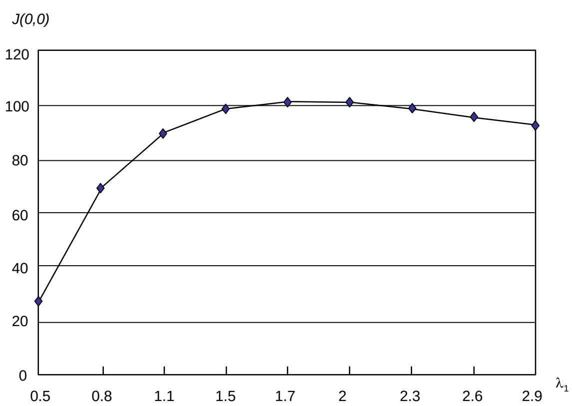

Figure 10. The effect of demand rates on the optimal profit

It is interesting to see that $J(0,\ 0)$ does not necessarily increase in λ_1 (see <Figure 10>). The intuition behind this is as follows. Increasing λ_1,

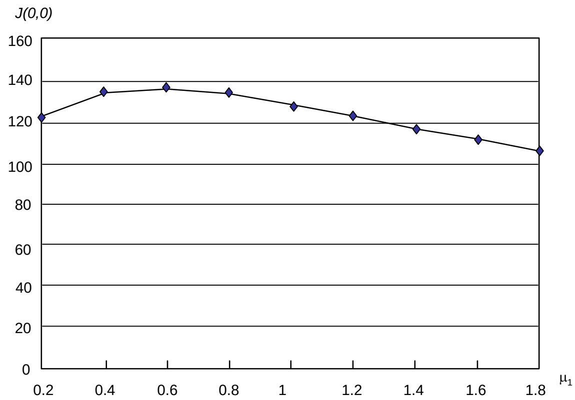

Figure 11. The effect of remanufacturing rates on the optimal profit

when λ_1/μ_1 is low, may contribute to enhancing the sales revenue. On the other hand, if λ_1 becomes increased when λ_1/μ_1 is high, the policy will accept more returned products for remanufacturing and thus sales revenue rate $\lambda_1 R$ will be offset by the increased remanufacturing cost and inventory holding cost of remanufacturable product. In fact, computational results show that $J(0,\ 0)$ is convex with respect to λ_1. Hence, it is conjectured that there exists λ_1^* which maximizes the firm's profit given system parameters.

It is also interesting to see that $J(0,\ 0)$, does not necessarily increase in μ_2(see <Figure 11>). The intuition behind this is as follows. As long as inventory of remanufacturable product is available, increasing μ_2 also increases the inventory level of serviceable products. Hence, when λ_1/μ_1 is not low, it can contribute to enhancing the sales revenue. However, with increasing μ_2 beyond some point, increased remanufacturing cost and inventory holding cost of serviceable products may offset the increased sales revenue. Computational results show that $J(0,\ 0)$ is convex with respect to μ_2 Hence, it is conjectured that there exists μ_2^* which maximizes the firm's profit given system parameters.

6. Conclusion

In this paper we considered a product recovery management where product returns occur randomly and can be accepted for remanufacturing or disposed of depending on the state of the system. While most of product recovery models in the literature deal with fixed disposal policies, we examined the structure of the optimal disposal policy. In particular, we examined the policy that utilizes the information of inventory of both serviceable and remanufacturable products. Under the assumption of Poisson demand and return processes and exponentially distributed manufacturing and remanufacturing processes, we studied the issue of when the firm should dispose of returned products and admit them for remanufacturing to maximize its profit subject to the system costs.

Using value iteration through the Markov decision process, we numerically characterized the structure of an optimal disposal policy as a monotonic threshold function. A disposal is allowed only when the remanufacturable inventory level exceeds the threshold which is the function of the serviceable inventory level and it is decreasing as the serviceable inventory level increases.

We also implemented a sensitivity analysis and observed many meaningful monotonic properties of the optimal disposal policy and the optimal profit with respect to system parameters. We believe that the properties and insights identified in this paper will be very useful for studying more realistic models with arbitrary probability distributions other than exponential ones, since it is not possible to identify the optimal policies under those conditions.

One of the major extensions to the current model is to control the production of new product as well as the disposal of returned product, since it is a more effective strategy in handling the inventory of remanufacturable and serviceable products. This issue has not been treated in the product recovery literature. The other direction of the future research is to study a disposal policy for a product recovery model that purchases new product from the outside source rather than manufactures inside.

Appendix

Proof of Theorem 1 : We denote by (*D*,*A*) the optimal action in state (x_1, x_2) where D and A respectively represent *Dispose of* and *Accept for remanufacturing* actions. For any real valued function J on z^{+2}, define the following:

$$T_1 J(x_1, x_2) = R1(x_1 > 0) + J(D(x_1, x_2)),$$
$$T_2 J(x_1, x_2) = J(x_1 + 1, x_2)$$
$$T_3 J(x_1, x_2) = J(I(x_1, x_2)) - c_M 1(x_2 > 0),$$
$$T_4 J(x_1, x_2) = \max\{\mathrm{J}(\mathrm{x}_1, \mathrm{x}_2 + 1),\ \mathrm{J}(\mathrm{x}_1, \mathrm{x}_2) + \mathrm{S}\},$$
$$TJ(x_1, x_2) = \frac{\beta}{\gamma}\left[-\sum_{i=1}^{2} x_i h_i + \lambda_1 T_1 J(x_1, x_2) + \mu_1 T_2 J(x_1, x_2) + \mu_2 T_3 J(x_1, x_2) + \lambda_2 T_4 J(x_1, x_2)\right].$$

(i) If $x_1 > 0$, $D_1 T_1 J(x_1, x_2) = D_1 J(x_1 - 1, x_2) \le \beta R$ (by (7)). If $x_1 = 0$,
$D_1 T_1 J(x_1, x_2) = R + J(0, x_2) - J(0, x_2) = \beta R$.
$D_1 T_2 J(x_1, x_2) = D_1 J(x_1 + 1, x_2) \le \beta R$ (by (7)).
$D_1 T_3 J(x_1, x_2) = D_1 J(x_1 + 1, x_2 - 1)1(x_2 > 0) + D_1 J(x_1, x_2)1(x_2 = 0) \le \beta R$ (by (7)).

To show that $D_1 T_4 J(x_1, x_2) \le R$, we focus on all possible combinations of actions in $(x_1 + 1, x_2)$ and (x_1, x_2).

For (A, A),
$D_1 T_4 J(x_1, x_2) = D_1 J(x_1, x_2 + 1) \le \beta R$ (by (7)).

For (D, D),
$D_1 T_4 J(x_1, x_2) = D_1 J(x_1, x_2) \le \beta R$ (by (7)).

For (D, A),
$D_1 T_4 J(x_1, x_2) = J(x_{1+1}, x_2) + S - J(x_1, x_2 + 1) \le J(x_1 + 1, x_2) + S - (J(x_1, x_2) + S) \le \beta R$ (by (7)).

For (A, D),
$D_1 T_4 J(x_1, x_2) = J(x_1 + 1, x_2 + 1) - (J(x_1, x_2) + S) \le J(x_1 + 1, x_2 + 1) - J(x_1, x_2 + 1) \le \beta R$ (by (7)).

Therefore, it follows that

$$\begin{aligned} D_1 TJ(x_1, x_2) &= \frac{\beta}{\gamma}[-h_1 + \lambda_1 D_1 T_1 J(x_1, x_2) + \mu_1 D_1 T_2 J(x_1, x_2) + \mu_2 D_1 T_3 J(x_1, x_2) \\ &\quad + \lambda_2 D_1 T_4 J(x_1, x_2)] \le \frac{\beta}{\gamma}(-h_1 + \gamma\beta R) \\ &\le \frac{\beta}{\gamma}(\gamma\beta R) \le \beta^2 R \le \beta R. \end{aligned}$$

(ii) The arguments similar to (i) can be applied here and we omit the detailed proof.

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