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Insight into the Development of Mathematics Teaching: A Comparative Study of Two Videotaped Lessons[1]

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From the perspective of phenomenongraphy (根据登的象式), two videotaped mathematics lessons were observed, compared and analyzed, and the features of those times were extracted. The quantitative and qualitative analysis of the global structure and the questioning in the lessons etc. s howed that in teaching behaviors there were some essential differences reflecting the impact of new beliefs and principles of teaching. Evidences also show that excellent teachers did keep and carry on the good tradition — teaching for basic knowledge and basic skills were seen as the most important thing in mathematics teaching in China

Keywords: c omparative study, qualitative research, teaching strategy, teaching behavior
ZDM Classification: C7 0, D40
MSC2000 Classification: 9 7D40

INTRODUCTION

Practical teaching is always a matter of the concern of mathematics educators. Many researches had pointed out that teacher's teaching strategies and behaviors imply his/her beliefs (Stigler & Hiebert, 1999; Jacobs & Morita, 2002), and also reflect the educational

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features and characteristics of the time. Resea rchers may use many ways to explore such beliefs and characteristics from different perspectives. Th e research reported here is an investigation of practical classroom teaching in Shanghai from a special angle.

OBJECT AND AIM

The object of the research was two videotaped lessons of plane geometry. The se lessons were videotaped at two key-point lower high school in Shanghai. Mr. A's lesson is in the late 1980s while and Mr. B's one is in the late 1990s. These lessons were used originally as cases of typical excellent lessons for in-service teacher training. Th e topics of these two lessons are both property theorem of mid-point connector of triangle. In this research, we tried to observe and analyze deeply these two lessons with a ten-year span. The aim was to look for and compare the differences and similarities of behaviors between these teachers. Then we would identify the changes and something unchanged, and insight into teacher's beliefs underlying the teaching. Becau se many new theories and principles were introduced since 1980s, we hoped to find the characteristics in the specific time, trace the trajectory, identify developing tendency and understand the achievements of the reform of mathematics education.

METHODOLOGY

The research was designed as case comparison, focused on the two teaching cases with the perspective of comparison. We first looked for the differences and similarities between two lessons with the qualitative and quantitative data, and then tried to reveal internal aspects, for examples, the tendency of the development of mathematics education, the changes of teachers' beliefs. In this research, we were "not to celebrate the uniqueness and oddity of a case", but studied these two lessons with same topic through qualitative research method "to explore the richness of a particular that may serve as an exemplar of something more general", "using a particular and concrete instance to suggest, evoke, and illustrate, if not describe, the general case", "to illuminate the general through the particular" (Ernest, 1997, p. 34). W e wished to present the basic characteristics and styles from specific background and to refine something for reflection.

The comparability of these two lessons was under consideration. In these lessons, the topics were the same, the content of the textbook similar. S tudents in both classes were eight graders in excellent schools in Shanghai, and the teachers were both leading teachers at that time. Actu ally, the teaching style of the two lessons looked at first glance in traditional way of teaching at those years.

The way of research was indirect observations. In the research process, we focused on the analysis from global to details, from teaching episodes to quantitative data, so that the difference and similarities in these two cases could be extracted and the implications for the reform of mathematics education refined. The global structures of the classroom teaching were explored to learn the frame and sequence of these lessons. Teachers' detailed behaviors were also compared. In according with the triangulation principle of research methodology, we hoped to look for some date to be analyzed quantitatively. So besides the teaching behaviors and episodes as qualitative data, three aspects were compared quantitatively. The time length of teaching stages, the time for student's autonomous activity, and teacher's questioning as a specific teaching strategy, were scrutinized and analyzed statistically.

Stigler, Fernandez & Yoshida (1996) developed some quantitative ways for research on classroom teaching, especially for videotaped lesson. The book "the Teaching Gap" (Stigler & Hiebert, 1999, pp. 30–31; see also Brown, 2002) provided some clues for analyzing and comparing lessons. Two ways for quantitative analysis were used in this research. One kind was time interval of every stage of the whole lessons and time interval of students' autonomous activity, which was for exploring the implications of the teachers' arrangement. The other is the comparison of teachers' questioning. W ith the reference of Wragg (1999, p. 41) introduced in "*Flanders' Interaction Analysis Categories*" (FIAC)[2], we classified all teachers' questionings into six types:

managerial,
mechanical,
remembering,
explaining,
reasoning, and
critical questioning.

With these types, all questionings were coded into some types and the frequency of every type was compared as the evidence for qualitative evaluation.

RESULTS

Teaching behaviors

Generally, we had strong expressions that the two lessons had a lot of same basic

[2] http:// www.kfmaas.de/lspr_fia.html

elements and similar teaching strategies: Same topic; similar teaching content, even some examples and problems solved were the same. Both teachers used structured teaching strategy with similar teaching stages: definition introduction, theorem introduction, theorem proof, problem solving and summary. Also we had seen the similar ways arranged for student's activity: students proving theorem in many ways, teachers questioning and students answering, student's experiments, discussions, problem solving and reading textbook, etc. Th ese teachers concentrated very much on the proof and problem solving, which could be seen as the core in the lessons. It was obviously that two teachers strongly controlled the global progress and every stage of the lessons.

At the same time, we also found some differences of the teaching behaviors (Table 1), which could be taken as essential indications of different beliefs teachers held.

Table 1. Compariso n of teaching behaviors

Teaching behaviors	Mr. A	Mr. B
Definition introduction	At the beginning of the lesson	After the proof of the Theorem
Proving	Just proved	From conjecture to prove
Situated problem	As application of theorem	As the introduction of the Theorem
Primary review the theorem	Recited loudly	Looked at textbook
Rephrasing the theorem	Word by word same as on textbook	Right but flexible
"How many MPC ... "	Told students	Hint
The difference to the median	Told students	Hint
Writing on chalkboard	Formal and detailed	Outline
Didactics	In-depth description	Less description & more Activities

They might imply the influences and effects of the new theories and ideas in the progress of mathematics education reform in general, and reflected practical and intrinsic changes within the span of the ten years. Here we only mentioned some aspects as examples.

1. Mr. B in his lesson in late 1990s let his students make a conjecture and mentioned the necessity of the deductive proof. So he emphasized the relation between conjecture and proof before considering proof and fostered students' understanding of the function of mathematics proof comprehensively. However Mr. A in his teaching only let his students think about the proof directly, even did not ask why the proof is needed.

2. Both teachers used a common situated problem, in which the distance between two points was hoped to be found but these points could not be measured directly since they were blocked by hill, pond or building. Mr . A used it as the application of the theorem: Theorem first and then the application. Bu t Mr. B dealt with the problem as the introduction of the conjecture. H e would pass such idea on to his students: Abstract mathematical theory comes from real situation.
3. After the theorem was proved, both teachers asked students to describe the theorem with their own words. In Mr. A's lesson, a girl stood up to say: "The midpoint connector of a triangle is parallel to the third side and is the half of the third side." Then Mr. A commented: You have mentioned the words "the third side" two times. Would you give us a more succinct sentence? Then the girl revised her sentence: "The midpoint connector of a triangle is parallel to the third side and is the half of IT." Mr . A felt satisfactory and might mean that a rigorous description have to be stated as the same as one in the textbook. It was coincidently that Mr. B confronted with the same case too. A boy in his lesson said as the same as the girl did above. Mr. B just commented that it was OK and even recited this statement again. It mi ght be understood that he did not think the different words "of the third side" or "of it" in the statement is crucial. S tudent's statement should be confirmed positively, provided he/she really knew what they pointed out was right in the context.

Time distribution

Although the general structures and some of the basic elements in the two classes were similar, we hope to explore the inner and substantive differences and similarities behind the phenomena. So we designed to collecting quantitative data in some aspects to refine meaningful results.

One of the aspects was the time distribution for different stages (Tables 2 and 3). Such data of two lessons compared might show us some interesting result.

It can be seen from Figure 1 that these two teachers behaved in different ways, say, they paid different time obviously for stages of definition introduction, theorem introduction and summary, while there was no great difference in the stages of theorem proving, and problem solving.

In the stage of definition introduction, Mr. A described the definition carefully and explained it thoroughly while Mr. A would not pay much time for it. Mr. B might believe that his excellent student could master such concept easily, and he would pay much time for theorem introduction with a situated problem.

If we saw the two stages — theorem proof and problem solving — as the center or core of lesson, we found the sum of the time is nearly the same (75.7% *vs.* 73.2%) (*cf.*

Figure 2). What did it mean?

Table 2. Mr. A's teaching stages

Total length：46min 41sec		
Teaching stage	Length	%
Definition introduction	5min 25sec	11.6
Theorem introduction	3min 18sec	7.1
Theorem proof	12min 58sec	27.8
Problem solving	22min 22sec	47.9
Summary	2min 38sec	5.6

Table 3. Mr. B's teaching stages

Total length：52min 17sec		
Teaching stage	Length	%
Definition introduction	1min 18sec	2.5
Theorem introduction	11min 43sec	22.4
Theorem proof	16min 45sec	32.0
Problem solving	21min 32sec	41.2
Summary	59sec	1.9

It would be concluded that such data reflected the fundamental feature of mathematics teaching belief in China. Both best teachers saw such stages as vital and believed they should strengthen students' learning of basic knowledge and basic skills in mathematics. In the ten years progression of mathematics teaching, such feature was kept and became a statically vital element. With such belief, practical teachings bright up a great number of excellent students with solid foundation in mathematics.

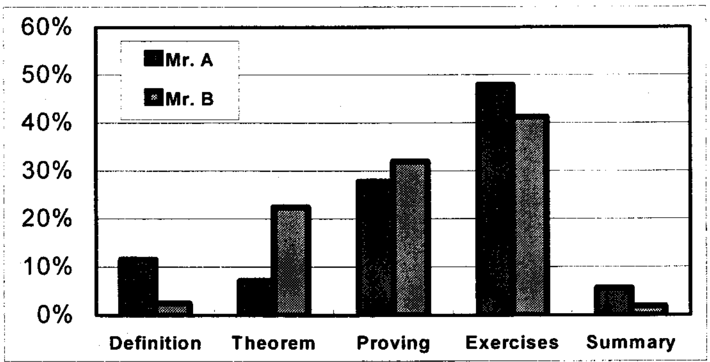

Figure 1. Comparison of teaching stages

Questionings

Questioning in classroom teaching was an important way for interaction between teacher and students. It was often thought as an effective strategy for encouraging students involving and promoting concept construction. Actually, the nature and function of teacher's questioning was worthy to insight into profoundly. Comparative study provides a special perspective.

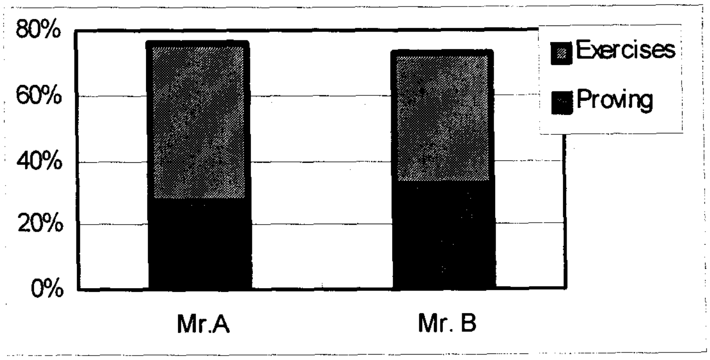

Figure 2. Comparison of the core of lessons

1. The classification of questionings

Based on the feature of mathematics teaching and practical situation in China, the questionings in the lessons was sorted into six types.

A. Managerial questioning: Hoped or encouraged students answering or explaining such as: "who has other method to share?" but teacher's question did not relevant to the mathematics content.
B. Mechanist questioning: simply asked question such as "Is it right?" or question which response is obvious: "How many auxiliary line are there in the figure now?"
C. Remembering questioning: Awaken the simple knowledge memorized and nearly had no time to think: How did we prove it in last lesson?
D. Explanative questioning: The answer to the question needed describe or discuss something, such as "What is the base side in a triangle? And what is the third side?"
E. Reasoning questioning: The answer required logical reasoning step by step: "Why do you draw this auxiliary line?"
F. Critical questioning: Its answer needed students think reflectively, or change the perspective of thinking.

Type A was not relevant to detailed mathematics content. Other five types were all questionings directly related to mathematics knowledge.

2. The number of questionings

We had seen that the great difference of the total number of two teachers' questionings. Mr. A asked questions much more than Mr. B did (93:46). It seemed that Mr. A expected to control the pace of the teaching with a lot of questions and guide students' thinking step by step. But the answers seemed spontaneous since students often had only some seconds for thinking. By contrast, Mr. B did not go this way. He would let students have more time to think deeply and answer questions comprehensively. This was more beneficial for develop mathematics thinking ability. We believed this should be confirmed as one of the evolutions in the teaching development in the ten years.

Table 4. M r. A's questioning analysis

Number of questionings: 93		
Type	Frequency	%
A. Managerial	16	17.2
B. Mechanical	14	15.0
C. Remembering	12	12.9
D. Explanative	38	40.9
E. Reasoning	13	14.0
F. Criticizing	0	0

Table 5. M r. B's questioning analysis

Number of questionings: 46		
Teaching stage	Frequency	%
A. Managerial	18	39.1
B. Mechanical	4	8.7
C. Remembering	2	4.3
D. Explanative	13	28.3
E. Reasoning	9	19.6
F. Criticizing	0	0

3. The nature of questioning

It is found in Figure 3 that there were obvious differences in the following three types: managerial, mechanical and remembering questioning (17.2% *vs.* 39.1%, 15.0% *vs.* 8.7 %, and 12.9% *vs.* 4.3% respectively). If the mechanical and remembering questioning was merged as simple questioning, there is Figure 4 for comparison. We explained the difference that Mr. B in the late 1990s had more managerial questioning and less simple questioning. He might hope manage his teaching and guild students involving with more managerial questioning and posed less simple questioning which seemed more likely as the behavior response.

We noticed that there was not great difference in the explanative and reasoning questioning (40.9% vs. 28.3% and 14.0% vs. 19.6% respectively). For further compare-son, these two classes were combined to the category of complicated questioning in contrast with simple questioning, and the result was showed in Figure 5. Mr. A's complicated questioning was 54.9% and Mr. B's was 47.9%. So, when they used the strategy of questioning, both of them did have a strong desire to manage student's higher-order thinking with the help of complicated questioning. It should be confirmed too as one of the static tradition and a specific technique in China to foster solid "two basics." Basic knowledge and basic skill in mathematics were always emphasized definitely.

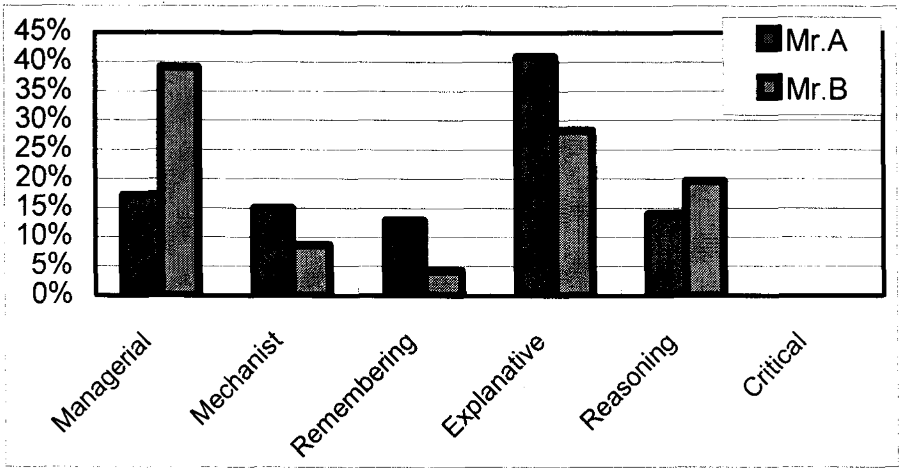

Figure 3. Questioning comparison

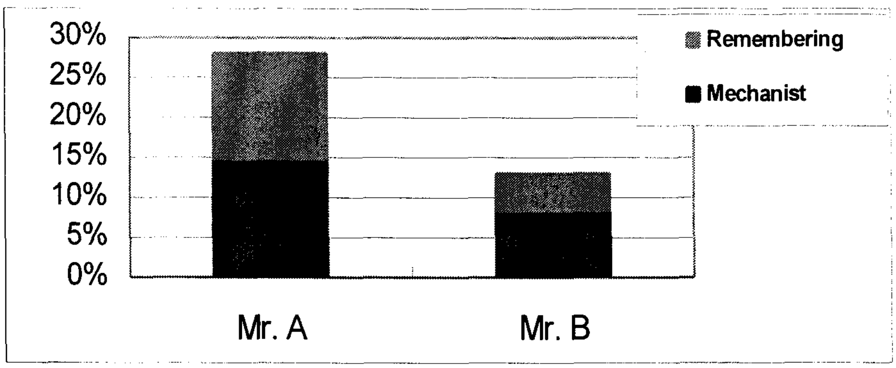

Figure 4. Comparison of simple questioning

Critical thinking is usually related to the conjecture, exploration and creativity. However, we had intuitive impression that both teachers had no requirement for student's critical thinking since there was no any such questioning at all (see Figure 3). So it was sorry that these two teachers had very weak belief in such aspect.

Students' autonomous activity

It was also designed to collect data of total time used for students' autonomous activity for quantitative comparison. So-called autonomous activity here was students' independent activity which time lasted more than 3 minutes interval, including experiment, group discussion, reading textbook, answering questions, proving theorem etc. In Lesson A, it was 26.3% (12min17sec), and 53.5% (28min) in lesson B. We had mentioned early that intuitive impression told us that the teaching process in both lessons were controlled globally and tightly by teachers, but the above data also showed us there was really some substantive and subtle difference in practical arrangements in these two lessons, in which students had different positions in teachers' mind.

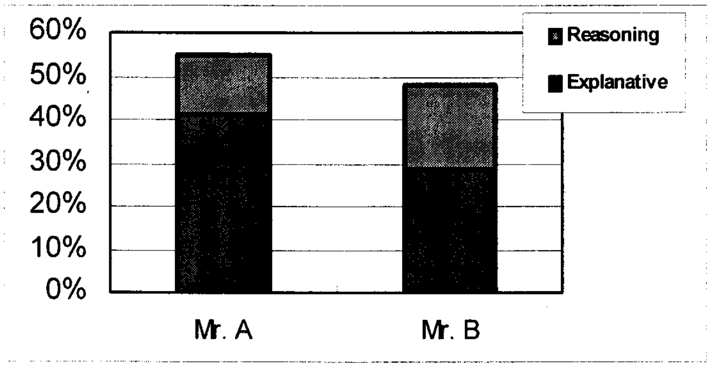

Figure 5. Comparison of complicated questioning

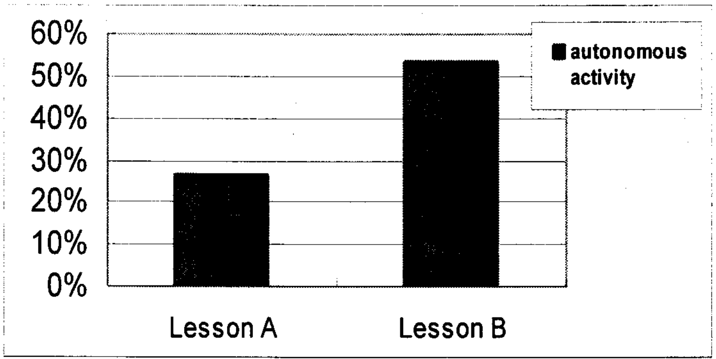

Figure 6. Comparison of autonomous activity

CONCLUSION

The last twenty years in last century was a special period in China. When the open-door policy was taken into practice, mathematics education community began its new development. Generally, Chinese mathematics educators and mathematics teachers experienced a reform process. We summarized from qualitative and quantitative data collected and findings in this study into two essential aspects: On one hand, a more open and flexible environment and atmosphere in classroom was really created in late 1990s. On the other hand, some positive and negative traditions were also inherited.

Analyzing from the perspective of longitudinal development, we should connect the teacher's behaviors and teaching episodes to the underlying beliefs the teachers might hold. Ac tually, constructivism theory had not been introduced in 1980s. The guiding principal of teaching was heuristics. Teachers made great efforts to practice the principle with their own understanding. In-depth description embodied the principle and formed comparatively conservative teaching style. In 1990s, new educational theories such as cognitive psychology and constructivist philosophy came in. Teachers realized that the most important thing for young generation was their ability of active learning. Students-center principle was accepted so that students had much room for autonomous learning. However, the principle was just accepted at some extent. It was just combined the traditional "teacher dominant" belief in China and formed a particular guide line called "teacher-guided AND student-centered," which is still confirmed now. The findings in this study really reflected these changes and the particular teaching style, which was seen as more effective and more reasonable.

At the same time, something which did not change in these lessons also caught our attention. Teaching for solid understanding of concepts and methods were observed in both lessons. Teachers and students made their best endeavors for talking mathematics concept and terms fluently, calculating accurately and reasoning precisely, which were seen as vital foundations for students' performance well academically. So positively, the tradition of mathematics teaching for two basics: basic knowledge and basic skills, was obviously kept. Bu t negatively, two teachers in the lessons did not take critical thinking as an essential aspect of teaching. Th is reflected the crucial weakness in mathematics education at that time.

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