

# An Optimum 2.5D Contour Parallel Tool Path

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*Although conventional contour parallel tool paths obtained from geometric information have successfully been used to produce desired shapes, they seldom consider physical process concerns such as cutting forces and chatter. In this paper, we introduce an optimized contour parallel path that maintains a constant material removal rate at all times. The optimized tool path is based on a conventional contour parallel tool path. Additional tool path segments are appended to the basic path to achieve constant cutting forces and to avoid chatter vibrations over the entire machining area. The algorithm was implemented for two-dimensional contiguous end milling operations with flat end mills, and cutting tests were conducted to verify the performance of the proposed method.*

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## NOMENCLATURE

- $B_M$  = viscous damping coefficient  
 $b$  = radial depth of cut  
 $F$  = cutting force  
 $I_M$  = spindle current  
 $J$  = moment of inertia of motor shaft  
 $K_t$  = spindle motor constant  
 $k$  = specific cutting energy  
 $r$  = radial depth  
 $R$  = corner radius  
 $s$  = step length  
 $T_d$  = torque of disturbance  
 $T_f$  = viscous friction torque of motor shaft  
 $t$  = axial depth of cut  
 $v$  = cutting speed

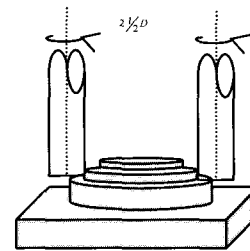


Fig. 1 2.5D milling path control

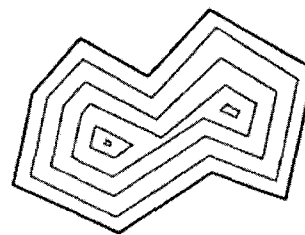


Fig. 2 Contour parallel tool path

## 1. Introduction

Two-and-a-half-dimensional (2.5D) milling represents a hybrid form of two- and three-dimensional milling in which only two axes are continuous-path controlled after one plane has been selected. This concept is illustrated in Fig. 1.

Generally, most CNC milling can be performed using 2.5D milling. According to Harenbrock,<sup>1</sup> more than 80% of all machined mechanical parts can be cut by applying this path control concept. This is partially due to the fact that a surprisingly large number of mechanical parts have 2.5 dimensions and more complicated objects are usually produced from a billet by 2.5D roughing and 3D-5D finishing. Hence, finding an efficient tool path for 2.5D milling tasks is one of the most important issues in CAM. This is known as the

pocket-machining problem. A popular tool path style in pocket machining is the contour-parallel path, which is generated by successive offsets of the input boundary, as shown in Fig. 2.

The most important issues in optimized 2.5D milling are geometrically efficient algorithms for tool path generation and physical considerations for better machining productivity that ensure the safety of the machining process. Nevertheless, most previous research has focused only on geometric algorithms. Although conventional tool paths obtained from geometric information have successfully produced the desired shapes, they seldom consider physical process concerns such as cutting forces and chatter. Therefore many problems are encountered in the actual machining process.

Tlustý *et al.*<sup>2</sup> noted that the radial engagement increased when a cutter moved into the corner region while the radial engagement

remained constant for straight path segments (see Fig. 3). Consequently, a momentary rise in the cutting forces and chatter are commonly encountered during cornering.

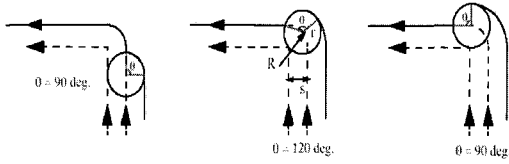


Fig. 3 Varying engagement arising with the contour path

In 2.5D milling, the tool path distance,  $s$ , and the angle of engagement between the tool and workpiece,  $\theta$ , must satisfy

$$\theta_{\max} = \pi - a \cos\left(\frac{2r^2 + 2R(s-r) - s^2}{2r(R-r)}\right) \quad (1)$$

Fig. 4 shows the relation between the engagement angle and radial depth of cut. The increase of the radial depth of cut with the engagement angle introduces excessive cutting forces that influence tool breakage, tool deflection, and chatter. Hence, tool paths with constant cutting forces are required from a physical point of view. In addition, Trusty *et al.*<sup>2</sup> and Smith *et al.*<sup>3</sup> addressed the issues of stability in end milling and the importance of the proper selection of axial and radial depths of cut for chatter-free machining. Since a given axial depth of cut is constant in 2.5D milling, the radial depth of cut plays an important role in determining the stability. Thus constant cutting forces and radial depths of cut are required for the entire machining area to generate optimum contour parallel tool paths in 2.5D milling.

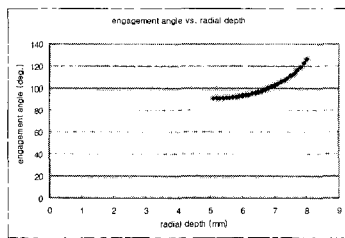


Fig. 4 Engagement angle vs. radial depth

The remainder of this paper will be presented in the following order. Previous studies are reviewed in Section 2. The necessity of a constant material removal rate (MRR) and the tool path modification method are described in Section 3. Cutting experiments are presented to verify the effect of the proposed method in Section 4. The final section contains our concluding remarks.

## 2. Previous Work

The literature on controlling cutting forces and machining stability can be roughly divided into two major approaches: feedrate scheduling and tool path modification. The first approach focuses on controlling the cutting forces by adjusting the feedrate so it does not exceed the reference cutting force and is based on a cutting force model.<sup>4-6</sup> If the model that predicts the cutting forces is exact, constant cutting forces can be achieved. The cutting force can be expressed as

$$F = kbt, \quad (2)$$

where  $F$  is the cutting force,  $k$  is the specific cutting energy,  $b$  is the radial depth of cut, and  $t$  is the axial depth of cut. Precise predictions of the cutting force are very difficult to obtain because the specific cutting energy,  $k$ , changes according to the cutting speed and radial

depth. Although cutting forces can be predicted exactly, it is impossible to keep the radial depth of cut constant using feedrate scheduling without tool path modifications.

Another approach for controlling cutting forces and machining stability is to modify the tool path geometry<sup>7-9</sup> as shown in Fig. 5. It can be more effective to use additional tool paths in corners to control the radial depth of cut and cutting forces. Iwabe *et al.*<sup>7</sup> attempted to reduce the chip load when cutting in a corner region by using a looped cutter path that removed the stock material incrementally in several passes. Tsai *et al.*<sup>8</sup> enhanced Iwabe's corner-looping tool path, but the improved tool path still could not deal with complicated corner shapes. In addition, a critical limit may be encountered when maintaining constant cutting forces over the entire machining area by inserting additional paths only in the corners.

The contour parallel tool path, which is generated by successive offsets of the input boundary, has degenerate local and global loops. Because degenerate local loops change the geometric shape after offsets, as shown in Fig. 6(a), additional paths only in the corners cannot make the cutting forces and radial depth of cut constant. Furthermore, the removed global invalid loops also cause excessive cutting loads and chatter in the region indicated in Fig. 6(b). Therefore contour parallel tool paths that consider both cutting forces and machining stability over the entire machining area are absolutely necessary.

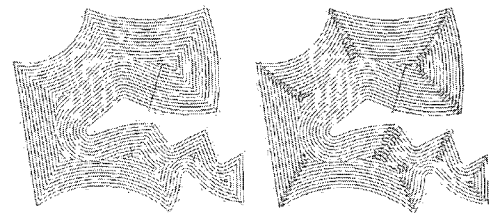


Fig. 5 Path insertion at the corners

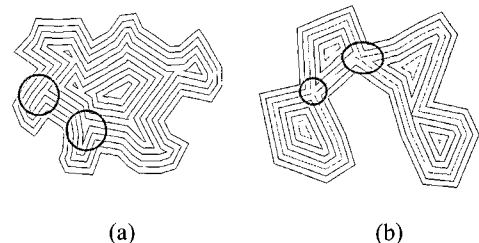


Fig. 6 Examples of excessive MRRs, except at the corners

## 3. Tool Path Modification

### 3.1 Tool Path Modification for a Constant MRR

For an optimum contour parallel tool path, both the cutting forces and the radial depth of cut must be maintained at constant values over the entire tool path to ensure machining stability. In the case of 2.5D milling, tool paths that sustain a constant MRR satisfy the conditions mentioned above. If the radial depth of cut is maintained at a constant value for a fixed feedrate, the MRR does not change because the axial depth of cut is also constant in 2.5D milling. This can be described by

$$Fv = kbtv = k \cdot MRR, \quad (3)$$

where  $F$  is the cutting force,  $k$  is the specific cutting energy,  $b$  is the radial depth of cut,  $t$  is the axial depth of cut, and  $v$  is the cutting speed. In other words, to obtain a constant MRR, one can maintain a constant radial depth of cut. The cutting forces are also kept constant because the specific cutting energy does not change. Therefore, it is important to maintain a constant MRR when using 2.5D optimum contour parallel paths while considering constant cutting forces and

machining stability.

In this paper, additional tool paths are inserted in the given offset curves that are used as contour parallel tool paths to maintain a constant MRR at all times. Since geometric information changes in degenerate parts, as illustrated in Figs. 5 and 6, the MRR cannot be computed analytically for arbitrary tool path trajectories. A pixel-based simulation method can be used to overcome this problem.

### 3.2 Pixel-based MRR Simulation

Fig. 7 illustrates the pixel-based simulation procedure for the MRR calculations. Both the geometry and tool are discretized and represented as pixel squares. In the figure, the dark gray region represents the material left to be machined. The light gray region has already been machined, and the present MRR is to be estimated from the pixel numbers in the blue region. The reference pixel numbers can be computed according to the nominal MRR as follows:

$$\text{reference\_MRR} = \left( \frac{\Delta s}{\text{pixel\_resolution}} \right) \times \left( \frac{s}{\text{pixel\_resolution}} \right) \quad (4)$$

where  $\Delta s$  is the feed per tooth and  $s$  is the step length.

The tool path is modified in the excessive MRR region, as shown in Fig. 8. The MRR calculations for linear motion are given in Fig. 9, and the detailed algorithm for path modification can be expressed as follows.

#### Tool path modification for constant MRR:

```
//input: points[] representing closed 2D lines//
//output: modified offset curves//
for(int i=1; i<points.size(); ++i)
{
    start_point=points[i-1];
    end_point=points[i];
    Linear_Motion(start_point, end_point, pixel_num);
    If(pixel_num>reference_pixel_num)
        Additional_Path_Insertion(start_point, end_point);
}
```

#### Additional\_Path\_Insertion(start\_point, end\_point)

```
{
    Repeat {
        Repeat {
            change end_point to the left of moving direction;
            Linear_Motion(start_point, end_point, pixel_num);
        } Until (pixel_num <= reference_pixel_num);
        insert an additional path from start_point to end_point;
        extend an additional path with same length;
        start_point=end_point;
        replace end_point with the end point of extension line;
        Linear_Motion(start_point, end_point, pixel_num);
    } Until (pixel_num <= reference_pixel_num);
    insert a return path;
}
```

#### Linear\_Motion(st\_pt, end\_pt, pixel\_num)

```
{
    make bounding box;
    while (lower_line < upper_line)
    {
        calculate intersection_pt;
        for (; st_x < end_x; ++st_x)
            if (pixel_map[st_x][lower_line] != visit)
                ++pixel_num;
            ++lower_line;
    }
}
```

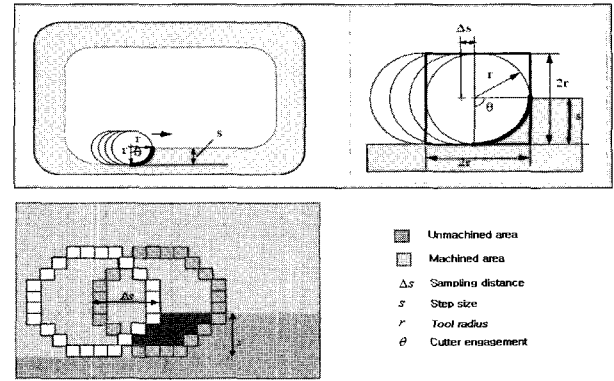


Fig. 7 Pixel-based MRR simulation

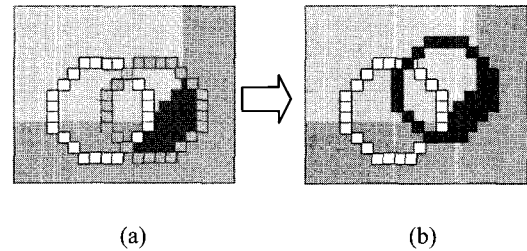


Fig. 8 Pixel-based tool path modification

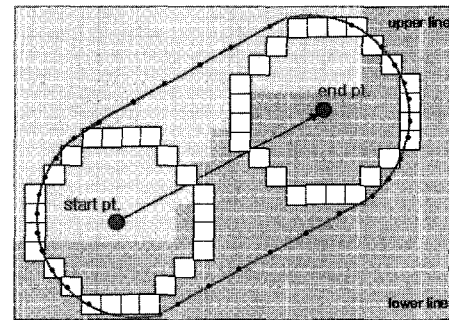


Fig. 9 MRR calculations for linear motion

### 3.3 Additional Tool Paths Obtained Using MRR Simulation

The proposed path modification algorithm was implemented using C++ on a PC. The result generated using our algorithm is shown in Fig. 10. Additional paths obtained using the simulation approach, such as those in Fig. 10, consisted of short segments for which a machine tool may spend the majority of its time accelerating and decelerating. Therefore, to reduce the accelerating and decelerating time, a circular interpolation was executed for the short segments, and spiral tool paths at the entrance were used to reduce the cutting forces. The result is shown in Fig. 11.

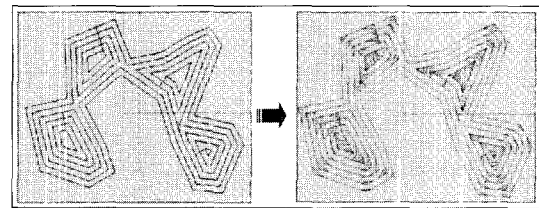


Fig. 10 Additional path insertion for constant MRR

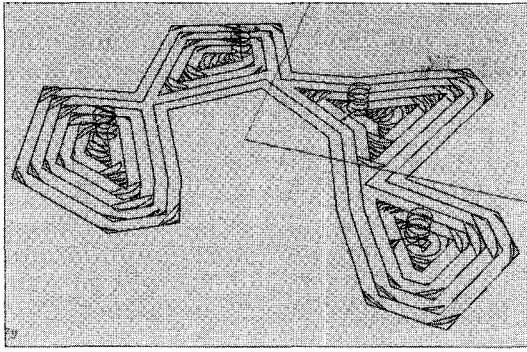


Fig. 11 Optimum path

#### 4. Experimental Verification

The spindle current was measured to investigate the real machining state and the MRR since the spindle current reflects influences of the tool material, characteristics of the machine tools, the workpiece material, and so on.<sup>10</sup> We investigated the relationships between the spindle current and cutting force, MRR, and  $Z_M$  from the equation of motion for machine tool spindles. The relationships can be linearized as follows (see Fig. 12):<sup>11,12</sup>

$$I_M = a + bF_c \quad (5)$$

$$I_M = p_1 + H \cdot p_2 Z_M \quad (6)$$

where  $I_M$  is the spindle current; parameters  $a$ ,  $b$ ,  $p_1$ , and  $p_2$  are constants related to the cutting tool, workpiece, CNC machine, and cutting conditions, respectively;  $F_c$  is the mean cutting force; and  $H$  is the coefficient due to the influence of hardness.

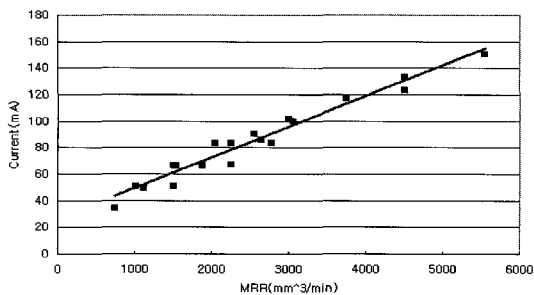


Fig. 12 Relationship between the current and MRR

Actual cutting experiments were performed using contour parallel tool paths before and after the simulation approach was applied in order to verify the constant MRR. The machining center was a Daewoo AL-40 with a Fanuc-0M controller. The spindle current was measured using a CNC spindle load meter. The experimental conditions are shown in Table 1; the pixel size was 0.3 mm.

Table 1 Experimental Conditions

| Experimental Conditions |  |
|-------------------------|--|
| Workpiece               | SM45C (200 x 200 x 50)                   |
| Spindle speed (RPM)     | 2000                                     |
| Axial depth (mm)        | 0.5                                      |
| Radial depth (mm)       | 3  |
| Feedrate (mm/min)       | 500                                      |
| Model size              | 120 x 85                                 |
| Tool                    | 10φ flat endmill (tungsten carbide tool) |

Fig. 13(a) shows the experimental results for the original contour parallel tool paths indicated in Fig. 10, and Fig. 13(b) shows the results for the optimized paths using the proposed algorithm. Without additional paths, there were large changes in the spindle current. However, when additional paths were added in the excessive MRR region, a constant MRR was maintained at all times, as shown in Fig. 13(b). Consequently, constant cutting forces and radial depth of cut were maintained for stable machining over the entire machining area.

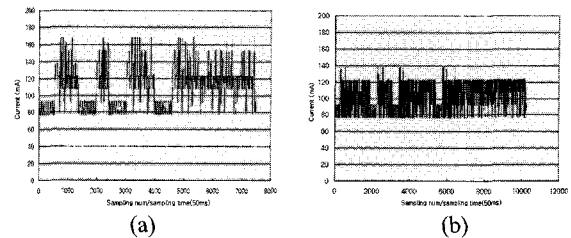


Fig. 13 Experimental results

#### 5. Conclusions

We presented a simple but novel approach that used a pixel-based simulation technique to maintain a constant MRR at all times during machining by considering the cutting forces and stability issues. When using conventional contour parallel tool paths, a varying radial depth of cut and MRR are accepted as inevitable by-products of the path generation, and the worst-case parameter selection is used inefficiently. Although feedrate modifications have been employed to overcome this problem, such alterations cannot change the radial depths, which affect the machining stability. A corner looping-based tool path, which is a more fundamental method that is sometimes used to mitigate this problem, still has limitations when used to maintain a constant MRR over the entire machining area because of degenerate regions. Therefore, we introduced and implemented a simulation technique to maintain a constant MRR over the entire machining area. We conducted cutting tests by measuring the spindle current, which is proportional to the MRR, and demonstrated that a constant MRR was maintained at all times.

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