Application of Spectral Properties of Basic Splines in Problems of Processing of Multivariate Signals

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ABSTRACT

The paper is devoted to problem of spline approximation. A new method of nodes location for curves and surfaces computer construction in multidimensional spaces by means of B-splines is presented. The criteria are which links a square-mean error caused by high frequency spline distortions and approximation intervals is determined and necessary theorem is proved. In this method use a theory of entire multidimensional spectra and may be extended for the spaces of three, four and more variables. Future work: application area such as digital contents like animation, game graphic.

Keywords: *B-spline, Multidimensional, Curve, Functions, Variables.*

1. INTRODUCTION

It is true that multidimensional B-spline theory is extremely effective in computer graphics [1, 2]. They are often used for design of curves and surfaces in imagination processes. For example, a function f(x,y,z) of three variables is described by expression:

$$f(x,y,z) \cong \sum_{i=-1}^{n_1+1} \sum_{k=-1}^{n_2+1} \sum_{l=-1}^{n_3+1} b_{ikl} B_{mi}(x) \times$$

$$\times B_{mk}(y) \times B_{ml}(z)$$

where b_{ikl} are coefficients of multidimensional approximation; m is B-spline power. One of the main problems for B-spline forms construction is the nodes points arrangement for exact space forms designing. New method which uses B-spline spectral singularities gives optimization way for node distribution.

2. MAJOR THEOREM

Let's prove following theorem.

Theorem: A square-mean error from multidimensional B-spline approximation is upper limited by value proportional to square root from interval product:

$$\varepsilon < C\sqrt{h_x h_y h_z}, \quad (1)$$

where C-coefficient depends on spline power m; h_x, h_y, h_z are node intervals.

3. ONE-DIMEMSIONAL B-SPLINES

At first stage let's establish this statement for a function of one variable that is given on [a.b] interval. A function f(x) can be represented as linear sum [1,4-,6]:

$$f(x) \cong \sum_{i=-1}^{n_1+1} b_i B_{m,i}(x),$$

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where b_k are B-spline coefficients

Initial B-spline spectrum is described by formula:

$$|F_B(\omega_x)| = B(0)h_x\left|\frac{\sin(\omega_x h_x/2)}{\omega_x h_x/2}\right|^{m+1},$$

where B(0) is B-spline magnitude;

$h_x = \pi/\omega_{cx}$;

ω_{cx} - cut-off frequency.

The B-spline consequence has a spectrum:

$$|F_{\Sigma B}(\omega)| = |F_B(\omega_x)|\left|\sum_{i=0}^{n_1} b_i \exp(-ji\omega_x(m+1))\right|. \quad (2)$$

According to Parceval equality we have:

$$\int_a^b f^2(x)dx = \frac{1}{\pi}\int_0^\infty F_f^2(\omega_x)d\omega_x \cong \frac{1}{\pi}\int_0^\infty F_{\Sigma B}^2(\omega_x)d\omega_x.$$

The last integral can be disjointed by two ones:

$$\int_0^\infty F_{\Sigma B}^2(\omega)d\omega = \int_0^{\omega_c} F_{\Sigma B}^2(\omega)d\omega + \int_{\omega_c}^\infty F_{\Sigma B}^2(\omega)d\omega \cdot \quad (3)$$

We can sign a high-frequency integral in Eq. (3) divided by π as $\varepsilon^2_\omega = D_\omega$ and name it as dispersion from high-frequency components of B-spline set. The last multiplier is a module of sum in Eq. (2) which includes a sum of complex exponents does not exceed the level:

$$C_1 = \left|\sum_{i=0}^{n} b_i\right|.$$

We can write following expression for square-mean error if take into account the well-known integral inequalities theorem [10,14].

$$\varepsilon_\omega^2 < C_1^2 h_X^2 \int_{\omega_C}^\infty \left(\frac{\sin(\omega h_X/2)}{\omega h_X/2}\right)^{2m+2} d\omega <$$

$$< C_1^2 h_X^2 \int_{\omega_C}^\infty \left(\frac{2}{\omega h_X}\right)^{2m+2} d\omega = \frac{2^{2m+2}}{(2m+1)\pi^{2m+1}} h_X$$

Therefore:

$$\varepsilon_\omega < \frac{2^{m+1}C_1}{\pi^m\sqrt{(2m+1)\pi}}\sqrt{h_x}. \quad (4)$$

4. TWO-DIMENSIONAL B-SPLINES

Now let's consider a function of two variables which describes any surface

$$f(x,y) \cong \sum_{i=-1}^{n_1+1}\sum_{k=-1}^{n_2+1} b_{ik}B_{m,i}(x)B_{m,k}(y).$$

Two-dimensional B-spline Fourier transform of the right part is separated by one-dimensional multipliers [7,10,11]. So initial B-spline spectrum is described by expression:

$$|F_B(\omega_x,\omega_y)| = B_x(0)B_y(0)h_xh_y \times$$

$$\times \left|\frac{\sin(\omega_x h_x/2)}{\omega_x h_x/2}\right|^{m+1}\left|\frac{\sin(\omega_y h_y/2)}{\omega_y h_y/2}\right|^{m+1}$$

Two-dimensional orthogonal B-spline structure has a spectrum:

$$F_{\Sigma B}(\omega_x,\omega_y) = F_B(\omega_x\omega_y) \times$$

$$\times \left|\sum_{i=0}^{n_1}\sum_{k=0}^{n_2} b_{ik}\exp(-ji\omega_x(m+1)h_x) \times\right.$$

$$\left.\times \exp(-jk\omega_x(m+1)h_x)\right| \quad (5)$$

Now we can conclude from Parseval equality

$$\int_a^b\int_c^d f^2(x,y)dxdy = \frac{1}{\pi^2}\int_0^\infty\int_0^\infty F_f^2(\omega_x,\omega_y)d\omega_x d\omega_y =$$

$$= \frac{1}{\pi^2}\int_0^\infty\int_0^\infty F_{\Sigma B}^2(\omega_x,\omega_y)d\omega_x d\omega_y, \quad (6)$$

where ω_x, ω_y - space frequencies.

The last double integral in Eq. (6) may be transformed to a sum:

$$\int_0^\infty\int_0^\infty F_{\Sigma B}^2(\omega_x,\omega_y)d\omega_x d\omega_y =$$

$$= \int_0^{\omega_{cx}}\int_0^{\omega_{cy}} F_{\Sigma B}^2(\omega_x,\omega_y)d\omega_x d\omega_y + \quad (7)$$

$$+ \int_{\omega_{cx}}^\infty\int_{\omega_{cy}}^\infty F_{\Sigma B}^2(\omega_x,\omega_y)d\omega_x d\omega_y$$

The last member in Eq. (7) is dispersion of high frequency distortions $D_\omega = \varepsilon^2_\omega$ that is multiplied by π^2. We can write:

$$D_\omega = \varepsilon_\omega^2 = \frac{1}{\pi^2}\int_{\omega_{cx}}^\infty\int_{\omega_{cy}}^\infty F_{\Sigma B}^2(\omega_x,\omega_y)d\omega_x d\omega_y \cdot$$

As one can see that the module of exponential sum in expression (4) not exceed a value:

$$C_2 = \left|\sum_{i=0}^{n_1}\sum_{k=0}^{n_2} b_{ik}\right|.$$

We want to underline that spectra $F_B(\omega_x)$ and $F_B(\omega_y)$ are separated and therefore following expression takes place:

$$D_\omega < C_2^2 h_x^2 h_y^2 \int_{\omega_{cx}}^\infty \left(\frac{\sin(\omega_x h_x/2)}{\omega_x h_x/2}\right)^{2m+2} d\omega_x \times$$

$$\times \int_{\omega_{cy}}^\infty \left(\frac{\sin(\omega_{yx} h_y/2)}{\omega_x h_{yx}/2}\right)^{2m+2} d\omega_y$$

According to above mentioned integral inequality theorem we see:

$$D_{\omega} < C_2^2 h_x^2 h_y^2 \int_{\omega_{cx}}^{\omega} \left(\frac{2}{\omega_x h_x} \right)^{2m+2} d\omega_x \times$$

$$\times \int_{\omega_{cy}}^{\infty} \left(\frac{2}{\omega_y h_y} \right)^{2m+2} d\omega_y$$

Under designations
$h_x = \pi/\omega_{cx}$, $h_y = \pi/\omega_{cy}$.

From here we conclude that:

$$\varepsilon_{\omega}^2 < \frac{4^{m+1} C_2^2}{(2m+1)\pi^{2m+1}} h_x h_y$$

and:

$$\varepsilon_{\omega} < \frac{2^{m+1} C_2}{\pi^m \sqrt{\pi(2m+1)}} \sqrt{h_x h_y} \qquad (8)$$

5. EXPERIMENTAL RESULTS

As a concrete example we shall consider the table of bidimentional measurements of increments of an induction of an electromagnetic field in horizontal and vertical directions in relation to a conditional zero level. Readout are taken during aeromagnetic measurements on the area 15x15 km through distances

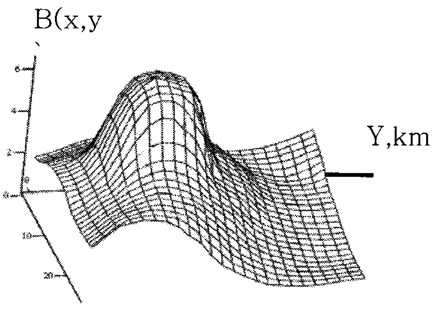

Fig.1. Example of a surface of a field of an electromagnetic induction above a surface of the ground

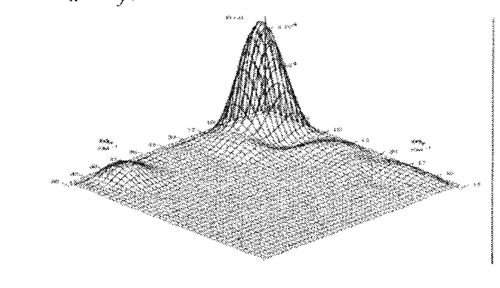

Fig.2.Spatial peak spectrum of sequence bilinear In-splines, an electromagnetic induction interpolating a field.

equal 1 km on each of horizontal coordinates x and y. Results of approximation of a field are resulted by bidimentional basic splines on Fig. 1. The power spectrum of this set B-splines is resulted on Fig. 2.

Computer modelling process of interpolation of an electromagnetic field by system of basic B-splines is lead. Spectral energy in space of frequencies up to and from up to represents the energy of a mistake caused by high-frequency components of approximation of a surface weeding by splines. At the values of steps of approximation equal 1 km on each of axes, this energy makes 1,4 % from full energy of a signal. At doubling each of steps the share of this energy increases up to 1,7

6. CONCLUSION

It is clear that using B-splines and their spectra as multidimensional function with separated components leads us to fact that proving of theorem may be easy extended to spaces of three and more arguments. The method is created for equidistant node location case. If nodes are not equidistant the problem of its choice is solved more effectively in comparison with the function finite spectrum theory methods [3]. Related animation such as character: background graphic data [8,9,12,13].

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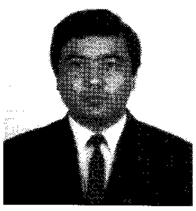

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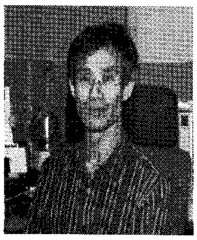

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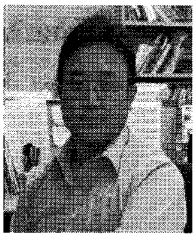

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