

ANALYSIS OF PLANETARY GEAR HYBRID POWERTRAIN SYSTEM PART 1: INPUT SPLIT SYSTEM

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ABSTRACT—In recent studies, various types of multi mode electric variable transmissions of hybrid electric vehicles have been proposed. Multi mode electric variable transmission consists of two or more different types of planetary gear hybrid powertrain system (PGHP), which can change its power flow type by means of clutches for improving transmission efficiencies. Generally, the power flows can be classified into three different types such as input split, output split and compound split. In this study, we analyzed power transmission characteristics of the possible six input split systems, and found the suitable system for single or multi mode hybrid powertrain. The input split system used in PRIUS is identified as a best system for single mode, and moreover we identified some suitable systems for dual mode.

KEY WORDS : Transmission efficiency, Hybrid electric vehicle, Planetary gear, Input split

NOMENCLATURE

PG	: planetary gear
MG	: motor/generator
α	: ratio of lever length of MG1 to lever length of input from output
R	: planetary gear ratio, the ratio between the number of teeth of a sun and ring gear
R_{ENG}	: reduction gear ratio of engine
R_O	: reduction gear ratio of output shaft
R_{MG1}	: reduction gear ratio of MG1
R_{MG2}	: reduction gear ratio of MG2
T_{ENG}	: engine torque
T_O	: output shaft torque
T_{MG1}	: MG1 torque
T_{MG2}	: MG2 torque
ω_{ENG}	: engine speed
ω_O	: output shaft speed
ω_{MG1}	: MG1 speed
ω_{MG2}	: MG2 speed
η_{MG21}	: charging efficiency of motor 1
η_{MG2}	: discharging efficiency of motor 2
γ_{TP}	: speed ratio of input speed to output speed when speed of MG1 or MG2 is zero
PSR	: power split ratio, power to the electric pass/total power

1. INTRODUCTION

As environmental and economic interests increase, the potential for hybrid electric vehicles also increases. Furthermore, many car makers produce hybrid electric vehicles or notify schedules of the production of hybrid electric vehicles. Various architectures of the hybrid powertrain have been studied, and some architectures showing merit in cost and efficiency have been selected. Recently, the parallel types that were studied by Honda and European car makers and the power split types by Toyota have become main trends in HEV powertrains.

The power split type performs shifts and exclusive functions of HEV without an additional transmission. Various kinds of cars, for example, PRIUS of TOYOTA, RX400H and GS450H of LEXUS, and ESCAPE of FORD, are equipped with powertrains of this type. The power split types used in HEVs are classified into three types; input split, output split and compound split by the method of power delivery. Among them, the input-split type is advantageous when it only uses an electric motor, for example, an electric vehicle mode and a regenerative braking mode. An input-split type shows good efficiency in all shifting ranges, therefore most cars that are sold use the input-split powertrain, including PRIUS.

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2. INPUT SPLIT SYSTEM

Generally, the input-split hybrid powertrain consists of one planetary gear and two motors. A planetary gear has three operating points, a sun gear, a carrier and a ring gear. Two of these points are connected to the engine and the first motor separately, and the other point is connected to the second motor and the output shaft of a transmission. A portion of an engine power splits and circulates through the first motor or the second motor, and the rest is delivered to the output shaft directly. Because the engine power inputted to the transmission splits into two power flows, we call this type of a powertrain architecture an input-split powertrain (Rizoulis, Burl and Beard, 2001). Figure 1 is a schematic diagram of THS (Toyota Hybrid System) of TOYOTA, which is the representative input-split hybrid powertrain.

It is possible to represent an input-split powertrain by lever diagrams (Benford and Leising, 1981). The input of powertrain is connected to the engine, and the output is connected to MG2 (Motor/Generator). The last point is connected to MG1, which splits the engine power.

We defined the ratio of lever length of MG1 to lever length of input from output as α . Equations of the velocity and torque of input, output, MG1 and MG2 are derived as functions of α (Conlon, 2005).

The lever length of MG1 α is determined by the ratio between the number of teeth of a sun and ring gear, R . To shorten the definition, it will be subsequently referred to as “planetary gear ratio.” It is also affected by the condition of connection between the planetary gear and the input, output, etc (Mucino, Smith, Cowan and Kmicikiewicz, 1997). There are six possible input-split combinations using one planetary gear and two motors, and Figure 2 shows these combinations and α of each combination.

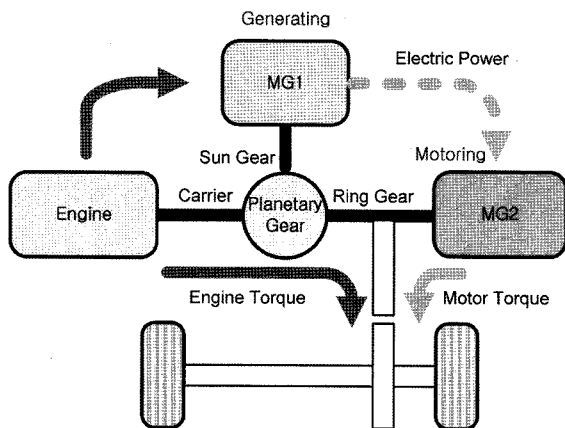


Figure 1. Power flow of input split type hybrid powertrain.

2.1. Torque and Speed Relationship

It is possible to express velocity and torque equations of input, output, MG1 and MG2 of an input split powertrain by the lever diagram (Figure 3 and 4). Using these lever diagrams, velocity and torque equations of steady state are derived as equation (2), (4) and (6) (Fussner and Singh, 2002).

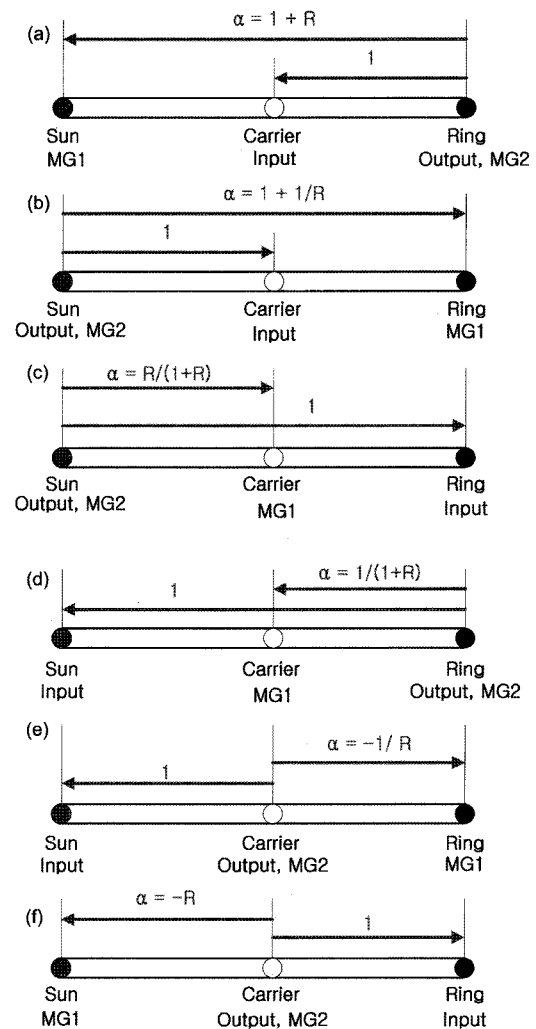


Figure 2. Possible input split combinations with single planetary gear system.

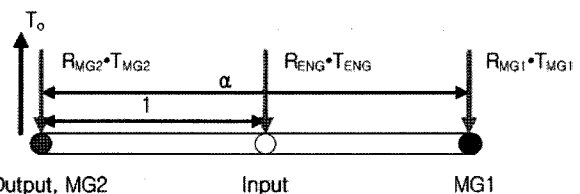


Figure 3. Lever diagram for steady-state torque relationship.

$$\Sigma M_{output} = R_{ENG} \cdot T_{ENG} + \alpha \cdot R_{MG1} \cdot T_{MG1} = 0 \quad (1)$$

$$T_{ENG} = -\alpha \cdot \frac{R_{MG1}}{R_{ENG}} \cdot T_{MG1} \quad (2)$$

$$\Sigma M_{input} = \frac{1}{R_O} \cdot T_O + (\alpha - 1) \cdot R_{MG1} \cdot T_{MG1} - R_{MG2} \cdot T_{MG2} = 0 \quad (3)$$

$$T_O = -(\alpha - 1) \cdot R_O \cdot R_{MG1} \cdot T_{MG1} + R_O \cdot R_{MG2} \cdot T_{MG2} \quad (4)$$

$$\frac{\frac{1}{R_{MG1}} \cdot \omega_{MG1} - R_O \cdot \omega_O}{\frac{1}{R_{ENG}} \cdot \omega_{ENG} - R_O \cdot \omega_O} = \alpha \quad (5)$$

$$\omega_O = \frac{1}{(\alpha - 1) \cdot R_O} \cdot \left(\frac{\alpha \cdot \omega_{ENG}}{R_{ENG}} - \frac{\omega_{MG1}}{R_{MG1}} \right) = \frac{1}{R_O \cdot R_{MG2}} \cdot \omega_{MG2} \quad (6)$$

The required torques of MG1 and MG2 for maintaining steady state and producing required output torque are derived by equation (2) and (4). Moreover it is possible to derive the engine speed and MG1 relative to the output speed by equation (6).

The speed of MG1 is determined by lever length α , transmission ratio, engine speed and output speed. The torque of MG1 is determined by α , the transmission ratio, and engine torque. However, the speed of MG2 is determined by the transmission gear ratio and vehicle speed and torque of MG2 is determined by α , transmission ratio, torque of MG1 and output.

Since MG2 is connected directly to the output shaft in an input split powertrain, the output torque and speed are controlled only by MG2, regardless of engine and MG1 motion. Therefore, an input-split powertrain has great advantages in electric vehicle mode and regenerative braking mode, which are operated during engine stop.

2.2. Direct Transfer Point

In case of the powertrain using planetary gear, the speed of MG1 or MG2 becomes 0 at a specific transmission

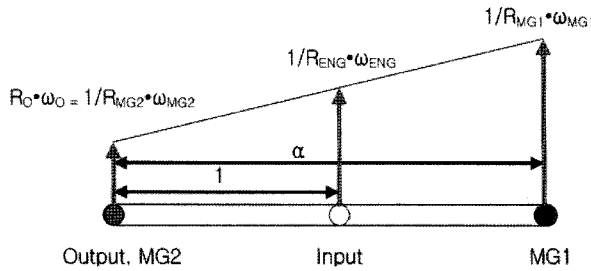


Figure 4. Lever diagram for steady-state speed relationship.

ratio. All power is delivered to the output through mechanical paths. This operating point is called 'Direct Transfer Point', and it is an important factor in the process of designing systems. Because MG2 is connected to the output in the case of an input-split powertrain, there is only one direct transfer point at which the speed of MG1 becomes 0, but the point at which the speed of MG2 becomes 0 does not exist. The direct transfer point is defined as equation (7).

$$\gamma_{DTP} = \frac{\omega_{ENG}}{\omega_O} = \frac{\alpha - 1}{\alpha} \cdot R_{ENG} \cdot R_O \quad (7)$$

2.3. Power Split and Transfer Efficiency

Engine power is delivered to output through two different paths. One is the electric path, which passes through MG1 and MG2, and the other is the mechanical path, which passes through the planetary gear. For this reason, the type of planetary gear powertrain is called 'series-parallel type powertrain'. In this powertrain the power delivered through the electric path is more inefficient than through the mechanical path because of electric and mechanical conversion efficiency. Therefore, the lower the power is delivered through the series path, the higher the system efficiency increases. The electric power of input-split powertrain is represented below.

The engine power equation can be written as:

$$P_{ENG} = T_{ENG} \cdot \omega_{ENG} \quad (8)$$

and the power split through MG1 as:

$$\begin{aligned} P_{MG1} &= T_{MG1} \cdot \omega_{MG1} \\ &= \left(-1 + \frac{\alpha - 1}{\alpha \cdot \gamma} \cdot R_{ENG} \cdot R_O \right) \cdot P_{ENG} \\ &= \left(-1 + \frac{\gamma_{DTP}}{\gamma} \right) \cdot P_{ENG} \end{aligned} \quad (9)$$

therefore the ratio of the power split through the series path is represented by:

$$PSR = \frac{-P_{electric}}{P_{ENG}} = \left(1 - \frac{\gamma_{DTP}}{\gamma} \right) \quad (10)$$

The efficiency of power delivery becomes the maximum 1 at transmission ratio $\gamma = \gamma_{DTP}$, that is, a direct transfer point.

3. TRANSMISSION EFFICIENCY

There are six possible input-split combinations according to the method of connection between planetary gear and power sources, and each combination has its own characteristics. In this chapter, the direct transfer point, motor torque, motor speed, and magnitude of power split are analyzed. For the simplification of analysis, there is an assumption that all reduction gear ratios connected to

each power source has a value of 1.

3.1. Direct Transfer Point

Because no power is split into the electric path at a direct transfer point, maximum system efficiency is available at this point. Furthermore, for input-split powertrain, the system efficiency rapidly decreases due to power recirculation, which occurs after the direct transfer point.

Therefore, selecting the location for a direct transfer point is very important in the design process. Generally, the direct transfer point should be located highly used areas. The direct transfer point is expressed by the lever length of planetary gear α and reduction gear ratios R_{ENG} R_O . It can be written as equation (7).

Because the direct transfer point γ_{DTP} is related to lever length α , it is possible to determine γ_{DTP} by changing α .

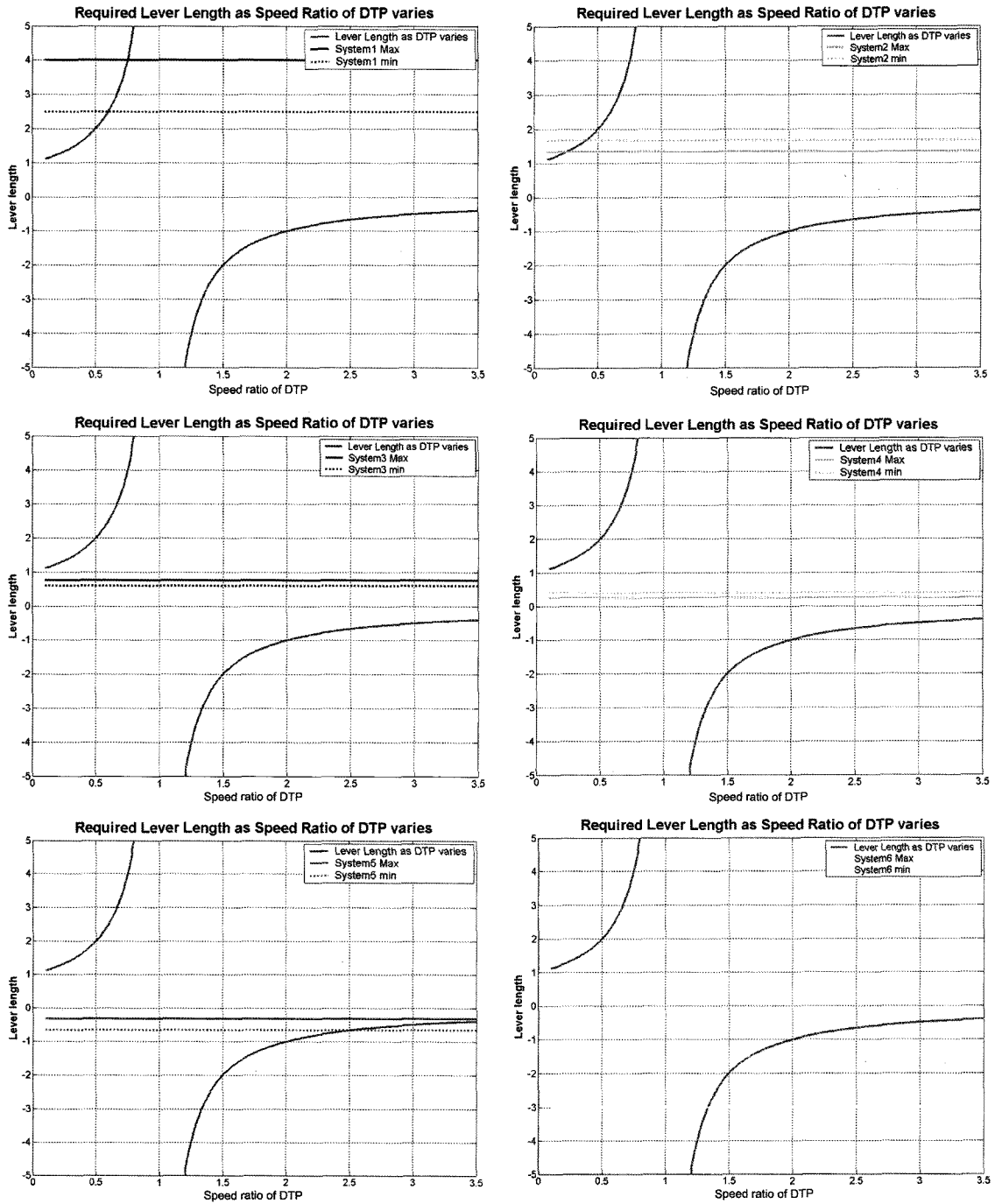


Figure 5. Required lever length of 6 different input split system.

However, the architecture of planetary gear restricts the planetary gear ratio R . In accordance with system architecture, the range of lever length α is also restricted. Figure 5 shows ranges of direct transfer point for six possible combinations. Red lines represent direct transfer points related to the lever length, and solid and dotted lines represent the maximum and minimum lever length of each combination.

Figure 5 shows that for systems 1 and 6, it is possible to change the location of direct transfer point by changing lever length. However, it can be seen that the variation in lever length is very restrictive for significantly the location of a direct transfer point. Hence, the process of design systems 1 and 6 has many degrees of freedom. Systems 3 and 4 have no direct transfer point despite their lever length, so these combinations are more inefficient relative to others.

3.2. MG1 Torque

Equation (2) shows that for maintaining engine speed MG1 absorbs torque proportional to engine torque. The proportional coefficient is determined by lever length α , so the torque required for MG1 relative to engine torque varies with the powertrain architecture. Figure 6 shows the required torque of MG1 as a result of variation in α , when the engine produces input torque. Each solid and dotted line represents the required torque of MG1 at maximum and minimum lever lengths for each system. The required torque of MG1 is determined by engine torque and lever length α .

Figure 6 shows that because Systems 3, 4 and 5 require higher torque than engine torque, these systems are disadvantaged in the magnitude of MG1 torque; however, system 1 is definitely superior to the others.

3.3. MG1 Speed

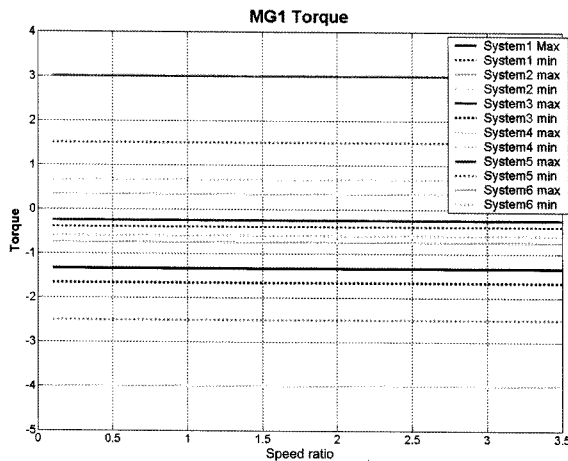


Figure 6. Required MG1 torques of 6 different input split system.

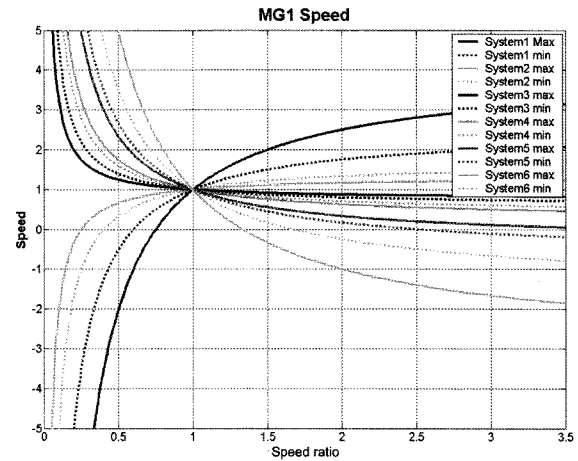


Figure 7. Required MG1 speeds of 6 different input split system.

In equation (6), it can be seen that MG1 speed relative to engine speed is determined by lever length α and output speed. In other words, MG1 speed is expressed as a function of lever length α and transmission gear ratio γ . Figure 7 shows required MG1 speeds of six different input-split systems normalized to engine speed. The MG1 speed can be expressed as:

$$\omega_{MG1} = \left(\frac{(\gamma - 1) \cdot \alpha + 1}{\gamma} \right) \cdot \omega_{ENG} \quad (11)$$

System 1 is an interior system because of the highest speed, but system 6 is superior to the others. Figure 7 shows that MG1 speed of all input split systems rapidly increases at low speed ratios. To prevent MG1 speed from increasing excessively, selecting a final gear ratio and controlling engine speed are necessary in situations where low speed ratios are required, specifically in situations of low engine and high vehicle speed.

3.4. MG2 Torque and Speed

MG2 torque can be written as equation (2). It is proportional to output torque T_o , therefore the ratio of MG2 torque to engine torque is proportional to the speed ratio. Moreover, because MG1 torque multiplied by $\alpha - 1$, lower MG2 torque results from lower MG1 torque and shorter lever length. As a result, the system requires the lowest MG1 torque and displays the lowest absolute value for MG2 torque.

Figure 8 shows that system 1 has the lowest MG2 torque demand and that system 4 and 5 needs the highest MG2 torque.

MG2 speed and output speed are similar, regardless of system architecture. This is shown as only one line, because all lines overlap.

That is, MG2 speed is in inverse proportional to speed ratio as Figure 9 shows.

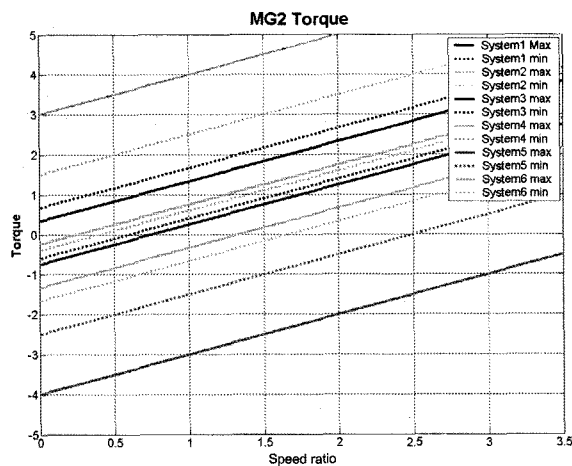


Figure 8. Required MG2 torques of 6 different input split system.

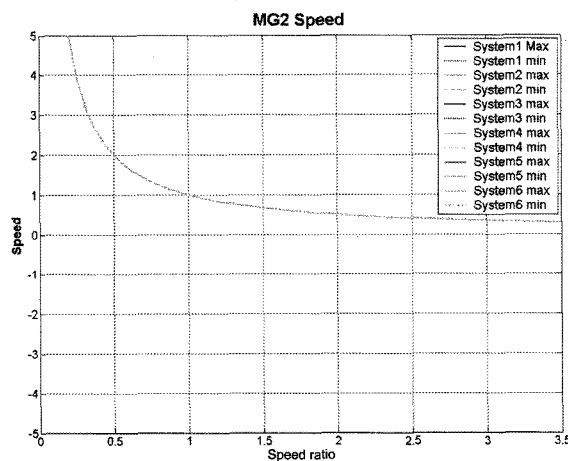


Figure 9. Required MG2 speeds of 6 different input split system.

3.5. Power Split to the Electric Path

For input split powertrain, the engine power splits into a mechanical and electric path. The more power splits into the electric path, the worse the system efficiency becomes. Figure 10 shows the power split ratio of the power split into the electric path to engine power of six different input split systems.

The sign of power split ratio represents the direction of power delivery (Pohl, 2002). That is, plus values denote a split in power minus the circulation. The system efficiency decreases rapidly in a power split. Figure 10 shows that systems 3 and 4 have a characteristic in which power is always split at the whole speed ratio and power is always circulated in some planetary gear ratios for system 5. For systems 1, 2 and 6, the power is split at low vehicle speeds, and power becomes circulated as the speed ratio decreases.

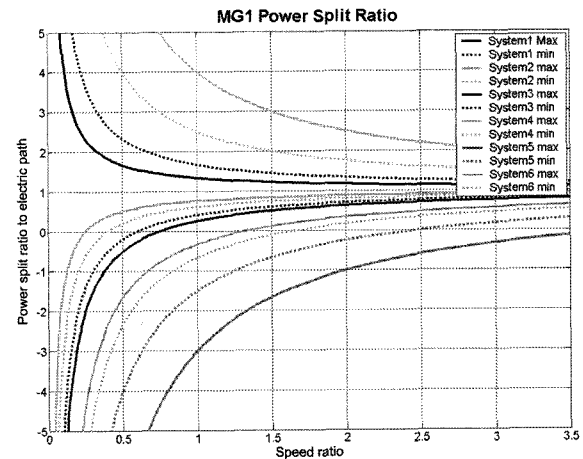


Figure 10. Power split ratio of the power split into the electric path to engine power of six different input split systems.

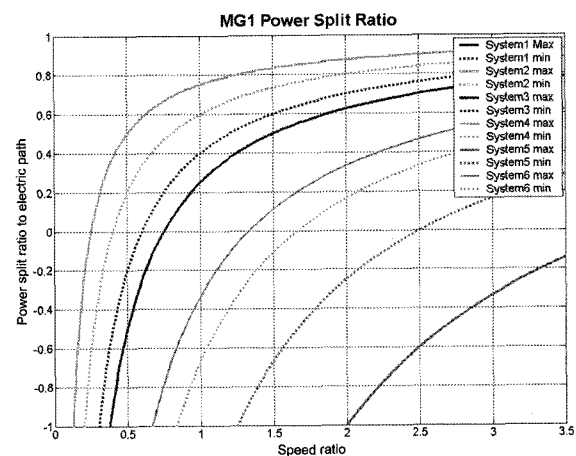


Figure 11. Possible transmission ratio with restriction of maximum motor power under the engine power.

The less power splits or circulates, the higher the potential for system efficiency (Fussner and Singh, 2002). Due to this characteristic, system 1 is superior to the others. System 6 shows high system efficiency, if it is used restrictively at high speed ratios, specifically low vehicle speed.

In Figure 10, the region in which power split ratio is more than 1 or less than -1 means that power recirculation has occurred. Therefore, the system efficiency is poor in this situation, and a large sized motor is necessary.

3.6. Transmission Ratio

When a lower speed ratio is used, the more power splits to the electric path, and consequently, a larger size motor is needed. Increase in motor size causes rising cost, increasing volume and eventually a problem with the

capability of loading. As a result, the capacity of possible transmission is restricted to a level of applicable capacity. Figure 11 shows possible transmission ratio with restriction of maximum motor power under the engine power.

Systems 1 and 2 have a wide possible transmission ratio but a large power split, except at a low speed ratio. However, systems 5 and 6 have small power split but a remarkably restrictive possible transmission ratio. If the transmission is used at whole vehicle speed, systems 1 and 2 are suitable. However, if used only at low vehicle speed, systems 5 and 6 are possible. In addition, systems 3 and 4, which are not drawn in figure 11, have no possible transmission ratio with restriction of maximum motor power under the engine power.

4. OPTIMIZED SYSTEM PERFORMANCES

In this chapter, we found optimal planetary gear ratios of six input-split systems for maximum fuel economy and acceleration performance, and calculated fuel economy and acceleration time from 0 kph to 100 kph in those ratios. Fuel economy and acceleration performance simulators programmed by matlab were used for this process. The planetary gear ratios from 1.5 to 3, which mean ratios of the number of ring gear to sun gear, were used for optimization. There is an assumption that there is no reduction gear connected to the engine and motors, because reduction gears do not affect the optimal planetary gear ratios. Motors used in this simulation are of the same type.

4.1. Fuel Economy Simulator

The fuel economy simulator calculates fuel economy at FTP72 cycle. The velocity profile of FTP72 cycle is shown in Figure 12.

In the simulation process, two assumptions are used for simplifying the calculation. One is that all power

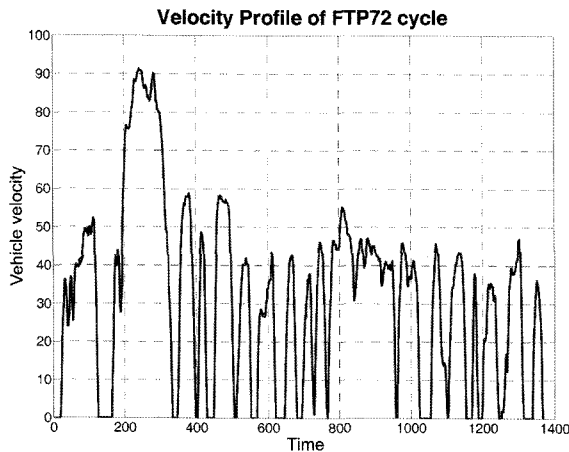


Figure 12. Velocity profile of FTP72 cycle.

absorbed by regenerative braking is used only under motor mode, and the other is that there is no power assist by motor.

Because power assist is not considered, planetary gears and motors are required only for transmission, and all power charged by a generator is used by the motor. As a result, all power charged by a generator is similar to the power discharged by a motor. This can be expressed by equation (12).

$$\eta_{MG1} T_{MG1} \omega_{MG1} + \eta_{MG2} T_{MG2} \omega_{MG2} = 0 \tag{12}$$

The input-split system has five variables of engine torque and speed, generator torque and speed and motor torque, and it is restricted by four constraint equations. So, this system has one degree of freedom. If one variable is determined, the rest of the variables are calculated automatically by constraint equations. The fuel economy simulator finds the optimal engine speed for the minimum fuel consumption at given driving conditions and determines other variables. Engine speed is restricted by motor speed ranges in the process of simulation (Cho and Vaughan, 2006). If the calculated engine torque exceeds maximum engine torque, the simulator finds other optimal points.

Figure 13 shows the process of determining the optimal engine operating points. The thick green line represents available operating points satisfying the restriction of motor speed. The simulator selects the point showing minimum fuel consumption among available operating points. This point is represented as the red points in Figure 13.

Optimal engine points at all steps in the driving cycle are determined, and the simulator calculates appropriate vehicle speed separating ZEV and motor mode. The vehicle speed separates ZEV and motor mode effects on

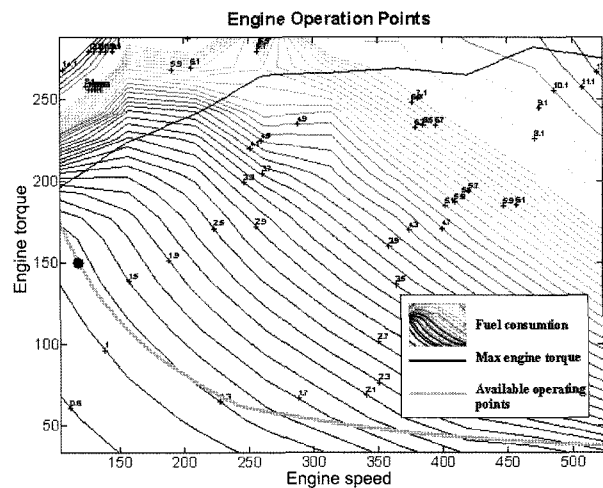


Figure 13. Determination of the optimal engine operating points.

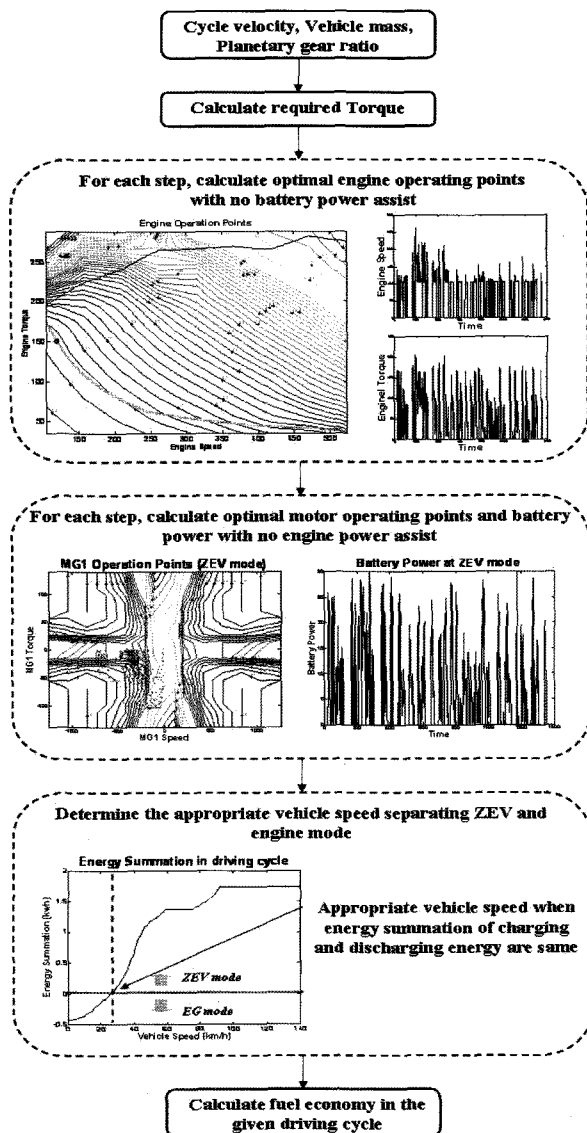


Figure 14. Flow chart of the fuel economy simulator.

used battery energy and final SOC of the battery, and therefore, the appropriate vehicle speed should be selected for the final SOC, which is similar to the initial SOC.

The steps for calculating fuel economy in given driving cycles are described below.

Step 1: Calculate the required torque by the velocity profile of given driving cycle.

Step 2: Determine operating points of engine and motors for maximum fuel economy with no power assist.

Step 3: Determine operating points for engine and motors with no engine power in the ZEV mode.

Step 4: Determine the appropriate vehicle speed separating ZEV and engine mode.

Step 5: Calculate fuel economy in given driving

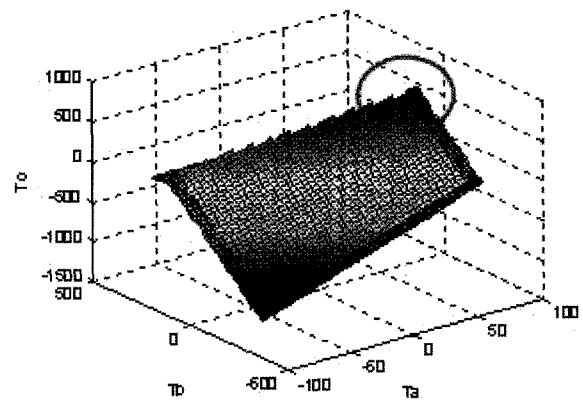
cycles.

These steps are outlined in figure 14 and subsequently discussed in more detail.

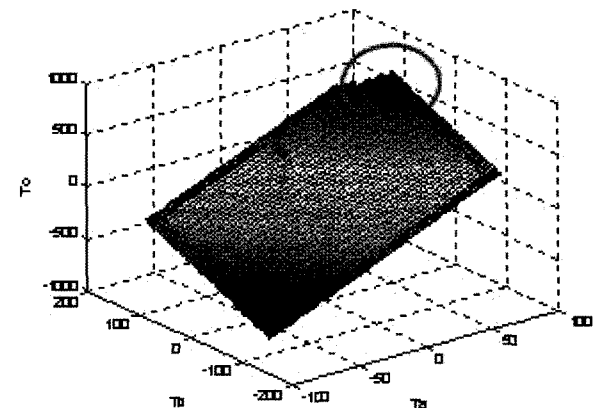
4.2. Acceleration Performance Simulator

Because a battery assumes the role for power assistance under full acceleration, the energy constraint equation, equation (12), is not useless. Because the vehicle speed is not provided, the output speed is added to the system variables. So the input-split system has three degrees of freedom under full acceleration. The acceleration performance simulator has three variables for engine speed, the motor and generator torque, and it finds the optimal operating point of engine, motor and generator for maximal output torque. Figure 15 shows the motor and generator torque for maximal output torque at each engine speed of 1000 rpm and 3000 rpm.

Figure 15 a) shows output torques as MG1 and MG2



(a) Maximum output torque at the engine speed of 1000 rpm



(b) Maximum output torque at the engine speed of 3000 rpm

Figure 15. Motor and generator torque for maximum output torque at the engine speed of 1000 rpm and 3000 rpm.

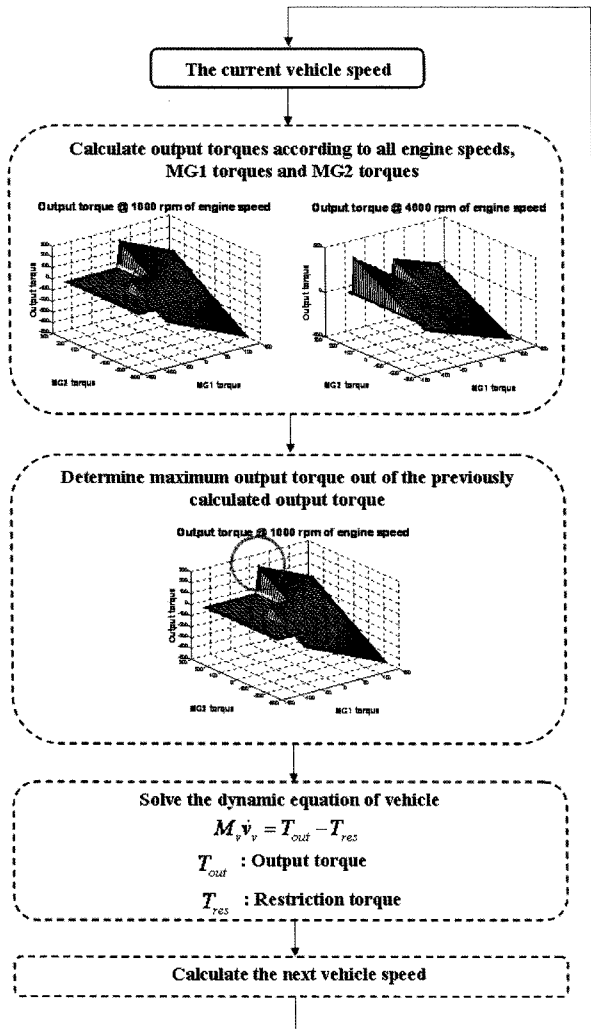


Figure 16. Flow chart of the acceleration performance simulator.

torque varies at the engine speed of 1000 rpm. The red circle indicates maximum output torque. Figure 15 b) shows output torques and maximum torques at an engine speed of 3000 rpm. The simulator finds the maximum output torque at every available engine speed and selects the maximum value among them. The maximum value is used for the output shaft torque, T_o , and engine speed, motor and generator torque at that time are used for the engine speed, motor torque and generator torque, ω_{ENG} , T_{MG1} , T_{MG2} .

The steps for calculating accelerating performance are described below.

Step 1: Calculate output torques according to all engine speeds, MG1 torques and MG2 torques at the current vehicle speed

Step 2: Determine maximum output torque

Step 3: Solve the dynamic equation of vehicle (Lu, Thompson, Mucino, and Smith, 2002)

Table 1. Optimization results for fuel economy and acceleration performance of each input-split system.

System	Performance	Optimal planetary gear ratio	Result
1	Fuel economy (km/L)	1.9	22.808
	0 to 100 time (sec)	3.0	5.75
2	Fuel economy	1.5	22.296
	0 to 100 time	1.5	6.73
3	Fuel economy	X	X
	0 to 100 time	1.5	7.28
4	Fuel economy	X	X
	0 to 100 time	3.0	6.55
5	Fuel economy	X	X
	0 to 100 time	1.5	5.23
6	Fuel economy	3.0	19.528
	0 to 100 time	1.5	4.43

Step 4: Calculate the next vehicle speed

Step 5: Execute these process over again until the aim of vehicle speed

4.3. Optimization Process

ISIGHT of Engineous company is used for optimization. The optimization parameter is a planetary gear ratio and the object function is fuel economy or acceleration performance. ISIGHT finds the optimal parameter for the minimum object function. The maximum fuel economy and the time from 0 kph to 100 kph of each input-split system are calculated by use of the optimal parameter.

4.4. Optimization Results

Optimal planetary gear ratios, maximum fuel economy and acceleration performance are shown at Table 1.

X signifies that the input split system cannot follow the velocity profile of the FTP72 cycle. Systems 3 and 4 cannot follow the FTP72 cycle and have the worst acceleration performance. So, it is impossible to use these system as single mode hybrid powertrains. Even if these system are used for one mode of dual mode, there are nearly no merits. Systems 5 and 6 cannot also follow FTP72, but they have the best acceleration performance among them. It is expected that the hybrid powertrain has excellent performance, if they are used for one mode of the dual mode system. Systems 1 and 2 are superior in fuel economy and acceleration, and especially system 1 is the best system for the single mode hybrid powertrain. Although system 2 has worse performance than system 1,

it has the best performance in fuel economy and acceleration at the same planetary gear ratio, and has an advantage in system design.

5. CONCLUSION

There are six possible input-split combinations in accordance with the connections between power sources and planetary gear. The research for the input split system used in PRIUS has been studied because this system is valued as the most suitable system for single mode powertrains. However, since the dual mode powertrain has been proposed, studies for other input split systems are necessary. In this study, we initially analyzed characteristics, advantages and disadvantages of these systems and optimized each system for fuel economy and acceleration performance. Next, we calculated the maximum fuel economy and minimum time from 0 kph to 100 kph. Because systems 1 and 2 have superior characteristics at whole vehicle speed, they can be used for both single mode and dual mode hybrid systems. Because system 6 has merits at low vehicle speed, it is expected that the hybrid powertrain performs well if they are used for one mode of the dual mode system.

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