

ADDENDUM TO THE PAPER: ON THE DEVELOPMENT OF THE INTERSECTION OF A PLANE WITH A POLYTOPE

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In [2], it was shown that the intersection of a plane with a polytope unfolds to a non-self-intersecting curve in the plane and conjectured that the developments of slice curves on the surface of a smooth convex body have no self-intersection as well. A *slice curve* is an intersection of a plane with the surface of a three dimensional convex body and its *development* is a trace of the slice curve when the body is rolled on a plane without slippage so that the slice curve is always the point of contact. We show that his conjecture is true:

Theorem 1. *Developments of a slice curve C of a smooth convex surface S have no self-intersection.*

The curve C is a smooth space curve lying on the smooth surface S . Let κ be the curvature of the space curve C and let κ_g, κ_n be the geodesic curvature and the normal curvature of C lying on the surface. Then we have

$$\kappa^2 = \kappa_g^2 + \kappa_n^2.$$

Now let C^* be the development of C and κ^* be the curvature of the plane curve C^* . Then, since

$$\kappa^* = \kappa_g,$$

see, for example, [3, p.159], we have

$$|\kappa^*| \leq |\kappa|.$$

Now, the proof follows from the following theorem of Schur [1, p.119].

Theorem 2. *Let C be a plane arc with the absolute curvature $|\kappa(s)|$ which forms a convex curve with its chord. Let C^* be an arc of the same length referred to the same parameter s such that its absolute curvature $|\kappa^*(s)| \leq |\kappa(s)|$. If d and d^* denote the length of the chords joining their end points, then $d \leq d^*$.*

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References

- [1] S. S. Chern, *Curves and surfaces in Euclidean space*, in : S. S. Chern (Ed.), *Global Differential Geometry*, MAA Studies in Mathematics, Vol. 27, Math. Assoc. America, 1989, 99–139.
- [2] J. O'Rourke, *On the development of the intersection of a plane with a polytope*, *Computational Geometry* **24** (2003), 3–10.
- [3] D. J. Struik, *Lectures on Classical Differential Geometry*, 2nd Ed., Dover, New York, 1988.

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