신호 복원을 위한 웨이브렛기반 알고리즘

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Wavelet-based Algorithm for Signal Reconstruction

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요 약

신호를 처리하는 과정에서 여러 가지 원인에 의해 잡음이 발생하고 있으며, 이에 따라 전송 데이터에서 오류가 발생하거나, 영상 및 음성 데이터의 인지도를 저하시킨다. 따라서 이러한 잡음들을 제거하여 신호를 복원하기 위한 다양한 방법들이 연구되고 있다. 현재, 시간-주파수 국부성을 갖고, 다중해상도 해석이 가능한 웨이브렛 변환이 많은 공학 분야에서 응용되고 있으며, 잡음 제거를 위해 문턱값과 상관관계를 이용한 방법 등이 제시되었다. 그러나 기존의 방법들은 여전히 많은 잡음들을 에지로서 판단하며, AWGN과 임펄스 잡음을 동시에 제거하기 위한 방법을 제공하지 않는다. 따라서 본 논문에서는 잡음에 의해 훼손된 신호를 복원하기 위하여, 새로운 웨이브렛기반 알고리즘을 제시하였으며, 기존의 방법들과 비교하였다.

ABSTRACT

Noise is generated by several causes, when signal is processed. Hence, it generates error in the process of data transmission and decreases recognition ratio of image and speech data. Therefore, after eliminating those noises, a variety of methods for reconstructing the signal have been researched. Recently, wavelet transform which has time-frequency localization and is possible for multiresolution analysis is applied to many fields of technology. Then threshold- and correlation-based methods are proposed for removing noise. But, conventional methods accept a lot of noise as an edge and are impossible to remove the additive white Gaussian noise (AWGN) and the impulse noise at the same time. Therefore, in this paper we proposed new wavelet-based algorithm for reconstructing degraded signal by noise and compared it with conventional methods.

키워드
reconstruction, wavelet transform, time-frequency localization, multiresolution analysis

Ⅰ. INTRODUCTION

Owing to the rapid development of high speed information and communication technology such as the computer and the Internet, the multimedia communication service is commercialized at home and abroad.

Accordingly, acquisition, transmission, and storage of multimedia data such as image and speech have become easy. But, noise still occurs due to various reasons during processing signals. In addition, this noise leads to an error during the transmission of data, or degrades the perception for speech and image data. Therefore, researches on various

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methods for removing noise so as to reconstruct signal have been actively performed.

So far, a method using the Fourier transform is widely used for removing noise. The Fourier transform is an important tool to analyze the signal and system, but it has a limited application because it does not have the time information for a certain frequency component.

Therefore, in order to overcome such a problem, the wavelet transform capable of the multi-resolution analysis is applied to many engineering fields. In order to reconstruct signal in the additive white Gaussian noise (AWGN) environment, methods such as one using the threshold and spatial correlation have been introduced. Methods using threshold suggested as 'wavelet shrinkage' by Donoho and Johnstone are currently the most commonly used and applied to remove AWGN [1],[2]. In addition, in case of methods using the spatial correlation, the correlation between wavelet coefficients in adjacent scale is used as a determining basis to separate edges of signal and noise [3],[4]. And, in the impulse noise environment, *B*-wavelet is proposed for a reconstructing method using wavelet [5]. But, wavelet-based signal reconstruction methods investigated so far have reflected only statistical features for noise. Accordingly, many small edges in a signal may be determined as noise, or relatively large noise may be determined as edge. In addition, the conventional methods cannot provide a method for simultaneously removing AWGN and the impulse noise.

Therefore, in order to reconstruct signal in the complex noise environment, a new wavelet-based algorithm is proposed in this paper, so that the separation performance between noise and edge component of the signal is improved. The results are compared with the conventional methods by means of the simulation.

Ⅱ. WAVELET TRANSFORM

The Fourier transform for a certain real or complex-value continuous-time function, $\psi(t)$ can be defined as following equation (1). In such a case, if $\Psi(\omega)$ satisfies equation (2), the function, $\psi(t)$ is called the mother wavelet.

$$\Psi(\omega) = \int_{-\infty}^{\infty} \psi(t)\, e^{-j\omega t}\, dt \tag{1}$$

$$C = \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|}\, d\omega \tag{2}$$

The baby wavelet of equation (3) can be obtained from the shift and dilation of $\psi(t)$.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\, \psi\left(\frac{t-b}{a}\right) \tag{3}$$

where, a is a scale or dilation variable, and b indicates time shift or translation.

If the signal $f(t) \in L^2(R)$, the continuous wavelet transform (CWT) can be defined as

$$Wf(a,b) = \int_{-\infty}^{\infty} f(t)\, \psi^*_{a,b}(t)\, dt \tag{4}$$

Ⅲ. RECONSTRUCTION ALGORITHM

3.1. Threshold-based method

Recently, many methods using the wavelet for reconstructing the signal in the AWGN environment have been investigated. Among these methods, the method using threshold, investigated most actively, has high denoising performance and a simple algorithm as follows.

Step 1. Transform the noisy signal so as to obtain the coefficient, W_g.

Step 2. Apply hard- or soft-threshold Th in each scale.

Step 3. Perform inverse wavelet transform so as to obtain the estimated signal.

In such a case, proper method and threshold level should be selected for applying the threshold at Step 2, and various methods are currently proposed.

Donoho applied the soft-threshold in the orthogonal wavelet transform (OWT), which is given as

$$\widehat{W}_f(n) = \operatorname{sgn}\{W_g(n)\}(|W_g(n)| - Th)_+ \tag{5}$$

where, $(\cdot)_+$ is a positive part operator which is defined as $(x)_+ = \max\{x, 0\}$. In addition, $Th = \sigma\sqrt{2\log N}$, and N is the length of the signal [1],[2].

3.2. Method using spatial correlation

The wavelet detail coefficient of noisy signal degraded by noise is composed of noise and edge components of signal in each scale. In addition, the edge component of signal has generally larger value than the wavelet detail coefficient by noise, and the noise component drastically decreases as the scale increases. Therefore, the correlation between adjacent scales using the above feature can be used to separate edge from noise.

The spatial correlation, given as equation (6), can enhance the edge of signal while suppressing small, sharp features of noise [3],[4].

$$C_l(m, n) = \prod_{i=0}^{l-1} W_g(m+i, n), \quad n = 1, 2, \cdots, N \tag{6}$$

where, m is a scale index, while n is a translation index. In addition, $l = 2$ is generally selected to calculate the correlation between two adjacent scales.

The spatially selective noise filtration (SSNF) algorithm can be explained as follows.

Step 1. Calculate $C_2(m, n)$ in every scale.

Step 2. Solve equation (7) using the wavelet coefficient, $W_g(m, n)$ and the spatial correlation, $C_2(m, n)$.

$$New\ C_2(m, n) = C_2(m, n)\sqrt{P_W(m)/P_C(m)} \tag{7}$$

where, $P_W(m, n) = \sum_n W_g(m, n)^2$,

$$P_C(m, n) = \sum_n C_2(m, n)^2.$$

Step 3. If $|New\ C_2(m, n)| \geq |W_g(m, n)|$, the point is determined as an edge.

3.3. Method using UDWT

In the undecimated discrete wavelet transform (UDWT) if the input signal, X is the white Gaussian noise $x \sim N(0, \sigma^2)$, the variance of the wavelet detail coefficients, W_1 in first scale could be expressed as

$$\begin{aligned}\sigma_1^2 &= Var(W_1) = Var(X * G_0/2) \\ &= \|g_n^0/2\|^2 \cdot Var(X) = \sigma^2\|g_n^0/2\|^2\end{aligned} \tag{8}$$

where, G is 1/2-band high-pass filter [4]. And from the pyramid decomposition structure, we can prove as follows.

$$\begin{aligned}\sigma_m^2 = \sigma^2\|&(h_n^0/2)*(h_n^1/2)*\cdots \\ &*(h_n^{m-2}/2)*(g_n^{m-1}/2)\|^2\end{aligned} \tag{9}$$

In the above equations, g and h are filter coefficients of the high-pass and the low-pass filter, respectively. In addition, the hard- threshold shown as equation (10) is applied in UDWT.

$$\widehat{W}_f(n) = W_g(n) \cdot I\{|W_g(n)| \geq Th\} \tag{10}$$

where, $I(\cdot)$ is an identification function, which keeps points having the wavelet coefficients larger than the threshold level.

3.4. Proposed method

The model of the noisy signal, $g(n)$ superposing signal, $f(n)$ with noise can be expressed as

$$g(n) = f(n) + \sigma z(n) + \gamma(n) \tag{11}$$

where, $z(n)$ is the Gaussian noise, and σ indicates the noise level. In addition, $\gamma(n)$ indicates the impulse noise. In such a case, $z(n)$ has an statistical average value of 0 regardless of σ, and $\gamma(n)$ does not have long duration time. Therefore, in case of the accumulation of wavelet detail coefficient, the noise component cannot largely affect the whole shape of the signal, and the edge component can be used for determining the shape of the signal. Accordingly, twice accumulated data for the detail coefficient in each scale can be used as the determining basis for separating the edge component of signal from noise. Equation (12) shows the accumulation function of the noisy signal.

$$F(n) = \sum_{i=1}^{n} W_g(i) + F(n-1), \quad n = 1, 2, \cdots, N \tag{12}$$

where, the accumulation function, $F(n) = 0$ $(\forall\, n \leq 0)$. In addition, for separating and detecting the edge component from the noisy signal, we calculate the approximated data expressed equation (13), using adjacent data of $F(n)$ at a certain point n.

$$p_2(x) = C_2 x^2 + C_1 x^1 + C_0 \tag{13}$$

where, C_i is coefficient of i-th order term, and each coefficient is determined in such a way that it can satisfy $F(n-k)$, $F(n)$ and $F(n+k)$.

And the edge point, which is presented as a great change of signal, has a huge difference between the accumulation function and the approximated data. Therefore, the edge of the noisy signal can be separated from error function $e_2(n)$ of equation (14).

$$e_2(n) = \sum_{i=n-k}^{n+k} |F(i) - p_2(i)| \tag{14}$$

The function $e_2(n)$, in somewhere n point, is the sum of the error for approximated data $p_2(i)$ and $F(i)$ which are estimated at the range between $[n-k,\ n+k]$.

Where, we estimate the local maximum point of the error function $e_2(n)$ as the edge. But, compared with the wavelet detail coefficient of fine scale, one of coarse scale is relatively exists in the low frequency band. That is, the increment of scale brings about slow changes of the accumulation function $F(n)$, which corresponds to the edge point of the signal. Therefore, the amplitude of the error function $e_2(n)$ rapidly decreases in all areas of the signal. So, in the coarse scale, in order to detect edge points of the signal from the accumulation function $F(n)$, after calculating the approximated data p_1 from the equation (15), the error function $e_1(n)$ of equation (16) is calculated.

$$p_1(x) = C_1 x^1 + C_0 \tag{15}$$

$$e_1(n) = \sum_{i=n-k}^{n+k} |F(i) - p_1(i)| \tag{16}$$

So in each scale, the error function $e(n)$, which is represented as the product of $e_1(n)$ and $e_2(n)$ in equation (17), is used for separating edge components from noise.

$$e(n) = e_1(n) \cdot e_2(n) \tag{17}$$

In above equations, the threshold Th, which is applied to $e(n)$ in order to remove noise, can be chosen in the wide range. Because, in the slowly changing part of a signal, amplitude of the error function generally has an influence by the noise, the $e(n)$ has a large number with a small value. But the edge point has a small number in the whole signal and the value of error function $e(n)$ affected by the amplitude of the edge. Also the histogram of the error function $e(n)$ as Fig. 1 can be used for determining the threshold Th.

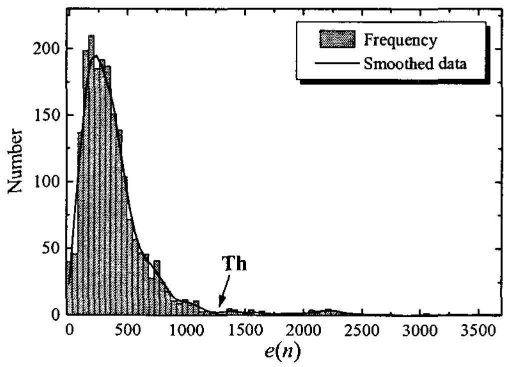

Fig. 1. The histogram of $e(n)$.

From the histogram of the error function $e(n)$, the low value of noise in the left shows a large number, on the other hand, the high value of edge in the right shows a small number.

Because the transition area exists in between the edge of high value and noise of low value, two parts are separated by distribution of the histogram. So from a approximated curved line of the histogram, when the local minimum value of a curved line is used as a threshold Th, noises are removed excellently. Finally, after appling the hard-threshold of equation (10), the inverse wavelet transform is performed to reconstruct the signal.

Ⅳ. SIMULATION AND RESULTS

In this paper, a new wavelet-based algorithm for reconstructing signal in the complex noise environment is proposed. In addition, the HeaviSine and Blocks signal are used as a test signal for the performance evaluation. In such a case, length of the signal is 2048 sample, and impulse noise having different signs and amplitude as shown in Fig. 2 and AWGN having SNR 12[dB] are superposed to the original signal and simulated.

Fig. 3 and Fig. 4 show the simulation results for the HeaviSine and Blocks signal, respectively. In the figures, (a) is the original signal, and (b) indicates the noisy version. In addition, (c) is the reconstructed signal by OWT, and (d), (e) and (f) show the reconstructed signals by SSNF, UDWT, and the proposed method, respectively. Here, SNR_G represents the enhanced SNR gain, which is the difference between SNR of the reconstructed signal and that of the noisy signal.

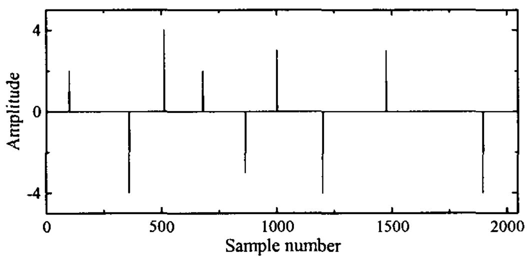

Fig. 2. Impulse noise.

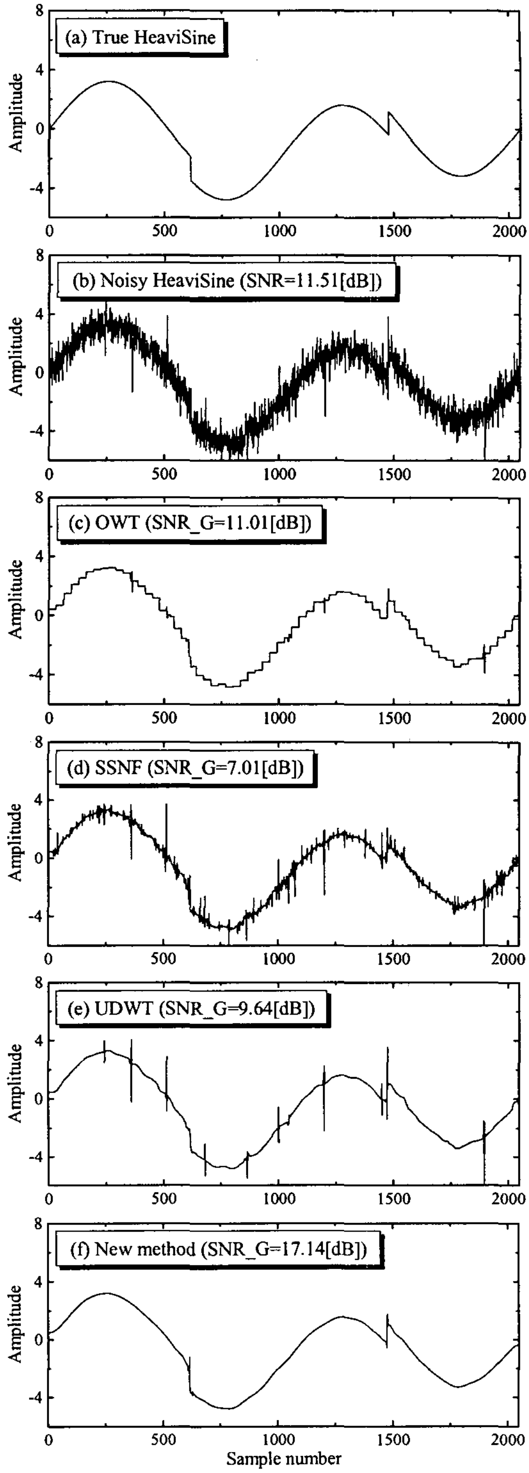

Fig. 3. Reconstruction of HeaviSine signal.

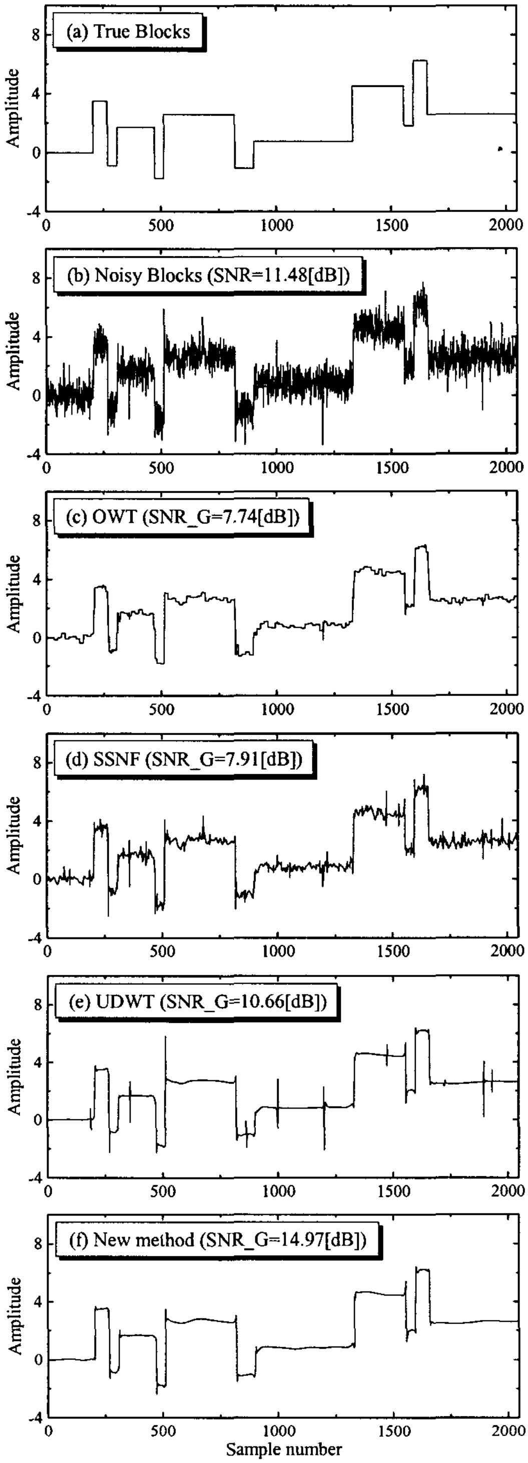

Fig. 4. Reconstruction of Blocks signal.

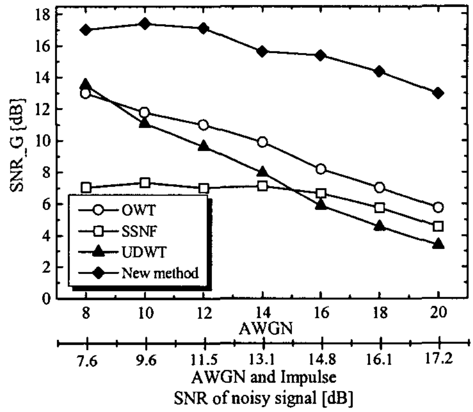

Fig. 5. SNR gain of HeaviSine.

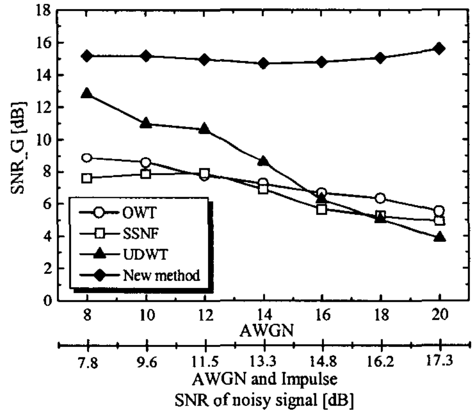

Fig. 6. SNR gain of Blocks.

According to the simulation results for the HeaviSine signal in Fig. 3, the OWT, SSNF, and UDWT showed SNR gains of 11.01[dB], 7.01[dB], and 9.64[dB], respectively. Moreover, the proposed method showed the enhancement effect of 17.14[dB]. From the result of the Blocks signal, the OWT, SSNF and UDWT showed SNR gains of 7.74[dB], 7.91[dB] and 10.66[dB], respectively. But the proposed method showed the enhancement effect of 14.97[dB].

Fig. 5 and Fig. 6 are a graph showing the SNR gain according to SNR for wholly comparing the signal reconstruction effect according to the denoising. In addition, two horizontal axes represent as SNR, respectively when SNR for AWGN and SNR for impulse noise and AWGN are superposed in complex.

According to the result, the conventional methods were not so good at reconstructing signal in the complex noise environment. In addition, UDWT showed excellent property at lower SNR region in which AWGN showed relatively large effect, but poor SNR gain property at higher SNR region in which impulse noise showed large effect. But, the proposed method showed excellent SNR gain property by removing most of the noise components in the entire SNR region.

V. CONCLUSION

In this paper, we proposed new wavelet- based algorithm for reconstructing degraded signal by noise, so that the separation performance between the noise and edge components of the signal is enhanced. In addition, this method is compared and analyzed with the conventional methods by means of the simulation.

According to the simulation result, OWT showed high denoising performance, but did not show smooth result due to the subsampling. In addition, the SSNF algorithm using the spatial correlation was more effective in detecting edge than OWT, but much noise still existed. Moreover, UDWT showed an excellent result in removing AWGN, but did not show the denoising effect for the impulse noise. Therefore, it showed lower SNR gain property at higher SNR region in which the impulse noise showed large effect. However, the proposed method showed excellent SNR gain property by precisely separating and detecting the edge component of signal from noise in the complex noise environment.

Therefore, the proposed method is excellent at reconstructing the degraded signal in the complex noise environment, so that it is thought to be applied to the various signal processing fields.

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