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Classification Rule for Optimal Blocking for Nonregular Factorial Designs*

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Abstract

In a general fractional factorial design, the n-levels of a factor are coded by the n^{th} roots of the unity. Pistone and Rogantin (2007) gave a full generalization to mixed-level designs of the theory of the polynomial indicator function using this device. This article discusses the optimal blocking scheme for nonregular designs. According to hierarchical principle, the *minimum aberration* (MA) has been used as an important criterion for selecting blocked regular fractional factorial designs. MA criterion is mainly based on the defining contrast groups, which only exist for regular designs but not for nonregular designs. Recently, Cheng *et al.* (2004) adapted the generalized(G)-MA criterion discussed by Tang and Deng (1999) in studying 2^p optimal blocking scheme for nonregular factorial designs. The approach is based on the method of replacement by assigning 2^p blocks the distinct level combinations in the column with different blocks. However, when blocking level is not a power of two, we have no clue yet in any sense. As an example, suppose we experiment during 3 days for 12-run Plackett-Burman design. How can we arrange the 12-runs into the three blocks? To solve the problem, we apply G-MA criterion to nonregular mixed-level blocked scheme via the mixed-level indicator function and give an answer for the question.

Keywords: Aliasing; indicator function; minimum aberration; nonregular design; word-length pattern.

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1. Introduction

Factorial experiments are conducted for simultaneously investigating a number of factors. Most of the background theory of designs is related with regular factorial designs. For *regular* designs, any two factorial effects can either be estimated independently of each other or fully aliased. A regular design is uniquely determined by independent defining words. Designs that do not possess this property are called *nonregular* designs, which include many mixed-level orthogonal arrays. For reasons of run size economy or flexibility, nonregular designs may be used. For nonregular designs, some factorial effects may neither be uncorrelated nor fully aliased, that is, they have an absolute correlation strictly between 0 and 1. In these designs, the aliasing of effects may have a complex pattern, and are therefore referred to as designs with complex aliasing. Because of complex aliasing, nonregular designs have traditionally been used for screening only main effects. However, complex aliasing actually may allow some interactions entertained and estimated without making additional runs as shown in Hamada and Wu (1992).

Blocking is a commonly used technique to control systematic noises in experiments. Such noises might come from day-to-day variation or batch-to-batch variation. Our primary concern is how to choose a good blocked factorial design. When we perform only a fraction of the complete factorial experiment, some factorial or block effects are aliased(or confounded) with some other factorial effects.

In this paper, we discusses the optimal blocking criteria for nonregular designs. Based on the hierarchical principle, the minimum aberration (MA, for short) (see, Fries and Hunter, 1980) has been used as an important blocking criterion for regular factorial designs. According to the MA criterion, to choose optimal blocked factorial designs we just sequentially minimize the wordlength. This criterion is basically same with sequentially minimizing the numbers of alias relations between factorial effects for blocked regular two-level factorial designs (see, Zhang and Park, 2000). MA criterion is mainly based on the defining contrast groups, which only exist for regular designs but not for nonregular designs.

Recently, Cheng *et al.* (2004) adapted the generalized minimum aberration (G-MA, for short) criterion discussed by Tang and Deng (1999) in studying 2^p optimal blocking scheme for nonregular factorial designs. The approach is based on the method of replacement by assigning 2^p blocks the distinct level combinations in the column with different blocks. However, when blocking level is not a power of two, we have no clue yet in any sense. Examples of nonregular factorial designs include Plackett-Burman (PB) designs, which are constructed from Hadamard matrices. Suppose we experiment during 3 days for 12-run PB deisgn. How can we arrange the 12-runs into the three blocks? To solve the problem, we apply G-MA criterion to nonregular mixed-level blocked scheme via the mixed-level indicator function and give the answer for the question.

The rest of this article is organized as follows. Section 2 introduces the mixed-level indicator function. Section 3 introduces the word ordering between pure-type words and mixed-type words according to the hierarchical assumptions and an appropriate ordering of the numbers of alias or confounding relations according to the generalized minimum aberration criterion. Section 4 gives an example to show how it works.

2. Mathematical Framework

2.1. Mixed-level indicator function

In an investigation on regular fractional factorial designs(FFD), the defining contrasts subgroups play a vital role. They define and characterize design matrices, describe how effects are aliased, and are used to develop criteria, such as resolution and MA, for ranking designs. However, such mathematical structures do not exist in nonregular designs.

In a general fractional factorial design, the n-levels of a factor are coded by the n^{th} roots of the unity. This device allows a full generalization to mixed-level designs of the theory of the polynomial indicator function which has already been introduced for two level designs by Ye (2003). Pistone and Rogantin (2007) showed that design D can be represented by a unique polynomial form in many aspects generalized the defining subgroup as follows;

Let $\mathbb{Q}^m$ and $\mathbb{C}^m$ be the set of all m-tuples of rational numbers and complex numbers respectively, and each set of points $D \subseteq \mathbb{Q}^m$ be the set of solutions of a system of polynomial equations. Each real valued function defined on D is a polynomial function with coefficients into the field of real number $\mathbb{R}$. Let $A_i = \{a_{ij} | j = 1, \ldots, n_i\}$ be factors and a_{ij} level codes by rational numbers or complex numbers so that $D = A_1 \times \cdots \times A_m$ in $\mathbb{Q}^m$ or $\mathbb{C}^m$ is to be full factorial design. Then D is the solution set of the system of polynomial equations rewriting rules:

$$\begin{cases} (X_1 - a_{11})\cdots(X_1 - a_{1n_1}) = 0, \\ (X_2 - a_{21})\cdots(X_2 - a_{2n_2}) = 0, \\ \vdots \\ (X_m - a_{m1})\cdots(X_m - a_{mn_m}) = 0 \end{cases} \quad \text{or} \quad \begin{cases} X_1^{n_1} = \sum_{k=0}^{n_1-1} \psi_{1k} X_1^k \\ \vdots \\ X_m^{n_m} = \sum_{k=0}^{n_m-1} \psi_{mk} X_m^k \end{cases}$$

We code A_j with the ${n_j}^{th}$ roots of the unity in $\mathbb{C}$, that is,

$$A_j = \Omega_{n_j} = \left\{ w_k = \exp\left(i\frac{2\pi}{n_j}k\right) \middle| k = 0, \ldots, n_j - 1 \right\}.$$

Note that the mapping

$$\mathbb{Z}_n \leftrightarrow \Omega_n \subset \mathbb{C} \quad \text{with} \quad k \leftrightarrow w_k$$

is a group isomorphism from the additive group of $\mathbb{Z}_n = \{0, 1, 2, \ldots, n-1\}(\text{mod n})$ to the multiplicative group $\Omega_n \subset \mathbb{C}$. The full factorial design D, as a subset of $\mathbb{C}^m$, is defined by the system of equations

$$\zeta_j^{n_j} - 1 = 0 \quad \text{for} \quad j = 1, \ldots, m.$$

The function $X_i : D \ni (\zeta_1, \ldots, \zeta_m) \mapsto \zeta_i$ is called factor and we call $X^a = X_1^{a_1} \cdots X_m^{a_m}$ monomial response or interaction where $a = (a_1, a_2, \ldots, a_m)$ belongs to the set $L = \mathbb{Z}_{n_1} \times \cdots \times \mathbb{Z}_{n_m}$.

With the Hermitian product $f \cdot g = (1/\sharp D) \sum_{\zeta \in D} f(\zeta)\overline{g(\zeta)}$, the set $\{X^a | \alpha \in L\}$ of all the monomial responses on D becomes an orthonormal basis of the Hilbert space $C(D)$ of all the complex valued functions on the full factorial design D. Each function defined on full factorial design is represented in a unique way by an identified complete regression model. (*i.e.* as a linear combination of constant, simple terms and interactions)

A *fraction* F is a subset of a full factorial design D, that is, $F \subset D$. It can be obtained by adding equations (generating equations) to restrict the set of solutions. The *indicator function* I of a fraction F is a response defined on the full factorial design D such that

$$I(\zeta) = \begin{cases} 1, & \text{if } \zeta \in F, \\ 0, & \text{if } \zeta \in D - F. \end{cases}$$

In a fraction with replicates F_{rep} the counting function R is a response on the full factorial design showing the number of replicates of a point ζ. They are represented as polynomials:

$$I(\zeta) = \sum_{\alpha \in L} b_\alpha X^\alpha(\zeta), \quad R(\zeta) = \sum_{\alpha \in L} c_\alpha X^\alpha(\zeta).$$

Since they are real valued, using the Hermitian product above, we can see that the coefficients b_α and c_α satisfy the following properties(see [6] for details):

$$b_\alpha = \frac{1}{\sharp D} \sum_{\zeta \in F} \overline{X^\alpha(\zeta)} = \overline{b_{[-\alpha]}}, \quad c_\alpha = \frac{1}{\sharp D} \sum_{\zeta \in F_{rep}} \overline{X^\alpha(\zeta)} = \overline{c_{[-\alpha]}}.$$

2.2. Word-length pattern

In a blocked indicator function, we can see that there are two types of polynomial terms with nonzero coefficients (*i.e.,* words), one involving teatment factors only and the other involving both block and treatment factors. Following the same language as used in Cheng and Wu (2002), we call the former *pure-type words* and the latter *mixed-type words.*

Throughout the paper, we make four usual assumptions for blocked factorial designs.

1. Lower-order factorial treatment effects are more likely to be significant than higher-order factorial treatment effect.
2. Treatment effects of the same order are equally important.
3. Treatment effects are more likely to be significant than block effect.
4. Interactions between block factors and treatment factors are negligible.
5. Interactions between block factors are as important as the main effects of block factors.

Under the assumptions, for blocked factorial designs, pure-type word and a mixed-type word have the following desirability as shown in Zhang and Park (2000).

$$ttt \gg ttb \gg tttt \gg ttttt \gg tttb \gg tttttt \gg ttttttt \gg ttttb \gg \cdots,$$

where $\gg$ means 'less desirable than'. In terms of the number and the order of treatment effects affected by the words, *ttb* should be more desirable than *ttt* and less desirable than *ttttt*. These relations are used for the rest of the paper.

According to the above word order desirability, let $\Gamma_{t,b}(F)$ be the number of words with non-zero coefficient in the indicator function where t is the number of treatment letters and b is the number of block letters of design F and define the *word-length pattern* $W(F)$ as

$$W(F) = (\Gamma_{3,0}(F), \Gamma_{2,1}(F), \Gamma_{4,0}(F), \Gamma_{5,0}(F), \Gamma_{3,1}(F), \Gamma_{6,0}(F), \ldots).$$

Definition 2.1 (Zhang and Park, 2000). *Suppose F_1 and F_2 are two blocked FFDs with word-length pattern $W(F_1)$ and $W(F_2)$, respectively. Let $m_l(F_1)$ and $m_l(F_2)$ be the l^{th} entries in $W(F_1)$ and $W(F_2)$, respectively. Let k be the smallest integer such that $m_k(F_1) \neq m_k(F_2)$. If $m_k(F_1) < m_k(F_2)$, it is said that F_1 has less aberration than F_2. If there is no design that has less aberration, then it is said to have minimum aberration.*

2.3. Generalized minimum aberration criterion

The aliasing(or confounding) relationship is reflected in its indicator function through the coefficient of the factorial effect, like b_{123}/b_0 in case of factorial effects X_1X_2 is aliased with X_3. In fact, for regular FFDs, the polynomial terms in the indicator function are the same words as used in the defining subgroup and all of their $|b_{i_1 \ldots i_l}/b_0|$'s equal to 1.

In the nonregular designs, it can be easily verified that some main effects are partially alised with some factorial effects. In the indicator function, it can be observed that $|b_{i_1 \ldots i_l}/b_0|$ is between 0 and 1, which is correlation (*i.e.,* inner product) when the contrasts are partially aliased. A larger value of $|b_{i_1 \ldots i_l}/b_0|$ indicates a stronger aliasing (or confounding) between effects associated with the polynomial term. The value is called the *aliasing index* for pure-type words and the *confounding index* for mixed-type words.

What should be mentioned here is the *J-characteristic* used by Deng and Tang (2002) as building blocks in defining their G-MA criterion. The J-characteristics of a design closely related to coefficients of its indicator function and can be viewed as its orthogonality measure.

Definition 2.2 (Deng and Tang, 2002). *A fractional design, regular or non-regular, is denoted by F with n runs and is regarded as a set of m columns $F = \{d_1, \ldots, d_m\}$ or as an $n \times m$ matrix $F = ((d_{ij}))$. For $1 \leq k \leq m$ and any k-subset $s = \{d_{j_1}, \ldots, d_{j_k}\}$ of F define the J-characteristic as follows:*

$$J_k(s) = J_k(d_{j_1}, \ldots, d_{j_k}) = \left| \sum_{i=1}^{n} d_{ij_1} \cdots d_{ij_k} \right|.$$

For mixed-level design with complex coding, the basic formula of J-characteristic is valid, but if one of d_j of the subset s is coded by complex coding, the value of $\sum_{i=1}^{n} d_{ij_1} \cdots d_{ij_k}$ will be a complex number, denoted by $a + bi$. Then, the absolute value is computed in a little different way. The fundamental idea of the absolute value is the length. On the complex plane, the length of $a + bi$ is computed as $||a + bi|| = \sqrt{a^2 + b^2}$. In this sense, we redefine the J-characteristic for mixed level with complex coding by

$$J_k(s) = J_k(d_{j_1}, \ldots, d_{j_k}) = \left\| \sum_{i=1}^{n} d_{ij_1} \cdots d_{ij_k} \right\|.$$

As a result, J-characteristic for mixed-level design implies that of symmetric designs. If a design has zero value of J-characteristic, it means that the design is orthogonal and design points are distributed uniformly. If not, it means the design points are distributed ununiformly somewhere. Moreover, as the J-characteristic value is higher, ununiformity is more serious.

Let $|s|$ be a cardinality of the set s. For a design F, regular or nonregular, let r be the smallest integer such that $\max_{|s|=r} J_r(s) > 0$, where the maximization is over all the subset of r distinct columns of F. We define its generalized resolution to be

$$R(F) = r + \left[1 - \max_{|s|=r}(J_r(s))/n\right].$$

Clearly, $r \leq R(F) < r + 1$. For orthogonal design, we have $R(f) \geq 3$. According to this criterion, a design with higher generalized resolution is preferred. As before, it is sometimes desirable to define resolutions for aliasing and confounding separately. We define the treatment resolution (R_t) as the generalized resolution over the pure-type words of the smallest word length and the block resolution (R_b) as the generalized resolution over the mixed-type words of the smallest word length.

For those words with the same order, G-MA compares the frequencies of words with the same J-characteristic value. Suppose that two nonregular factorial designs F_1 and F_2 have the same generalized resolution. If the frequency of combinations of r distinct columns that attain $\max_{|s|=r} J_r(s) > 0$ in F_1 is lower than that in F_2, then the design F_1 is preferred. If the two frequencies are the same, compare the frequency of combinations of r distinct columns that attain the second largest $J_r(s)$ value of the two designs. This process is continued until the two designs can de distinguished.

Since designs have finite points with finite levels, the number of all the possible values of $J_r(s)$ is finite. Therefore we can list up the frequencies of combinations of r distinct columns that attain the possible values of $J_r(s)$ in descending order. In this way, we define the word *length frequency vector* by $\Gamma_{t,b}(F) = (f_{t,b,1}, \dots, f_{t,b,j})$ for the words of same order with t the number of its treatment letter and b the number of its block letter, where $f_{t,b,k}$ is the frequency of combinations of r distinct columns that attain the k^{th} value of $J_r(s)$. Note that the number j of the component $f_{t,b,j}$ varies but finite. Finally the *confounding frequency vector* of the design F is defined as follow

$$W[F] = [\Gamma_{3,0}(F); \Gamma_{2,1}(F); \Gamma_{4,0}(F); \Gamma_{5,0}(F); \Gamma_{3,1}(F); \Gamma_{6,0}(F); \cdots].$$

Definition 2.3 (Tang and Deng, 1999). *Let F_1 and F_2 are two fractional designs and $f_l(F_1)$ and $f_l(F_2)$ be the l^{th} entries in the confounding frequency vectors of F_1 and F_2, respectively. Let k be the smallest integer such that $f_k(F_1) \neq f_k(F_2)$. If $f_k(F_1) < f_k(F_2)$, it is said that F_1 has less generalized aberration than F_2. If there is no design that has less generalized aberration, then it is said to have generalized minimum aberration.*

3. The Optimal Blocking 12-run PB Design into Three Blocks

In this section we consider p treatment factors from the 12-run PB design and assign them into three blocks with complex coding, denoted by 1, w_1 and w_2. A pair of columns in a matrix are called *orthogonal* if all possible combinations of levels in the two columns appear equally often. If the orthogonality between treatment effect and blocking effect of a design is violated, it implies that a main factorial effect and a block effect are confounded for the given blocking scheme and it is not desirable. Therefore we focus only on the blocked designs whose block factor and treatment factors form an orthogonal array.

There are 34,650 possible arrangements in three block case. Among them, we only focus on the orthogonal arrangements. We programmed by <Microsoft visual C++ 6.0> for the choices. As a result, we found that the possible choices of blocking schemes with orthogonality with $p = 3$ are 300 and with $p = 4$ are 36 respectively. However, there are no blocking scheme having orthogonality with $p \geq 5$. As we mentioned, we should only focus on the cases of designs whose treatment main effect and block effect are orthogonal. Thus, in this paper, we only consider the mixed-level designs with $p = 3$ and $p = 4$. Even if there are many arrangements for each case, there exist two types of indicator functions when $p = 3$, whereas only one type exists when $p = 4$ with respect to the word-length and the corresponding coefficients. When $p = 3$, two $2^3 3^1$-designs are given in Table 3.2 and Table 3.3. Also, a design in Table 3.6, denoted by $2^4 3^1$, is an example of only type when $p = 4$.

3.1. Optimal blocking scheme with $p = 3$

Suppose we take first three ($p = 3$) columns from the 12-run PB design and assign into three blocks with complex coding, denoted by 1, w_1 and w_2. Table 3.2 and Table 3.3 represent possible two different blocking schemes.

Table 3.1: 12-run PB design

run \ factor	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}
1	1	1	−1	1	1	1	−1	−1	−1	1	−1
2	−1	1	1	−1	1	1	1	−1	−1	−1	1
3	1	−1	1	1	−1	1	1	1	−1	−1	−1
4	−1	1	−1	1	1	−1	1	1	1	−1	−1
5	−1	−1	1	−1	1	1	−1	1	1	1	−1
6	−1	−1	−1	1	−1	1	1	−1	1	1	1
7	1	−1	−1	−1	1	−1	1	1	−1	1	1
8	1	1	−1	−1	−1	1	−1	1	1	−1	1
9	1	1	1	−1	−1	−1	1	−1	1	1	−1
10	−1	1	1	1	−1	−1	−1	1	−1	1	1
11	1	−1	1	1	1	−1	−1	−1	1	−1	1
12	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1

Assign the block factor, denoted by B, into the last column of F_1 and F_2. Then the 12 runs in F_1 and F_2 are divided into three blocks, each of size four. In its (unblocked) indicator functions, we replace x_4 by B to indicate the factor that is used for blocking. Then the indicator functions for the blocked design F_1 and F_2 are as follows:

$$\begin{aligned} I_{F_1}(x) &= \frac{1}{24}\{12 - 4X_1X_2X_3 + (-4w_1 + 4w_2 - 4)X_1X_2X_3B \\ &\quad +(4w_1 - 4w_2 - 4)X_1X_2X_3B^2\}, \\ I_{F_2}(x) &= \frac{1}{24}\{12 - 4X_1X_2X_3 + (4w_1 - 4w_2)X_2X_3B \\ &\quad +(-4w_1 + 4w_2)X_2X_3B^2 - 4X_1X_2X_3B - 4X_1X_2X_3B^2\}. \end{aligned} \quad (3.1)$$

The word-length patterns are

$$W(F_1) = (1, 0, 0, 0, 2), \quad W(F_2) = (1, 2, 0, 0, 2).$$

Table 3.2: 12-run $2^3 3^1$ design F_1

run \ factor	X_1	X_2	X_3	B
1	1	1	−1	1
2	−1	1	1	1
3	1	−1	1	1
4	−1	1	−1	w_1
5	−1	−1	1	w_1
6	−1	−1	−1	1
7	1	−1	−1	w_1
8	1	1	−1	w_2
9	1	1	1	w_1
10	−1	1	1	w_2
11	1	−1	1	w_2
12	−1	−1	−1	w_2

Table 3.3: 12 run $2^3 3^1$ design F_2

run \ factor	X_1	X_2	X_3	B
1	1	1	−1	1
2	−1	1	1	1
3	1	−1	1	1
4	−1	1	−1	w_1
5	−1	−1	1	w_1
6	−1	−1	−1	1
7	1	−1	−1	w_1
8	1	1	−1	w_1
9	1	1	1	w_2
10	−1	1	1	w_2
11	1	−1	1	w_1
12	−1	−1	−1	w_2

Both of two designs have only one *ttt*-type word , but F_2 has more *ttb*-type words than F_1. Therefore F_1 has less aberration than F_2, and as the minimum aberration criterion, F_1 can be referred to the optimal design for mixed-level with $p = 3$. To find the optimal design, we only need to compare word-length pattern rather than compare the confounding frequency vectors to apply G-MA criterion.

Table 3.4: Coefficients and J-characteristics of F_1

word	coefficient	$J_k(s)$
$X_1X_2X_3$	-4	4
$X_1X_2X_3B$	$-4w_1 + 4w_2 - 4$	8
$X_1X_2X_3B^2$	$+4w_1 - 4w_2 - 4$	8

Table 3.5: Coefficients and J-characteristics of F_2

word	coefficient	$J_k(s)$
$X_1X_2X_3$	-4	4
X_2X_3B	$4w_1 - 4w_2$	$4\sqrt{3}$
$X_2X_3B^2$	$-4w_1 + 4w_2$	$4\sqrt{3}$
$X_1X_2X_3B$	-4	4
$X_1X_2X_3B^2$	-4	4

The J-characteristics are the index of how uniformly design points are distributed. For the design F_1, we can find out one ttt-type word is distributed more uniformly than $tttb$-type words as shown in Table 3.4. Whereas for F_2, the words of type ttt and $tttb$ are distributed uniformly with the same degree, but ttb-type words are distributed less uniformly as shown in Table 3.5. Table 3.4 and Table 3.5 show the coefficients for terms in the indicator functions (3.1) and their J-characteristics. We can easily see that $R(F_1) = R_t(F_1) = 3+(1-4/12) = 3.67, R_b(F_1) = 4+(1-8/12) = 4.33, R_t(F_2) = 3+(1-4/12) = 3.67$, and $R(F_2) = R_b(F_2) = 3 + (1 - 4\sqrt{3}/12) = 3.42$.

3.2. Optimal blocking scheme with $p = 4$

Table 3.6: 12 run $2^4 3^1$ design F_3

run \ factor	X_1	X_2	X_3	X_4	B
1	1	1	−1	1	1
2	−1	1	1	−1	1
3	1	−1	1	1	1
4	−1	1	−1	1	w_1
5	−1	−1	1	−1	w_1
6	−1	−1	−1	1	w_2
7	1	−1	−1	−1	w_2
8	1	1	−1	−1	w_1
9	1	1	1	−1	w_2
10	−1	1	1	1	w_2
11	1	−1	1	1	w_1
12	−1	−1	−1	−1	1

The indicator function of F_3 becomes

$$\begin{aligned} I_{F_3}(x) = \frac{1}{24}\{&12 - 4X_1X_2X_3 + (4-4w_1)X_1X_4B + (4-4w_2)X_1X_4B^2 \\ &+(4w_1 - 4w_2)X_2X_3B + (-4w_1 + 4w_2)X_2X_3B^2 \\ &-4X_1X_2X_3B - 4X_1X_2X_3B^2 - 4w_2X_1X_2X_4B \\ &-4w_1X_1X_2X_4B^2 + 4w_2X_1X_3X_4B + 4w_1X_1X_3X_4B^2 \\ &-4X_2X_3X_4B - 4X_2X_3X_4B^2 \\ &-4w_1X_1X_2X_3X_4B - 4w_2X_1X_2X_3X_4B^2\}. \end{aligned} \tag{3.2}$$

All possible arrangements of block factor that satisfying orthogonality have the same type of indicator function as in (3.2). Therefore it is not necessary to derive the word length pattern or J-characteristics. The word length pattern is derived as a reference.

$$W(F_3) = (1, 4, 0, 0, 8, 0, 0, 2)$$

Also J-characteristics are derived in Table 3.7 below. All the words except for *ttb*-type are aliased in the same degree, and *ttb* type words are distributed less uniformly than the others. We also see that $R_t(F_3) = 3 + (1 - 4/12) = 3.67$ and $R(F_3) = R_b(F_3) = 3 + (1 - 4\sqrt{3}/12) = 3.42$.

Table 3.7: Coefficients and J-characteristics of F_3

word	coefficient	$J_k(s)$	word	coefficient	$J_k(s)$
$X_1X_2X_3$	-4	4	$X_1X_2X_4B$	$-4w_2$	4
X_1X_4B	$4-4w_1$	$4\sqrt{3}$	$X_1X_2X_4B^2$	$-4w_1$	4
$X_1X_4B^2$	$4-4w_2$	$4\sqrt{3}$	$X_1X_3X_4B$	$4w_2$	4
X_2X_3B	$4w_1-4w_2$	$4\sqrt{3}$	$X_1X_3X_4B^2$	$4w_2$	4
$X_2X_3B^2$	$-4w_1+4w_2$	$4\sqrt{3}$	$X_2X_3X_4B$	-4	4
$X_1X_2X_3B$	-4	4	$X_2X_3X_4B^2$	-4	4
$X_1X_2X_3B^2$	-4	4	$X_1X_2X_3X_4B$	-4	4
			$X_1X_2X_3X_4B^2$	-4	4

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