

A Comparison between Editions A and B of Experimental Textbooks in Ordinary Senior High Schools in China¹

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In order to study the new curriculum, as well as to help teachers understand new textbooks well, the authors make horizontal comparisons between Editions A and B of Experimental Mathematics ② textbook in ordinary senior high schools (Grades 10–12) of PEP Edition, which focuses on the topic of line slope. The interview was made with 11 selected teachers who are teaching the content in the light of Edition B. It was found that there are obvious differences in the introduction of concepts and the exercises. The interviews showed that teachers were familiar to the contents of Edition A and saw it reasonable, because few differences are found between Edition A and the old Edition. The teachers also viewed that though the contents of Edition B were easy and fit to self-educated, but its reasoning is not clear, and the examples are not enough so that the teachers need to add additional examples.

Keywords: mathematics textbook in ordinary senior high school, line slope, comparison.

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I. PREFACE

We enter the gate of 21st century with a new mathematics curriculum reform followed. Since 2004, six editions of mathematics textbooks (People's Education Publishing House (PEP) Edition A, and PEP Edition B, Beijing Normal University Press Edition, Jiangsu

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Province Press Edition, Hunan Province Press Edition, and Hubei Province Press Edition) which were developed by Curriculum Standard in Senior High School have been experimented in China. Comparing and analyzing them horizontally can help us find out their advantages and get acquainted further with the headway of the course reform and help senior high school teachers understand and teach new textbooks well. These six textbooks vary in many aspects (Peng & Liu, 2005). In this context, the authors take line slope in two editions of PEP, Editions A and B, of Experimental of Mathematics ② of Textbook in Ordinary Senior High School, as an example to make a horizontal comparison.

Many researches indicated that whether or not success in implementing the new mathematics curriculum, using new technology and new teaching methods are influenced by the teachers' beliefs about how to understand and implement the new curriculum (Clarke, 1986; Jin, 2001; Romberg, 1986). So, we selected 11 teachers from four senior high schools who are teaching the contents in the light of Edition B, and interviewed with him with regard to the contents of slope in order to investigate teachers' attitudes about using the textbooks. Three questions are given in this interview: How did the teachers think about the concept introducing this content? How did the teachers teach examples and exercises? What was the students' thinking about the contents?

The purpose of which we compare different kinds of textbooks is not to value which textbook is more excellent and inferiority, but to help teachers adopt the best way to carry on effective teaching.

II. CONCEPT GUIDING PROCESSES OF EDITIONS A AND B

In order to help readers get acquainted with the contents of these two editions of textbook, next we simply introduce the chapter and section which contains the concept of slope. Chapter three of Edition A — Lines and Equations — is divided into three sections:

- 3.1 Angle of inclination and line slope,
- 3.2 The equation of a line,
- 3.3 The coordination of the intersection point and distance formula.

The concept of slope appears in the first paragraph of 3.1.

3.1.1 Angle of inclination and slope

In Edition B, the second chapter is plane analytic geometry preliminary, of which the second section is the equation of a line. There are four paragraphs in this section, and the concept is in

2.2.1 The concept of the equation of a line and the slope

1. The Guiding Process of Edition A

There is a question in this content of line slope, “Thinking? For a line l in the plane coordinates, how can we determine its location?” after which there are four questions put forward one by one.

- ① Can one point determine the location of a line?
- ② If point P is on l , and can we locate l ?
- ③ There are numerous lines l_1, l_2, l_3, \dots (they compose a bunch of lines), what are their differences?
- ④ It is obviously that the degrees of their inclination are different. How can we describe the degree of inclination?

The definition of angle of inclination follows these questions, which is divided into two cases:

Case 1: “When l crosses x-axis, we could take x-axis as the standard and define angle of inclination of a line l by the angle α between the positive direction of x-axis and the upward of l ”. There are pictures of acute and obtuse angles in the textbook.

Case 2: “When l is parallel or coincide with x-axis, we would define its angle of inclination by 0° . So the range of angle of inclination is $0^\circ \leq \alpha < 180^\circ$ ”.

Then the textbook explains the relationship between the degree of inclination and the angle of inclination. Finally, the elements that can determine a line in the coordinates are pointed out.

Edition A also sets the second question, “Thinking? In the daily life, are there any variables which can describe the degree of inclination?” after which follows an example of gradient that is used to derive the concept of slope. “We define line slope by tangent of its angle α of inclination. It is denoted by a miniscule of k , then $k = \tan \alpha$.” In the same page on the left margin there is another question, “Does the line with angle of inclination 90° have slope?” Immediately after this, two examples illustrate how to calculate the slope of angles of 45° and 135° .

As for the case of obtuse angle, the textbook utilizes another formula: If α is an acute angle, then $\tan(180^\circ - \alpha) = -\tan \alpha$ holds.

Edition A makes further explanation about the relationship between slope and angle of inclination. “Each line has a slope except for the angle α of 90° ones. Besides, the slope varies as angle changes.” In the following paragraph, the textbook discusses “how to calculate the slope using two given points’ coordinates on a line.”

2. The Guiding Process of Edition B

In Edition B, the concept appears in

2.2.1 *The concept of the equation of a line and the slope*

And it is composed of two paragraphs:

- ① The concept of the equation of a line;
- ② The line slope.

In the first part, the graph of linear function of one variable is reviewed using an example of $y = 2x + 1$, which explains the relationship of a line and the points on it. If a point is on the line, its coordinate will satisfy the function of the line. Whereas, if a point's coordinate satisfies the function of a line, it is just on the line. The conclusion is "the function of $y = 2x + 1$ could describe the character of line l , therefore

$$l = \{(x, y) \mid y = 2x + 1\}.$$

Now let us study the case of $k = 0$. For example, y remains 2 whatever x is for the equation of $y = 2$ (Constant function). Although it is not a linear function any more, its graph is a line that parallels with x-axis and with point $(0, 2)$ on it."

(Hint: Edition B directly uses the concept of slope, the reason of which is that slope is defined in

2.2 *Linear function and quadratic function:*

of Chapter II of Edition B Mathematics ①: "The graph of linear function $y = kx + b$ ($k \neq 0$) is a line, therefore we can note easily as line $y = kx + b$ and call k as the slope of this line."

The textbook also points out that, "the expression of a linear function is only a special case of equation", and they may not absolutely have functions. For instance, x and y in the equation of $x^2 + y^2 = 1$ are not functional relationship in the real number systems. The concepts of equation of line and line of equation are introduced from the conclusion that the solution of $y = kx + b$ and the points on its graph are one-to-one correspondent.

Line slope is directly defined on account of "Lines are determined by any two different points on it." Supposing two arbitrary points' coordinates on the line and taking them into the equation of it, then calculating out

$$k = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_1 \neq x_2).$$

Replacing the residual quantity with Δx and Δy , "as a result,

$$k = \frac{\Delta y}{\Delta x} \quad (\Delta x \neq 0)$$

which is called quotient. Usually, we call the coefficient k in $y = kx + b$ slope of the line. The lines which are vertical to x-axis don't have a slope (why?)".

The definition of angle of inclination in this Edition comes after the concept of slope. "We might perceive from direct observation that slope k determines the degree of inclination of line. The angle between the positive direction of x-axis and the upward of the line is called the angle of inclination. We let the angle which parallels or coincides with x-axis be angle of zero." Later, the textbook states the relationship between the sign of the slope and the angle as well as the transformation of angle when k increases. At the same time, it mentions that "The relationship between slope and angle will be discussed in Mathematics ④."

III. THE COMPARISON OF CONCEPT OF SLOPE

Next, we will compare the concept of slope from five aspects that are concept guiding pattern, the guide of angle of inclination, the guide in the two points' coordination formula, instances in the guide, and examples and exercises.

1. C concept Guiding Pattern

Generally, there are two patterns in the guiding line slope in senior high school teaching. The former is making use of tangent of the angle to define slope after defining the angle of inclination. The latter directly defines slope by two points' coordinates on the line. In other words, for two points $A_1(x_1, y_1)$, $A_2(x_2, y_2)$ on a line, we can define slope

$$k = \frac{y_2 - y_1}{x_2 - x_1}$$

Then, we get the concept of angle of inclination based on the equation

$$\frac{y_2 - y_1}{x_2 - x_1} = \tan \alpha$$

As the second pattern is more complex, the former pattern is generally used in the textbook of senior high schools (Collaborate Group in Thirteen Universities, 1981). We can analyze from the foregoing that edition A and B choose respectively the first and the second concept introducing pattern mentioned in the above.

Edition A firstly defines angle of inclination, and uses tangent of the angle to define slope. This arrangement makes reference to the mathematics textbook² for ordinary full-time senior schools (it is just the old edition of mathematics textbook). For experienced teachers, using Edition A can probably make them teach proficiently. Edition B adopting the second pattern aims at explaining the relation between equation and curve.

We know from the interview all teachers prefer the first introducing pattern (defining slope by angle of inclination). The reasons are that students can more explicitly grasp the relation between slope and angle of inclination, and the teachers are familiar with the introducing pattern of the old Edition of textbook. In spite of these, all teachers showed respect to Edition B in the practice and didn't reject the introducing pattern of it. This means that the teachers understand and grasp the compiling idea of "quotient" in the curriculum standard although they do not completely approve of it.

In teaching with Edition B, it showed that the students had no question in comprehending the concept of slope, but they misunderstood the concept "line of equation" and "equation of line", for example, some students can not see the differences between these two concepts, and some can not understand the function of "line of equation", etc. ... But Edition B did not give the exercises about the two concepts of "line of equation" and "equation of line", so the students have no opportunity to probe the differences further.

2. The Guide of Angle of Inclination

Angle of inclination of a line is one of the variables which describe the degree of inclination of the line, and it has relationship with the slope. We know that the slope of a line can be defined by the tangent of angle of inclination of it.

Combined with "the introducing the process of Edition A" paragraph, let we see the guiding angle of inclination. We notice that there's no picture till this time before the first question, and students can imagine a line and a point on it in the space arbitrarily, in the meantime they could also use the knowledge of plane geometry — two points determine a line to do a negative answer. Question ② seems homology to question ①, but its abstraction degree rises. It has abstract mathematical sign such as using l and P to sign line and point. Question ③ guides students to consider the difference about the lines of the bunch. Students will easily find if, but they may not describe it with suitable mathematical terms. Given the fourth question, students can describe the difference using the term-- inclination degree of line, but they don not know how to draw with a corresponding mathematics term. Till, students naturally realize the necessity about

² It was compiled according to mathematics teaching syllabus for full-time ordinary senior high schools.

introducing the term of angle of inclination. In this guiding process, students gradually feel the necessity about creating of mathematical concept, and it fosters the students' abstract thinking ability in mathematics.

Compared with Edition A, the guide in Edition B seems more sudden. In the process given from "we might perceive from directly observation that slope k determines the degree of inclination of a line" to "the angle between positive direction of x-axis and the upward of the line is called the angle of inclination", there is no explanation about the relationship between the inclination of a line and k , why we introduce the concept of "the angle of inclination" in here and what the usage of it. Students can not consider the angle of inclination unless teachers point out it. Therefore, teachers should notice this when they are analyzing.

In explaining the relation between the slope and angle of inclination, Edition A gives the concrete explanation of "Each line has slope except for the angle of inclination α of 90° . The slope changed with change in the angle of inclination. Therefore, we can use the slope to show the inclination degree of a line." This indicates the function of angle of inclination and the relation between angle of inclination and the slope.

Edition B discusses the slope k from different signs, in addition to explaining the angle which is an acute angle or obtuse-angled, it also presents the conclusion of the transformation of angle when slope enlarges. The fact could be used by the students to judge the graph of a line drawn correct or not. Besides the usage, we can not notice its other usage in the exercises. Though Edition B has more contents than Edition A, teachers need to handle appropriately based on different contents.

According to the above analyses, we conclude that the guide of angle of inclination in Edition A seems more fluent than that in Edition B. The questions are designed more simple and direct in Edition A, and let the students think actively. Whereas Edition B involves more contents, and the relation between contents isn't tight.

Correspondently, all the interviewed teachers raise doubts for the introduction of angle of inclination in Edition B. They also think that the introduction is more or less sudden and which explains about the relation between the slope and the angle of inclination abstractly.

In teaching corresponding contents, all the interviewed teachers give the added explanation based on the textbook at different degrees. Some teachers directly taught their students of the formula $k = \tan \alpha$ and asked them to grasp several tangents of acute angles. More teachers let students observe, think, and conclude the relation between slope and angle by drawing line graphs of different slopes, but the observed conclusion is limited to the textbook. The teachers also said that "I can not explain in details if I teach completely according to the textbook of Edition B. But if I don't enlarge, students have difficulty in doing the corresponding exercises."

3. The Guide of Two Points' Coordination Formula

Another method describing the slope quantitatively is using two points' coordinates on the line. Then, let me see in Edition A and B how to describe it.

Having defined the slope with the tangent of the angle of inclination, Edition A discusses how to use two points' coordinates to present the slope. After discussing four cases that a line can be in rectangular coordinate system and is paralleling or coinciding with x-axis, Edition A gives the formula

$$k = \frac{y_2 - y_1}{x_2 - x_1} (x_1 \neq x_2).$$

The deduction of the formula in Edition A is slightly tedious as students' lack of enough knowledge. The formula as a tool in Edition A is used to calculate the slope.

But in Edition B, the formula is used in the definition of the slope. And the deduction of the formula in Edition B is much easier, and students could make clear of the content as long as they understand the definition of linear equation of two variables and the technique of solving it. In Edition B the slope is denoted by the quotient, and that is preparing for teaching derivative which keeps accordance with the compiling idea of quotient. Also, the idea is embodied in all Edition B.

In the practice, all of the teachers let students deduce the formulas, and most students could deduce this formula by themselves. This is why the teachers think that Edition B is easier and appropriate for students to do self-educated.

4. Instances in the Guide

Many mathematical concepts have very high degrees of abstraction. In order to grasp the concepts, the students usually needed to ask instances in the daily life or other concrete things. The concept of slope is abstract that instances are demanded for understanding.

Through analogy, Edition A utilizes gradient as an instance to make students understand the rationality of introducing tangent of angle of inclination. Students may query that when the angle of inclination is an obtuse angle, how to calculate its tangent?

Edition A uses a formula "If α is an acute angle, then $\tan(180^\circ - \alpha) = -\tan \alpha$ holds" to solve this problem. The example of slope just provided a circumstance where the angle is an acute angle that students can't understand the concept overall. Teachers should utilize more appropriate instances to guide students.

There is no concrete instance in Edition B at all. The deduction is only limited to explain in abstract mathematical signs. Students' comprehension inclines to only grasp the application within mathematical problems and they can't apply them into daily life. Besides, it is difficult for students reminding slope as when they solve problems about

location of line and other curves.

5. Examples and Exercises

Example and exercise are the key elements in mathematics textbook. They directly affect the studying quality of students. Here, we compare the examples and exercises with amount, type and knowledge function.

① Amount

In Edition A the exercises are divided into three types: practices, exercises (divided into A and B set), and reviews (also divided into A and B set). In Edition B the exercises consist in: practices (divided into A and B), exercises (divided into A and B), consolidation and enhancement, calculation on the computer, test and valuation. Set A is easier than Set B which is more synthetic and flexible. The arrangement is to fit for students of different levels.

If we put each type exercises (practice, each set of exercises, reviews and consolidations and enhancement) that concerning the slope concept together, there are 37 topics in Edition A and 42 topics in Edition B. The examples in two Editions are both 2.

We find that the topics of exercises in Edition B are more than A.

② Type

Mathematics exercises can be classified according to the different classification. Dai Zaiping wrote in *Mathematics Exercises Theories* that “geometry problems can be divided into four kinds according to the form: proving problems, calculating problems, graphing problems and tracking problems” (Dai, 1991).

We carry on a comparison according to this category.

Example aspect. In Edition A example 1 is a calculation problem, and example 2 is to make a diagram. In Edition B example 1 is calculation, and example 2 is drawing graphs. Obviously, the examples in two Editions are of the same type and amount.

Exercises aspect. Among 37 topics of Edition A, 31 topics are calculation which account for 84% of the whole amount, 5 topics are proving problems which account for 14%, only 1 topic requires drawing graphs that account for 2%. As for Edition B, there are 40 topics of calculation which account for 95.5%, 2 graphing problems, 0.05%, but there is no proving problem in it. And the calculation problems almost account for the whole exercises. Differences are so evident that the proving problems in Edition A are of 14% whereas Edition B is none. Both textbooks involve graphing problems but that their proportion is not very large.

Thus, it can be seen that Edition A and B are both intended to foster students’

calculating and graphing ability.

③ *Knowledge Function*

The function of exercises can be divided into: knowledge function, education function, evaluation function (Dai, 1991) and development function (Mao, 1994). Knowledge function is that “making students acquire systematic mathematics knowledge and necessary mathematics technique” (Dai, 1991). The exercises in the textbook should have these functions, particularly knowledge function.

Example 1 in Edition A and Edition B are both calculation exercises which requesting students to calculate slope. However, in addition to computing the slopes of three lines, Edition A also requests “Judging the angles of inclination of these lines are acute angles or obtuse ones.” Compared with Edition A, Example 1 in Edition B requests only for calculating the slope of one line that there is no embodiment of the knowledge of relation between slope and angle of inclination in the teaching material.

Although the second examples of these two textbooks are both to draw graphs of lines, differences are easy to find. Edition A’s request is to draw “a line whose slope is respectively 1, -1 , 2, and -3 ”, and Edition B is “drawing the graph of the equation of $3x + 6y - 8 = 0$ ”. The purpose of Edition A is to let students do the exercise using the formula for two points’ coordinates adversely. Edition B considers making students comprehend better of the concept of lines of equations. And Edition A seems to be more difficult for students.

Through the interview with seven teachers we know only two teachers completely referred to and demonstrated the examples of the textbook on class, one teacher only taught example 2 and students exercised Example 1, one took these two examples for students’ exercises, and the other three gave different examples about calculation and graphing choose from other exercise book.

For example, the authors observed the lesson of one of the three teachers, and the examples on it are as following:

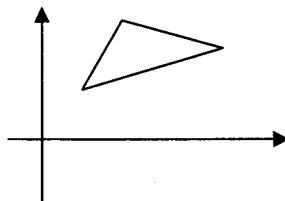


Figure 1. *Mathematical Tasks Framework*

Example 1. Let students give quantities to the points’ coordinates themselves, then calculate the slope of the lines which are correspondent to the edges of the triangle (see

Figure 1).

Example 2. Calculate the slopes and the angles of the lines.

- ① $P_1(2, 1), P_2(3, 1)$
- ② $P_1(1, 2), P_2(1, 3)$
- ③ $P_1(2, 3), P_2(3, 4)$

IV. CONCLUSION

The article firstly makes a detailed description to line slope in Editions A and B, then compares them from four aspects of concept guiding pattern, the guide of angle of inclination, the guide of two points' coordination formula, the instances in the guide and the examples and exercises in the textbooks. In the aspect of introducing pattern, Edition A defines the slope by the angle of inclination. Edition B utilizes the concept of equation of line to deduct the slope. In the aspect of introducing angle of inclination, Edition A appears natural, whereas Edition B is more or less not smooth. The instances of slope in Edition A are concrete, and it in Edition B is rather abstract. As for the two points' coordinate formula, Edition B explains clearer and easier than Edition A. The amount of examples and exercises in Edition B is more than Edition A, but they are both aimed to foster students' ability of calculating and graphing, and the exercise of Edition A seems difficult than Edition B.

This paper only carries on a comparison between Edition A and B compiled by People's Education Publishing House with the line slope, instances, examples and exercises. We just did the interview with eleven teachers from four senior high schools in Dalian City from Liaoning Province about the usage with Edition B. In order to understanding actual reform situation among senior high school mathematics teaching in China well, we should further compare with these two Editions or other editions of textbooks systemically, by another comparison methods(Hong & Cha., 2005). And based on the comparison with various textbooks, through observing the classroom and surveying the teachers' beliefs about mathematics and mathematics teaching-learning, we would further study how and why the teachers implement the new curriculum in the practice.

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