■ 論　文 ■

생산기반 가정아래서의 통행선택행위분석

A Production-Based Approach to Travel Choice Modeling

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목　차

Key Words : 통행선택, 체감비용, 간접효용, 가계생산이론, 동차생산함수
travel choice, implicit price, indirect utility, household production theory, homogeneous production function

요　약

이 논문은 통행선택행위를 간접효용함수 대신 체감비용을 비교하는 평가방법을 제시하였다. 체감비용 비교방법은 가계생산이론을 통행자가 통행비용과 시간 등 정량적 요소와 함께 안락성과 안전성 등 정량적 요소를 동시에 고려하는 경우에 적용하여 도출되었다. 이 분석에서는 통행행위가 통행목적 달성과 함께 여러 정성적 요소를 동시에 산출하는 과정을 기술하는 특수한 형태의 동차 공동생산함수를 고안하여 이용하였다.

통행대안별 체감비용은 금전적인 통행비용에 시간가치 중에서 정량적 요소에 대해 통행자가 느끼는 기회비용을 제한 값을 합하여 산출하였다. 이러한 체감비용은 통계적 측정가능조건을 충족시킬뿐더러 통행선택행위를 보다 합리적으로 설명할 수 있어, 통행선택에 대한 통계분석에서 간접효용함수에 비해 통계측정 대상으로서 보다 우수하다.

This paper suggested an approach to characterize travel choice behaviors using the implicit price instead of the indirect utility. The choice criterion to compare the implicit prices of available trip options was developed from the utility maximization problem of a trip maker which is supposed to choose the best option from the available ones differentiated by only by the quantitative attributes such as travel cost and time but also by qualitative attributes such as comfort and safety. The utility maximization problem is constructed under household production theory, and is incorporated with a special kind of joint homogeneous production functions.

The implicit price of a certain trip option is the sum of the monetary price and the multiple of travel time and the value-of-travel-time, and the value-of-travel-time refers to the portion of wage, which can be assignable to the trip-making activity. This choice criterion is statistically identifiable, and behaviorally plausible. Moreover, this criterion has the expression simpler than the indirect utility, and therefore could be an effective target of the statistical estimation for travel choice behaviors.

Ⅰ. Introduction

Modeling of a trip maker's choice behaviors for applications to qualitative choice models has mainly consisted of searching for an indirect utility function that estimates the level of satisfaction attainable by choosing a certain trip option. Early research focused on identifying the adequate explanatory variables of the indirect utility function for statistical estimations (*e.g.*, Domencich and McFadden 1975, Charles River Associates, Inc. 1976). More recently, some researchers have searched for the appropriate functional form of the indirect utility based on microeconomic foundation (e.g. McFadden 1998, Jara-Diaz and Videla, 1989, Kockelman 2002).

These previous studies however have a common shortcoming in that they do not systematically accommodate the peculiar aspect of travel choice problem. This choice problem is significantly affected by various qualitative attributes such as comfort, safety, etc, as well as quantifiable attributes such as monetary price and travel time. One typical example that illustrates this peculiar aspect could be the choice of trip modes that are endowed with different qualitative attributes each other. Another example could the choice of shopping locations that offer different services in terms of the price and quality of goods.

The travel choice problem of consumers can hardly be accommodated by neoclassical consumer demand theory. This traditional theory can only be applicable to the consumption decision for goods and services which fulfill the following two restrictive requirements: first, each consumption item constitutes only one argument of consumer's utility; second, all the items are traded in markets at the prices predetermined by suppliers. For this reason, it is difficult for this theory to adequately accommodate the qualitative attributes of transportation service, which certainly affect the utility of consumers, in spite that they not have market prices.

One candidate theory, which has the potential to reasonably describe the travel choice decision, is household production theory initiated by Becker (1965) and Lancaster (1966). This theory postulates the decision-making process of a consumer in two steps. The first step involves the efficient household production of commodities with the inputs of goods and services traded in markets together with the consumer's time being a valuable human resource. The second step involves the utility-maximizing decision to choose the optimal bundle of commodities under the budget constraint that the production cost of commodities does not exceed the income.

However, household production theory has a serious shortcoming in the case when the production jointly yields the multiple commodities. The joint production leads to the implicit prices of multiple outputs, which usually do not fulfill the consistency condition such that the implicit prices are independent of the production amount of commodity bundles (Hall 1973, Pollak and Wachter 1975). Because of this limitation, the demand analysis based on this theory yields the indirect utility function that can hardly be applicable to the econometric test to measure the effect of changes in price on the utility and travel choice decision.

The reason for the above assertion is as follows. The implicit price constituting the indirect utility function is the target of statistical estimations for the analysis of choice behaviors. Also the implicit price which does not fulfill the consistency condition is the function of commodity bundles being the independent variables. Therefore, unless the consistency condition is fulfilled, the statistical estimation of the indirect utility function is not applicable to the forecast of the revealed preference. That is, the estimation result for a certain price systems cannot correctly reflect the effect of changes in price systems on the change in the indirect utility (Muellbauer 1974).

Unfortunately, the application of household production theory to the travel choice problem calls for the employment of the production function yielding multiple commodities. One commodity is the yield of trip-making activity to pursue a certain trip purpose common to all the *trip options*, and is termed trip output. The other commodities are the qualitative attributes of services, such as comfort and safety, and are called *hedonic commodities*. For this reason, the application of the theory has the possibility to yieldthe implicit prices that do not fulfill the consistency condition.

Another candidate approach could be random utility theory which was firstly proposed in McFadden (1973). The theory expresses the revealed preference of consumer with the indirect utility being the function of the implicit price of commodities. The theory has widely accepted as the microeconomic rationale for the statistical estimations of the qualitative choice model such as logit and probit models.

It could be said that the significance of random utility theory basically consists in the advantage of qualitative choice models which can effectively estimate the effect of qualitative attributes on the indirect utility with disaggregated data. Also the critical factor determining the descriptive power of the estimated choice models might be the microeconomic rationale employed in statistical tests. For this reason, there have been a lot of studies to search for the appropriate functional form of the indirect utility based on microeconomic foundation (*e.g.*, Kockelman 2002, McFadden 1998).

However, the previous studies have paid little attention to construct the indirect utility function by applying household production theory. Such a lack of attention in the previous studies might be an outcome of the consideration that the analysis based on the theory usually gives the indirect utility function not satisfying the consistency. However, no one can deny that household production theory is the effective microeconomic foundation being able to accommodate the effect of qualitative attributes.

Then one critical question could be whether there is really no *joint homogeneous production function* which satisfies two conditions simultaneously the joint production required for accommodating the effect of qualitative attributes on trip demands, and the consistency condition required for securing the implicit prices amenable to statistical estimations. In this regard, it is true that there is no joint homogeneous function for the production of multiple outputs with multiple inputs. However, such an assertion is not always applicable to the production of multiple outputs with one input, as will be demonstrated in this study.

The objective of this paper is to introduce an approach to analyze travel choice behaviors under household production theory, termed *the production-based approach*. The choice criterion for a certain trip option is developed from the utility maximization problem of a consumer who faces multiple trip options differentiated by the various attributes of service quality. The difference among the available options in the service quality is accommodated by employing the homogeneous production function for the production of multiple outputs with one input of the travel service offered by an option.

The production-based approach of this paper examines the travel choice behavior with the two-step utility maximization problem constructed in line with household production theory. The first-step model involves the cost minimization problem. This decision-making model estimates the minimum cost accrued in producing a given bundle of trip outputs and hedonic commodities. The second-step model is about the utility maximization problem. This model searches for the optimal bundle of commodities under the budget constraint that the production cost of commodities does not exceed the full income introduced in Becker (1965).

We begin the analysis to find the choice criterion with one specific form of joint homogeneous production functions, called *the basic production* *function.* This homogeneous production function has the functional form which can reasonably describe various types of travel choice problems. The analysis for this homogeneous production function focuses on showing that the choice criterion resulted from the utility maximization problem is expressed by the implicit prices of trip outputs for all the available options, instead of the indirect utilities. It is also shown that the implicit prices of service outputs satisfy the consistency condition.

Subsequently, we apply the analysis for the basic production function to number of specific travel choice problems. One group of specific choice problems deals with the mode, route, and destination choice problems. Each of these choice problems employs the simplified version of the basic choice problem, which reasonably depicts the decision-making environment of the specific choice problem. Another group of the problems covers the location choice problems which can accommodate the case when hedonic commodities are produced from the multiple sources such as service or goods to be purchased, the service process of supplier, and access and egress trips. Each choice problem is analyzed using the extended version of the basic production function, which satisfies the requirement of jointness and homogeneity.

Ⅱ. Travel Choice Problem

1. Trip options

A consumer generally makes the trip toachieve a certain trip purpose, such as working, schooling, shopping, recreational activities, etc. To achieve a certain trip purpose, the consumer has to make a choice on the trip option. *Each option* generally is characterized by destination, time of day, mode and route. Such a choice problem under the condition that trip purpose is predetermined is termed *the travel choice* problem.

A trip option could be differentiated by two categories of trip service attributes quantitative and qualitative attributes. *The quantitative attributes* can be measured in an objective scale, irrespective of the consumer's perception. The attributes in this category are assumed to consist of price (or monetary cost) and travel time only. In contrast, *the qualitative attributes* cannot be quantified in an objective manner. Their quantities depend upon the subjective perception of consumers. The attributes include comfort, safety, privacy, dignity, etc.

We denote an option by an integer mn. The first index m designates the combination of destination, time of day, and mode. Each group m is usually differentiated from the other groups by qualitative attributes. On the other hand, the second index n denotes travel route. This index distinguishes one option only by quantitative attributes from the other options belonging to the same service group m.

At this stage, it might be relevant to note that one possible way to denote an option could be to use the index $dtmn$, where d and t represent destination and time of day. This representation has the advantage in that each option is denoted more precisely. However this representation calls for the use of more complex index without improving the rigor the demand analysis.

We differently accommodate the quantitative and qualitative attributes in the demand analysis. We hypothesize that the quantitative attributes of an option such as its price and service time are the variables determining the implicit price of the corresponding option. These attributes are therefore called quantitative variables. In contrast, we assume that each qualitative attribute is an independent argument of the utility function. Each qualitative attribute is therefore termed a hedonic commodity. For example, comfort is assumed to a hedonic commodity different from the other hedonic commodities such as safety.

Specifically, we postulate that the arguments of the utility function are composed of two groups; trip outputs and the hedonic commodities. The trip output for a certain trip purpose is the prime product of the activity to make the trip. In contrast, the hedonic commodities are the byproducts of the trip-making activity. Moreover a certain hedonic commodity can be produced by consuming the other kinds of qualitative services. For example, the comfort perceived in trip could be produced by consuming the other kinds of services such as entertainment.

2. Production functions

The production function specifies the production process of commodities being the arguments of the utility with the inputs of goods and services traded in the market. The key concern here is to construct a joint homogeneous production function which can be applicable to the travel choice problem. This production function, termed *the basic production function*, is presented below.

The production yields two groups of commodities. One group composed of only one commodity is the tripoutput which refers to the yield of trip-making activities pursuing a predetermined trip purpose, and its quantity is denoted by y. The other group isthe hedonic commodities being the byproducts of the activity to consume the qualitative service, called the hedonic commodities, and their quantities are expressed by $z \equiv (z_1, ..., z_k)$..

The production consumes two groups of inputs. One group is the travel services offered by multiple trip options, denoted by mn or $m'n'$, where $m \in <1, M>$ and $n \in <1, N_{m>}$. The quantities purchased from the available options indicate the number of trips, and are expressed by $q \equiv (q_{11}, ..., q_{MN})$ [1]. Another group are J services and goods consumed for the production of hedonic commodities only, and each of which is denoted by $j \in <1, J>$. The quantity of these inputs to commodity k is expressed by $x_k \equiv (x_{k1}, ..., x_{kj})$.

Assumption 1: The production function of a consumer satisfies the following.

(1) The production function of the trip output, denoted by Y', is

$$Y'(q;y) = y - \sum_{mn} a_m q_{mn} = 0$$ [2]

where $a_m > 0$ is the yield per the trip using group m.

(2) The production function of the hedonic commodities, denoted by Z'_k, is

$$Z'_k(q, x_k, t_k; z_k) = z_k - \sum_{mn} b_{km} t_{mn} q_{mn} - Z_k(x_k, t_k) = 0, \ \forall k$$

where b_{km} is the production coefficient of commodity k for option mn, t_{mn} the travel time of mn, t_k the consumer's time inputted to production of k, and Z_k the substitute production function of k.

(3) Every production function Z_k exhibits constant returns in input x_k. The input x_k is non-joint to the other functions F_i, for every $i \neq k$. Moreover, the production decision always satisfies the condition that $x_k > 0$.

A series of conditions in Assumption 1 specifies the basic production function constructed so as to fulfill the following two requirements. First, the trip-making activity is the joint inputs to the

1) Throughout this study, we denote all the available options by $(11, ..., MN)$ instead of $(11, ..., M_N)$, only to simplify the expression.

2) The first study to use the expression similar to $Y'(q; y)$ might be Moses and Williamson (1963), which specifies the yield of work trip with the formula $y = \sum_m q_m$ in analyzing the mode choice behavior of commuters.

multiple commodities, one kind of trip output and multiple kinds of hedonic commodities, as formulated in Assumptions 1(1) and 1(2). Second, this joint production function is homogeneous of degree one in both inputs and outputs, as shown below.

Lemma 1: The joint production function defined in Assumption 1 is homogeneous of degree one in both inputs and outputs; that is,

$$\begin{aligned} & Y'(\alpha q;\alpha y) + \sum_k Z_k'(\alpha q, \alpha x_k, \alpha z_k) \\ & = \alpha Y'(q;y) + \alpha \sum_k Z_k')q, x_k, t_k : z_k), \end{aligned}$$

where α is a positive real number.

Proof. The production function is homogeneous of degree one in outputs, when it holds that

$$y\frac{\partial Y'}{\partial y} + \sum_k z_k \frac{\partial Z_k'}{\partial z_k} = y + \sum_k z_k$$

On the other hand, the function is homogeneous of degree one in inputs, when it holds that

$$\sum_{mn} q_{mn} \frac{\partial Y'}{\partial q_{mn}} + \sum_k \left(\sum_{mn} q_{mn} \frac{\partial Z_k'}{\partial x_{kj}} + \sum_j x_{kj} \frac{\partial Z_k'}{\partial x_{kj}} + t_k \frac{\partial Z_k'}{\partial t_k} \right) = y - \sum_k z_k$$

The proof of the above two equalities are presented in Appendix A. Q.E.D.

A set of conditions in Assumption 1 approximates the functional relationships between the two groups of inputs and outputs. These conditions are formulated under number of premises for the trip-making environment and implicit assumptions about the perception of consumer, which are clarified below.

First, Assumption 1(1) expresses the process to produce a trip output from the group of trip options, under the premise that there is no substitute for the travel service, which yields the same kind of trip output. It also postulates that the yield per service offered by option mn is a constant a_m common to all the options belonging to group m. It also hypothesizes thattotal amount of service output is simply the additive sum of yields of all the individual services.

The trip output of service group m, denoted by a_m, might be different from that of the other groups. Such a difference is taken place when group m has a different trip destination and/or time of day from the other groups. For example, a reasonable specification for the trip output of shopping trips could be the purchasing amount of groceries per trip. Also it is usual that the purchasing amount of groceries per trip usually differs by location. Therefore it can be said that the quantity a_m for shopping trips could differs from the quantity $a_{m'}$, for some $m' \neq m$. Another example of the trip output could be the psychic reward of sightseeing trips. This psychic reward could differ by the season to visit a certain place.

Second, the term $b_{km} t_{mn} q_{mn}$ in Assumption 1(2) estimates the amount of hedonic commodity k yielded from the service of option mn. The yield of hedonic commodity from non-durable service is linearly proportional to the service time that is, the amount of commodity k is estimated by multiplying the production coefficient b_{km} to the total service time $t_{mn} q_{mn}$. This coefficient b_{km} is different by group m, but common to all the options belonging to the same group m.

One example that could well explain this aspect of the coefficient b_{km} is the air passenger service. It is certain that the coefficient differs by seat. For example, a passenger feels that the first class seat is more comfortable than the economy. Then, it can be said that the first class has a larger coefficient of comfort than the economy class. In contrast, it could be asserted that a particular seat, e.g., economy class, has the identical coefficient across the carriers or travel routes.

Third, the production coefficient b_{km} represents the perception of consumer, under the assumption that the perception does not change throughout the analysis period. In reality, this coefficient is affected by many factors constituting the decision-making environment, such as physical and psychic condition, personal schedule, available information, etc. Therefore, the assumption that the coefficient b_{km} is deterministic implies that the decision-making environment is not changed throughout the analysis period.

Fourth, Assumption 1(2) depicts the production process of hedonic commodities, under the hypothesis that the consumer supplements each commodity k with a substitute production process, expressed by $Z_k(x_k, t_k)$. This substitute production consumes the two different kinds of inputs; ordinary goods and services being traded in markets, and consumer's time spent. Nonetheless, the substitute production Z_k yields the hedonic commodity, which identical byproduct k produced in the process to make the trip.

There are number of examples that could support the above hypothesis. For example, comfort is an important attribute of transportation services. One way of substituting this attribute could be taking rest at home. Another important attribute of transportation services is safety. One substitute way of enhancing a level of safety from accidents could be to purchase a more expensive insurance.

Fifth, Assumption 1(3) leads to the implicit price of every hedonic commodity, which depends only upon the price of goods and services traded in the market, but independent of the travel choice decision and the consumption amount of the hedonic commodity. Specifically, the two assumptions that the production Z_k is non-joint to the others and that its input satisfies the condition $x_k > 0$ guarantee that the implicit price is determined solely by the function Z_k. Also, the assumption that the function Z_k exhibits the constant returns gives the implicit price which is independent of the amount of commodity k produced.

3. Utility maximization problem

A trip maker who follows household production theory is supposed to choose the optimal bundle of commodities being the outputs of the production defined in Assumption 1, under the budget constraint that the production cost of the commodity bundle does not exceed the full income.Such a decision-making mechanism of the trip maker could be postulated as below.

Assumption 2: The decision of a consumer who is willing to maximize the utility satisfies the following.

(1) The utility of a trip maker, denoted by U, depends not only upon a trip output y but also upon multiple hedonic commodities $z=(z_1,\ldots,z_k)$, and strictly concave and differentiable in the relevant region of y and z_k, for every k.

(2) The production of a commodity bundle, expressed by (y,z), is undertaken under the following budget constraint:

$$\sum_{mn}(p_{mn}+wt_{mn})q_{mn}+\sum_k\left(\sum_j p_j x_{kj}+wt_k\right)\leq \overline{M}=I_o+wT_o,$$

where p_{mn} is the monetary price of option mn, w the value-of-time equal to wage, p_j the monetary price of input j, I_o the non-labor income, T_o the evaluation period of the choice decision problem, and $\overline{M}$ the full income.

(3) In addition, it is assumed that the price P_j, for every j, is fixed, and that service output is a normal commodity.

The decision-making problem defined in Assumption 2 is specified in the fashion widely accepted in the literature for household production theory. The utility function proposed in Assumption 2(1) has the structure identical to that of neoclassical consumers, except for one difference that the arguments of the former consist of commodities. Also, the budget constraint defined in Assumption 2(2) is constructed by employing the concept of the full income.

The budget constraint in Assumption 2(2) is formulated under the conditionthat the consumer earns the labor income at the fixed wage rate w without any binding constraint on working hours. By this assumption, the full income $\overline{M}$ is expressed by the sum of the non-labor income I_o and the value of available time wT_0(Pollak and Wachter 1975). Likewise, the price of q_{mn} is estimated by $p_{mn}+wt_{mn}$, and the price of x_{kj} is expressed by P_j+wt_j.

Assumption 2(3)is equivalent to hypothesize that the implicit price of every hedonic commodities is constant, since this implicit price depends only upon the price of inputs to the substitute production Z_k, as noted previously. This assumption will greatly simplify the forthcoming demand analysis for qualitative service, since it is not the main concern to analyze the impact of the change in the implicit price of hedonic commodities on the demand for qualitative service.

Combining the two sets of Assumptions 1 and 2 gives the utility maximization problem, which can be expressed by the Lagrangian L_o such that

$$L_o(q,x,y,z,\lambda,\mu,\varnothing,\eta)=\max U(y,z)+\lambda\Big(\sum_{mn}a_m q_{mn}-y\Big) \quad (1)$$
$$\sum_k \mu_k\Big(\sum_{mn}b_{km}t_{mn}q_{mn}+Z-k(x_{k,}t_k)-z_k\Big)+\sum_{mn}\varnothing_{mn}q_{mn}$$
$$\eta\Big(\overline{M}-\sum_{mn}(P_{mn}+wt_{mn})q_{mn}-\sum_k\Big(\sum_j p_j x_{kj}+wt_k\Big)\Big)$$

where $x\equiv\sum_k x_k$ is the sum of inputsto the substitute production $Z_k(x_k)$, $\forall k$, and $\lambda>0$, $\mu\equiv(\mu_1,\ldots,\mu_k)>0$, $\varnothing\equiv(\varnothing_{11,}\ldots,\varnothing_{MN})\geq 0$ and $\eta>0$ are Lagrange multipliers.

The utility maximization problem Eq. (1) is too complex to carry out the demand analysis to figure out the structure of the optimal choice decision. For this reason we conduct the demand analysis with the two sub-optimization problems obtained by decomposing the utility maximization problem, as in the case of previous studies for household production theory. Such a demand analysis, called the two-step demand analysis, is used to sequentially evaluate the optimality conditions of the two sub-optimization problems interrelated with each other, aswill be presented in the subsequent section.

Ⅲ. Optimal Travel Choice Decision

1. Consistency of implicit prices

The first sub-optimization problem of the utility maximization problem in Eq. (1) is t*he cost minimization problem* which estimates the optimal input of the production under the condition that the amounts of the commodities are arbitrarily given. The analysis for this cost minimization problem is used to identify the structure of the implicit price of the commodities, which generally depends upon the functional form of the production function incorporated into the cost minimization problem. The analysis puts emphasis on how the homogeneity and jointness of the basic production function constructed in Assumption 1 exert the effect on the structure of the implicit price.

The cost minimization problem is to used search for the optimal bundle of inputs (q,x), which gives the minimum cost necessary for the production of an arbitrary commodity bundle $(y,\ z)$. The Lagrangian of this minimization problem, denoted by L_1, is

$$L_1(q,x,\pi,\varphi,\gamma)=\min\Big\{\sum_{mn}(p_{mn}+wt_{mn})q_{mn}+\sum_k\Big(\sum_j p_j x_{kj}+wt_k\Big)\Big\}$$
$$+\pi\Big(y-\sum_{mn}a_m q_{mn}\Big)+\sum_k\varphi_k\Big(z_k-\sum_{mn}b_{km}t_{mn}q_{mn}-Z_k(x_{k,}\ t_k)\Big)-\sum_{kn}\gamma_{mn}q_{mn} \quad (2)$$

where $\pi>0$, $\varphi\equiv(\varphi_1,\ldots,\varphi_K)>0$ and $\gamma\equiv(\gamma_{11},\ldots,\gamma_{MN})\geq 0$ are Lagrange multipliers.

The optimalsolutions of the Lagrange coefficients π and $\varphi$are the implicit prices of service output and the hedonic commodities, respectively. Also the structures of those implicit prices are delineated by the basic production function in the cost minimization problem, which is homogeneous of degree one in both inputs (q, x) and outputs (y, z), and also has the joint input $q$to multiple outputs (y, z). Such implicit prices of commodities are estimated in the special case of only one option being available below.

Lemma 2: Under the condition that only one available option mn, the cost minimization problem in Eq. (2) gives the implicit price of service output, denoted by π_{mn}, and the implicit price of hedonic commodity k,denoted by $\overline{\varphi_k}$, such that

$$\pi_{mn}(p_{mn,}t_{mn}) = \frac{1}{a_m}(p_{mn} + v_m t_{mn}),$$
$$\overline{\varphi_k} = p - j \Big/ \frac{\partial Z_k(\overline{x}_{k,} \overline{t}_k)}{\partial x_{kj}} = w \Big/ \frac{\partial Z_k(\overline{x}_{k,} \overline{t}_k)}{\partial t_k}, \forall k, j$$

where

$$v_m = w - \sum_k \overline{\varphi}_k b_{km}$$

Proof. The results can be proved by evaluating the Kuhn-Tucker conditions of L_1, as shown in Appendix B. Here note that the assertion that $\overline{\varphi}_k$is constant is the consequence of Assumption 2(3), as pointed out previously. Q.E.D.

Lemma 2 confirms that the implicit prices of all the commodities satisfy the consistency condition that the prices are independent of commodity bundle (y, z) produced[3)]

The lemma well depicts how the joint input q affect the structure of the implicit prices of commodities. The joint input q does not exert any impact on the implicit price $\overline{\varphi}_k$. This implicit price depends solelyupon the price of inputs to the substitute production Z_k which uses the non-joint input x_k. In contrast, the implicit price π_{mn}, depends not only on the price and service time of the option but also the implicit price of hedonic commodities.

The implicit price π_{mn}, expressed by $a_m^{-1}(p_{mn} + v_m t_{mn})$, has the following structure. First, the implicit price that reflects the difference in the yield of trip output per trip, denoted by a_m, among differentiated service groups. Second, it estimates the total cost per trip output in the full income term, which is the sum of monetary price P_m and the value for service time $v_m t_{mn}$. Third, the time value $v_m t_{mn}$ reflects the perception of consumer about the value for the qualitative attribute of differentiated service group m.

The term v_m, called *the value-of-travel-time*, is smaller than the value-of-time w by margin $\sum_k \overline{\varphi}_k b_{km}$. This margin estimates the sum of the perceived values for the hedonic commodities jointly produced in trip. Also each perceived value $\overline{\varphi}_k b_{km}$ represents the opportunity cost of the byproduct amounting to b_{km}, which equals the cost saving of substitute productions Z_k. Therefore, the value-of-travel-time v_m corresponds to the portion of the value-of-time assignable to the trip-making activities to pursue the predetermined trip purpose.

Proposition 1: Under the condition that multiple options are available, the cost minimization problem in Eq. (2) gives the following implicit prices of commodities.

(1) The implicit price $\overline{\varphi}_k$ is identical to the one in Lemma 2.

(2) The implicit price of trip output, denoted by $\overline{\pi}$, is estimated by

$$\overline{\pi}(p,t) = \min_{mn} \{\pi_{mn}(p_{mn}, t_{mn})\}.$$

(3) The optimal solution $\overline{q}_{mn}$ is estimated by

3) The consistency condition defined in Muellbauer (1974) refers to the condition that the implicit income function $\sum_k \hat{\pi}_k Z_k$is linearly proportional to the cost function $C(p,t;,y,z)$. Such a consistency condition holds, if and only if the implicit price is independent of y and z, as shown in Muellbauer.

$$a_m \bar{q}_{mn} \begin{cases} y, \quad \mathrm{if} \bar{\pi} = \pi_{mn} < \pi_{m;n;}, \forall m'n' \neq mn \\ = 0, \quad \mathrm{if} \bar{\pi} < \pi_{mn} \\ \leq \sum_{m'n' \in I_{mn}} a_{m'} \bar{q}_{m'n'} = y, \mathrm{if} \bar{\pi} = \pi + mn = \pi_{m'n'}, \\ \qquad \mathrm{for}\, some n'n' \neq mn \end{cases}$$

where $I_{mn} = \{m'n' | \pi_{m'n'} = \bar{\pi}\}$ has more than one element of mn.

Proof. The proof is presented simultaneously in the proof of Lemma 2 in Appendix B.

Proposition 1 depicts the optimal choice decision of a trip maker in the case when multiple options are available, and can be interpreted as follows. First, Proposition 1(2) shows that the implicit price of trip output, denoted by $\bar{\pi}$, is equal to the implicit priceof trip output for the option having the least value of this implicit price. Second, the condition in Proposition 1(3) depicts the converse that the consumer chooses the option that offers the least implicit price. Third, the implicit prices $\bar{\pi}$ and $\bar{\varphi}_k$ for every k, estimated in Proposition 1(1), satisfy the consistency condition that is, they are independent of y and z, as in the case of only one option being available.Fourth, in the case when $\bar{\pi} = \pi_{mn} = \pi_{m'n'}$, the cost minimization problem has the degenerate optimal solutions of q_{mn}.

The condition in Proposition 1(2), which compares the implicit prices of available options, is called *the choice criterion*. The implicit price in the choice criterion satisfies the consistency condition. Therefore, the choice criterioncan be applicable to an arbitrary choice of output bundle (y, z), including the optimal bundle for the utility maximization problem in Eq. (1). This implies that the option chosen by a utility maximizer can be identified by simply comparing the implicit prices of available options.

2. Revealed preference condition

Here we analyze the second sub-optimization problem of the utility maximization problem in Eq. (1), which is termed *the reduced utility maximization problem*. This optimization problem is used to estimate the optimal commodity bundle maximizing the utility of the trip maker. The analysis of the reduced utility maximization problem focuses on showing that the decisionto choose the least implicit price option is amenable to econometric estimations.

The utility maximization problem is used to search for the optimum bundle of (y, z), which maximizes $U(y, z)$ under the budget constraint that the production cost of (y, z) does not exceed the full income. ItsLagrangian, denoted by L_2, is expressed by

$$L_2(y, z, \eta) - \max U(y, z) + (\overline{M} - C(p, t; y, z)) \tag{3}$$

where $\eta > 0$ is a Lagrange multiplier, $P \equiv (P_{11}, \ldots, P_{mn})$ the prices of available trip options, and $t \equiv (t_{11}, \ldots, t_{MN})$ the travel times of the options. Here the cost function $C(p, t; y, z)$, which estimates the minimum production cost of (y, z), has the following expression:

$$C(p, t, ; y, z) - \sum_{mn} (p_{mn} + wt_{mn}) \bar{q}_{mn}(y, z) + \sum_{kj} (p_j + wt_j) \bar{x}_{kj}(y, z) \tag{4}$$

By virtue of the consistency of the implicit prices, the cost function is simplified as below.

Lemma 3: The cost function for the cost minimization problem in Eq. (2) satisfies the following equality:

$$C(p, t; y, z) = \bar{\pi}(p, t) y + \sum_k \varphi_k z_k$$

Proof. The above equality can be proved using the Kuhn-Tucker conditions of the cost minimization problem in Eq. (2), as shown in Appendix C. Q.E.D.

Substituting the cost function in Lemma 3into the utility maximization problem in Eq. (3) gives

$$L_2(y,z,\eta)=\max U(y,z)+\eta\left(\overline{M}-\overline{\pi}(p,t)\,y-\sum_k \overline{\varphi}_k\, z_k\right). \tag{5}$$

This utility maximization problem has the structure identical to that of neoclassical utility maximization problem, except one difference that the independent variablesof the former are commodities. Using this property, the utility-maximizing decision is characterized below.

Proposition 2: Suppose that the cost minimization problem in Eq. (2) leads to the choice of option mn. Then this choice decision satisfies *the revealed preference condition* such that

$$\overline{\pi}=\pi_{mn}\ \langle\ \pi_{m'n'} \Leftrightarrow$$
$$U(\overline{y},\overline{z})=U(\overline{y}_{mn},\overline{z}_{mn})\ \rangle\ U(\overline{y}_{m'n'},\overline{z}_{m'n'}),\quad \forall m'n'\neq mn,$$

where $(\overline{y},\overline{z})$ is the optimal solution of $(y,\ z)$, when all the multiple options are available whereas, $(\overline{y}_{mn},\overline{z_{mn}})$ is the optimal solution, when only one option mn is available.

Proof. See Appendix D

Proposition 2 well depicts the revealed preference of the utility maximizer with use of the implicit price of available options. "The left side implies the right" indicates that the choice of option mn having the least implicit price can attain a higher utility than the choice of the other options. Conversely, "the right side implies the left" shows that option mn chosen by a utility maximizer is the one offering the minimum implicit price.

Another important aspect of Proposition 2 is that the optional solution estimated by directly solving the utility maximization problem in Eq. (1) is identical to the optimal solution obtained from the two-step analysis which solves the cost minimization problem in Eq. (2) and the reduced utility maximization problem in Eq. (5) sequentially, as proved in Appendix D. Specifically, the equality $\overline{\pi}=\pi_{mn}$ in the left sideof Proposition 2 implies that the two different analyses yield the identical solutions of the implicit price together with the chosen option. Also the equality $U(\overline{y},\overline{z})=U(\overline{y},\overline{z}_{mn})$ in the right side shows that the two different analyses give the identical optimal solutions of commodities y and z.

3. Statistical identifiability

The statistical identifiability of a certain function implies that the parameter of the statistical estimation is not a function of the independent variables. This requirement of the statistical identifiability is met by the choice criterion $\pi_{mn}<\pi_{m'n'}$ for every $m'n'\neq mn$ in Propositions 1 and 2. This desirable property of the choice criterion stems from the homogeneity of the basic production function, and is proved below.

It is certain that one necessary condition for the statistical identifiability is the consistency of the implicit price π_m. That is, the implicit price is independent of y and z, of which optimal solutions to the utility maximization problem are the functions of p and t. Moreover, the statistical identifiability calls for another requirement that the unobservable term v_m in the implicit price π_m is also independent of p and t.

Proposition 3: The choice criterion in Proposition 2, which compares the implicit prices π_{mn} for every mn fulfills *the statistical identifiability condition* that all the parameters of the implicit price are independent not only y and z but also of p and t.

Proof. Every the implicit price $\overline{\varphi}_k$ in $v_m\equiv w-\sum_k\overline{\varphi}_k b_{km}$ satisfies the consistency conditions that is, it is independent of (y, z), as shown in Lemma 2. Moreover, the implicit price $\overline{\varphi}_k$ estimated in Lemma 1.1 depends only upon the input price $p_j + wt_j$ assumed to be a constant. Hence it follows the assertion. Q.E.D.

Proposition 3 indicates that the implicit price π_{mn} is amenable to statistical estimations. Specifically, one unknown term of the implicit price π_{mn} is the value-of-service-time v_m, besides the constant a_m. This term v_m being the target of the statistical estimation is an identical value, irrespective of the value of (p, t) being the independent variables of the statistical estimation. Moreover, this term v_m has an identical value, whether option mn is chosen or not.

Finally, the choice criterion to compare the implicit price of service output could be an effective target for the statistical estimation of choice behaviors. This choice criterion is sufficient to express the revealed preference, as proved in Proposition 2, and also fulfills the statistical identifiability, as shown in Proposition 3. Moreover, the statistical estimation of the implicit price does not necessitate any information about the structure of the production function of hedonic commodities in Assumption 1(2).

Specifically, the value-of-service-time v_m, expressed by $w - \sum_k \overline{\varphi}_k b_{km}$, is a function of $\overline{\varphi}_k$ and b_{km}. However the statistical estimation of the term v_m is feasible, without resource to information about $\overline{\varphi}_k$ and b_{km}. For this reason, the statistical estimation of this single value v_m is free from the specification of the production function, which can hardly be formulated in an objective manner.

4. Illustration of travel choice decision

Here the analysis of the utility maximization problem in Eq. (1) is illustrated with the following simple example. Suppose that a trip maker plans to make to trip to a certain destination to achieve a predetermined trip purpose such as working, shopping, etc. Suppose also that only undetermined choice component is the trip mode, and that two trip modes are available; auto and transit, denoted by 1 and 2, respectively. Suppose further that the outputs of the production are composed only of two commodities: a trip output referring totrip-purpose, and a hedonic commodity being defined to be comfort.

Suppose now that the cost minimization problem in Eq. (2) results in the choice ofoption 1. Then, by Proposition 1, this choice decision satisfies the following:

$$\pi_1 = p_1 + v_1 t_1 \ \langle\ \pi_2 = p_2 + v_2\, t_2 \tag{6}$$

Here, the value-of-service-times v_1 and v_2 are constants different each other, since the qualitative attributes of the two modes are different.

Consider next the case when only one mode is available. Then, by Lemma 3, the implicit prices of commodities, all of which fulfill the consistency condition, yields the cost function C_m expressed by

$$C_m(y, z) = \pi_m\, y + \overline{\varphi}\, z\ ,\quad m = 1, 2 \tag{7}$$

where $\overline{\varphi}$ is the implicit price of comfort. Note here that the implicit prices π_1, π_2 and $\overline{\varphi}$ are constant being independent of y and z.

Therefore, in case of only one option being available, the reduced utility maximization problem in Eq (5) has the budget constraint such that

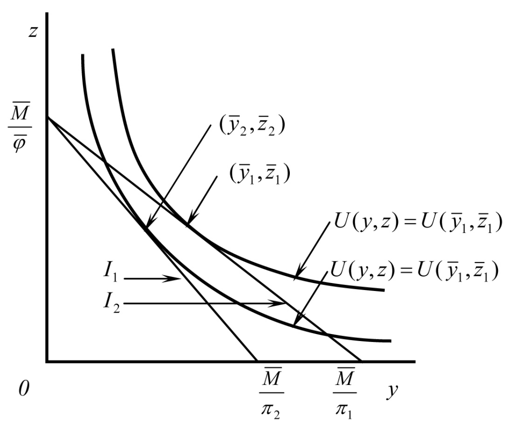
$$\overline{M} = \pi_m\, y + \overline{\varphi}\, z\ ,\quad m = 1, 2 \tag{8}$$

The above budget line for a certain mode equals the production possibility frontier of y and z under the condition that only one mode is available.

The budget line for mode 1 is located below the line for mode 2, as depicted in Figure 1. Such a result is the direct consequence of inequality (7) which holds for any value of (y, z). This result guarantees the right-side inequality of Proposition 2 that is,

$$U(\bar{y},\bar{z}) = U(\bar{y}_1,\bar{z}_1) \rangle U(\bar{y}_2,\bar{z}_2) \tag{9}$$

where $(\bar{y}, \bar{z})$ is the optimal solution in the case when the two modes are available, and $(\bar{y}_1,\bar{z}_1)$ is the optimal solution in the case when only mode 1 is available.



〈Figure 1〉 Illustration of revealed preference with an example of trip mode choice

Ⅳ. Applications Of Production-Based Approach

1. Application of the basic production function

Generally the travel choice problem is used to choose the best option among the combinations of independent choice components such as trip destination, departure period, mode, and route. This travel choice is a typical example of qualitative choice problems. We illustrate below this aspect of the travel service with a number of decision-making problems to choose the best option for a particular choice component.

To begin with, we consider the following mode choice problem of a trip maker. First, each mode has qualitative attributes different from the others; that is, each mode offers the differentiated service, and therefore is designated by option $m \in <1, M>$. Second, the destination to attain a certain trip purpose is predetermined; that is, trip output am, $\forall m$, is common to all the options. Third, each mode has only one travel route; that is, $N=1$.

In this case, the basic production functioncan be amended as follows:

$$Y'(q;y) = y - \sum_m q_m = 0, \tag{10}$$

$$Z'_k(q,x_k;z_k) = z_k - \sum_m b_{km} t_m q_m - Z_k(x_k) = 0, \quad \forall k. \tag{11}$$

This production function is actually a simplified version of the basic production function, which also is homogeneous of degree one.

It is immediate from Proposition 2that the utility maximization for this production function has the optimal solution satisfying the following condition: the chosen mode m has a smaller implicit price of service output π_m than the competing modes; that is,

$$\pi_m = p_m + v_m t_m \langle \pi_{m'} = p_{m'} + v_{m'} t_{m'}, \quad \forall m' \neq m, \tag{12}$$

where $v_m = w - \sum_k \bar{\varphi}_k b_{km}$.

One example that can well illustrate the advantage of the above mode choice criterion is an air passenger who buys a first class ticket, instead of a business or economy class ticket. One common approach might be to compare the total price of the service, estimated by $p_m + wt_m$, where w is the wage equal to the value-of-time. In contrast, the above criterion compares the implicit price of service, estimated by $p_m + v_m t_m$, where v_m is the value-of-service-time specific to option m.

It is certain that the total price of first class is larger than that of the other options, because the travel time is identical across all the options. Therefore, there is no reason for the travelerwho prefers the cheapest option in terms of the total price to choose the first class ticket. In contrast, the choice criterion to compare the implicit price provides the following plausible reasoning: the choice of the first class ticket is the outcome of the judgment that the additional fare does not exceed the additional value assignable to the better services, such as comfort, privacy, prestige, etc.

Subsequently, we consider the choice problem for the trip departure time with an example of recreational trip. Suppose that a traveler has already decided the resort area and the trip mode. Suppose also that the service output of recreational trip in terms of psychic rewards differs by the departure period of trip. For example, the psychic reward achievable at a sea beach usually differs by the period to visit the place. Suppose however that all the options have no significant difference in qualitative attributes of in-site service and access trip.

In this case, the production function for service output has to reflect the difference in the amount of trip output by period, expressed by a_m. Hence the production function for trip output could be expressed by

$$Y'(q;y) = y - \sum_m a_m q_m = 0 \qquad (13)$$

On the other hand, the production function for hedonic commodities could be assumed to be identical across all the options, since all the options have no significant difference in the qualitative attributes of in-site service and access trip. Therefore, the production function can be formulated as follows:

$$Z'_k(q, x_k; z_k) = z_k - \sum_m b_k t_m q_m - Z_k(x_k) = 0, \quad \forall k \qquad (14)$$

where the production coefficient b_k is common to all the options.

The production function defined above is also homogeneous of degree one, as in the case of the basic production function. Hence it follows from Proposition 2 that the chosen departure time m satisfies the following:

$$\pi_m = \frac{1}{a_m}(p_m + vt_m) \langle \pi_{m'} = \frac{1}{a_{m'}}(p_{m'} + vt_{m'}), \quad \forall m' \neq m \qquad (15)$$

where$v = w - \sum_k \overline{\varphi}_k b_k$. Here the value-of-service-time v is common to all the options.

The choice criterion in Eq. (15) can well explain the decision of the traveler who chooses the busiest period, denoted by m. The period m has the possibility to require the higher in-site service charge and the longer travel time than the other options. Therefore the option $m$has the possibility to call for the larger cost than the others in terms of the total price per trip, expressed by $p_m + wt_m$, and the implicit price per trip, estimated by $p_m + vt_m$. However the option m can give the minimum implicit price per one unit of on-site psychic reward achievable, estimated by $(p_m + vt_m)/a_m$, since the service output am is larger than that of the other period.

Finally, we analyze the route choice of a trip-maker under the condition that the trip maker has already determined the trip destination and mode. In this case, the choice problem of the trip maker is to choose the best route among multiple options differentiated only by quantitative attributes of travel cost and time, each of which is designated by $n \in <1, N>$. The production function for this choice problem can be expressed as follows:

$$Y'(q;y) = y - \sum_n q_n = 0, \qquad (16)$$

$$Z_k'(q,x_k;z_k) = z_k - \sum_n b_k t_n q_n - Z_k(x_k) = 0, \quad \forall k, \quad (17)$$

where the factor b_k is common to all the routes.

Then the decision to choose route n is expressed by

$$\pi_n = p_n + vt_n \langle \pi_{n'} = p_{n'} + vt_{n'}, \quad \forall n' \neq n, \quad (18)$$

where $v = w - \sum_k \overline{\varphi}_k b_k$. This choice criterion can be applicable to the route choice of highway users. It could also be applicable to the location choice of gas stations for the auto user who concerns the distance to the gas station and the price of oil together.

2. Application of the amended basic production function to location choice

The location choice problem of a consumer is generally affected not only by the service quality of the destination but also by the transportation cost to reach the destination. One way to analyze this location choice problem could be to employ the production which has two independent sources generating the hedonic commodities. Such an approach is illustrated with the location choice problem searching the the best location of services such as groceries, entertainment, recreation, hotel, and dining services.

Suppose that the production source of hedonic commodities consists of the in-site activity to purchase the service and the activity to make access and egress trips. Suppose also that the amount of hedonic commodities for the in-site service is linearly proportional to the in-site service time, as in the case of access and egress trips. Suppose further that certain location have more than one trip option differentiated by mode and route.

In this case, it might be necessary to identify the options by combining the two kinds of choice components, location and access trip options. Also each location could be denoted by mn, where m represents the option offering a certain heterogeneous service, and n represents the options differentiated only by quantitative attributes. Furthermore, one location has to be designated by multiple options if the multiple trip modes and routes are available.

Then the production function for trip output could be applied without any modification; that is,

$$Y'(q;y) = y - \sum_{mn} a_m q_{mn} = 0 \quad (19)$$

In contrast, the production function for hedonic commoditiescould be amended as follows:

$$Z_k'(q,x_k;z_k) = z_k - \sum_{mn}\left(b_{km}^s t_{mn}^s + b_{km}^a t_{mn}^a\right) q_{mn} - Z_k(x_k) = 0, \quad \forall k \quad (20)$$

where t_{mn}^s and t_{mn}^a are the in-site service time and the access/egress time, respectively, and b_{km}^s and b_{km}^a are the production coefficients of commodity k for the in-site service time and the travel time, respectively.

The cost minimization problem for the production function defined above, which finds the optimal value of inputs (q, s) necessary for the production of outputs (y, z), can be expressed as follows:

$$\begin{aligned} & L_1(q,x,\pi,\varphi,\gamma) \\ & = \min\left\{ \sum_{mn}(p_{nm}^s + wt_{mn}^s) q_{mn} + \sum_{mn}(p_{nm}^a + wt_{mn}^a) q_{mn} + \sum_{kj}(p_j + wt_j) x_{kj} \right\} \\ & + \pi\left(y - \sum_{mn} a_m q_{mn} \right) \\ & + \sum_k \varphi_k \left(z_k - \sum_{mn}\left(b_{km}^s t_{mn}^s + b_{km}^a t_{mb}^a\right) q_{mn} - Z_k(x_k) \right) - \sum_{mn} \gamma_{mn} q_{mn} \quad (21) \end{aligned}$$

where p_{mn}^s is the in-site service charge of mn, and p_{mn}^a is the access/egress travel cost of mn.

The production function in the above cost minimization problem is homogeneous of degree one in both inputs and outputs. Therefore, the implicit price π_{mn} is independent of the values of y and z. Moreover the implicit price of the chosen option mn satisfies the condition that

$$\pi_{mn} = \frac{1}{a_m}\left(p^s_{mn} + v^s_m t^s_{mn} + p^a_{mn} + v^a_m t^a_{mn}\right)$$
$$\langle \pi_{m'n'} = \frac{1}{a_{m'}}\left(p^s_{m'n'} + v^s_{m'} t^s_{m'n'} + p^a_{m'n'} + v^a_{m'} t^a_{m'n'}\right),$$
$$\forall\, m'n' \neq mn \quad (22)$$

where $v^s_m = w - \sum_k \bar{\varphi}_k b^s_{km}$ is value-of-service-time for in-site time, and $v^a_m = w - \sum_k \bar{\varphi}_k b^q_{km}$ is value-of-service-time for access/egress time.

The above choice criterion could be applicable to shopping trips to purchase the predetermined amount of non-durable goods such as groceries. Suppose that there are two options; one is a shop at the suburban shopping mall, denoted by 1, and a near-by shop, denoted by 2. Suppose also that the shopper plans to purchase the same amount, irrespective of the location choice; that is, $a_m = 1$. Suppose further that the shopper chooses option 1, which is located far from the home than option 2.

For this case, the choice criterion in Eq. (22) is simplified as below:

$$\pi_1 = p^s_1 + v^s_1 t^s_1 + p^a_1 + v^a_1 t^a_1 \,\langle\, \pi_2 = p^s_2 + v^s_2 t^s_2 + p^a_2 + v^a_2 t^a_2 \quad (23)$$

This implies that the price of goods at the shopping mall, denoted by $P_1{}^s$, is significantly lower than that of the nearby shop, denoted by P^s_2, considering that the shop at the shopping mall requires a larger access/egress cost and time than the nearby shop.

Subsequently, we analyze the location choice problem of a traveler under the following conditions. First, the traveler considers a number of options for resort areas. Second, the service output of each option is proportional to the diversity of facilitiesfor various recreation and entertainment activities, which determines the psychic reward of the visit. Third, the yield of relevant hedonic commodities by option is affected by the landscape and supporting facilities. Fourth, the traveler plans to use the same trip mode to visit the resort area, irrespective of locations.

Then the decision to choose location m could be expressed as follows:

$$\pi_m = \frac{1}{a_m}(p^s_m + v^2_m t^s_m + P^a_m + v^a t^{a)}_m$$
$$< \pi_{m'} = \frac{1}{a_{m'}}(p^s_{m'} + v^s_{m'} t^s_{m'} + p^a_{m'} + v^a t^a_{m'})$$
$$,\ \forall\, m' \neq m \quad (24)$$

Here the term v^s_m differs by location, since each location is endowed with different qualitative attributes. In contrast, the term v^q is common to all the options, since the traveler uses the identical trip mode irrespective of locations.

The implicit price estimated in Eq. (24), which determines the location choice, accommodates the service quality of resort areas in following three ways. First, the implicit price of a location becomes smaller as the diversity of recreation and entertainment facilities, denoted by a_m, is larger. Second, the implicit price of a location also becomes smaller as the value of hedonic commodities, expressed by v_m, is larger; that is, the location which has better landscape and supporting facilities has a smaller value of v_m. Third, the implicit price of a location becomes smaller as the trip distance determining the values of p^a_m and t^a_m is shorter.

Ⅴ. Summary And Concluding Remarks

This paper applied household production theory to the travel choice problem of a trip maker who has multiple options differentiated not only by quantitative attributes such as travel cost and time but also by qualitative attributes such as comfort and safety. The choice criterion of an option was developed from the utility maximization problem constructed in line with household production theory. The key distinct feature of the utility maximization problem is to use a special kind of joint homogeneous production functions, called the basic production function.

The basic production function specifies the production process such that a single input of trip-making activity yields the multiple outputs of trip output and number of hedonic commodities, all of which constitute the arguments of the utility function. The trip output refers to the outcome of trip-making activity to pursue a certain trip purpose; whereas, each hedonic commodity represents a specific kind of qualitative attribute. Such a joint production process is formulated into the homogeneous function of degree one.

The utility maximization problem incorporated with the basic production function yields the implicit price of trip output for every option mn, denoted by π_{mn}, such that

$$\pi_{mn}(p_{mn},t_{mn})=\frac{1}{a_m}(p_{mn}+v_m t_{mm}),\quad \forall mn,$$

where v_m is value-of-travel-time of service group m, p_{mn} the monetary price of mn, t_{mn} theservice time of mn, and $a_m>0$ the yield per service of group m. Here the term v_m is the value-of-time equal to the wage minus the value of qualitative attributes of differentiated service group m. Therefore, the value-of-service-time v_m represents the portion of the value-of-time assignable to the trip-making activity.

Moreover the utility maximization problem gives an expression of the revealed preference, such that

$$\bar{\pi}=\pi_{mn}\langle\pi_{m'n'}\Leftrightarrow$$
$$U(\bar{y},\bar{z})=U(\bar{y}_{mn},\bar{z}_{mn})\rangle U(\bar{y}_{m'n'},\bar{z}_{m'n'}),\quad \forall m'n'\neq mn$$

where $(\bar{y},\ \bar{z})$ is the optimal solution of (y,z), when all the multiple options are available whereas, $(\bar{y}_{mn},\bar{z}_{mn})$ is the optimal solution, when only one option mn is available. This revealed preference condition can reasonably explain the choice behavior of consumers, and can be applicable to the econometric estimation of the choice behavior, as explained below.

First, the implicit price π_{mn} reasonably depicts the valuation of qualitative service by consumers. It estimates the total cost of service output for option mn in the full income term, which is expressed by the sum of monetary price p_m and the value for service time $v_m t_m$. Moreover the time value $v_m t_m$ reflects the perception of consumer about the value for the qualitative attribute of heterogeneous service m.

Second, the inequality $\pi_{mn}<\pi_{m'n'}$ is sufficient to express the revealed preference of trip makers, without recourse to the other terms constituting the indirect utility function. The revealed preference condition shows that the choice of option mn having the least implicit price can attain a higher utility than the choice of the other options. The condition also shows that the converse holds; that is, option mnchosen by a utility maximizer is the one offering the service at the minimum implicit price.

Third, the criterion to compare the implicit price π_{mn} is amenable to statistical estimations using qualitative choice models. The implicit price fulfills the statistical identifiability; that is, the value-of-service-time v_m being the target of statistical estimations is a constant. Moreover, the value v_m does not change whether option mn is chosen or not. Furthermore, the term v_m is free from the arbitrary choice of the specification for the production function of hedonic commodities.

Subsequently, we demonstrated that the analysis approach of the paper has the flexibility enough to handle various kinds of choice problems. For example, we were able to deduce the reason why a trip maker chooses the first-class ticket charging more expensive fare than business and economy classes, in spite that all the options require the identical travel time. We were also able to explain the reason why the traveler who visits the recreation or entertainment facilities at the peak period when in-site service charge is the most expensive and the congestion is the most severe. Moreover, we introduced the choice criterion that can explain the decision of a shopper who chooses a shop in the far-distance shopping mall instead of a near-by shop.

However, the travel choice problem evaluated in this study has the shortcoming in that the expression of the household production function is too simple to adequately reflect the real decision-making environments faced by trip makers. Such aspects which call for the further studies are as follows.

First, the household production function used in this study has the expression free from trip purpose. One way of improving such an expression could be to add the term formulating the relationship betweenthe activity to pursue a trip purpose and its derived demand for trips. Such a specification could yield the implicit price of trip output, which contains additional information about the cost incurred by the activity to pursue the trip purpose. Also, the additional information could be a useful input in better understating the choice behaviors.

Second, the analysis is confined to the homogeneous production function. In other words, we do not address the question of whether the non-homogeneous production function, which can depict more accurately the decision-making environment for a certain type of travel choices, yields results compatible with those of the homogeneous one. Moreover, we do not explain in detail the reasoning why the non-homogeneous production function results in statistically unidentifiable choice criteria.

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【Appendix A: Proof of Lemma 1】

It suffices to prove the following:

$$y\frac{\partial Y'}{\partial y}+\sum_k z_k\frac{\partial Z_k'}{\partial z_k}=y+\sum_k z_k \tag{A.1}$$

$$\sum_{mn}q_{mn}\frac{\partial Y'}{\partial q_{mn}}+\sum_k\left(\sum_{mn}q_{mn}\frac{\partial Z_k'}{\partial q_{mn}}+\sum_j x_{kj}\frac{\partial Z_k'}{\partial x_{kj}}\right)=-y-\sum_k z_k \tag{A.2}$$

The proof of (A.1) is trivial. The equality in (A.2) is the consequence of the following:

$$\sum_{mm}q_{mn}\frac{\partial Y'}{\partial q_{mn}}=-\sum_{mn}a_m q_{mn}=-y\text{, by A.1.1,} \tag{A.3}$$

$$\sum_{mn}q_{mn}\frac{\partial Z_k'}{\partial q_{mn}}=-\sum_{mn}b_{km}t_{mn}q_{mn}\text{,} \tag{A.4}$$

$$\sum_j x_{kj}\frac{\partial Z_k'}{\partial x_{kj}}=-\sum_j x_{kj}\frac{\partial Z_k}{\partial x_{kj}}=-Z_k\text{, by A.1.3,} \tag{A.5}$$

$$\sum_{mn}b_{km}t_{mn}q_{mn}+Z_k=z_k\text{, by A.1.2.} \tag{A.6}$$

【Appendix B: Proof of Lemma 2 and Proposition 1】

(1) The Kuhn-Tucker conditions of L_1 in Eq. (2): To begin with, we construct theKuhn-Tucker conditions of the cost minimization problem L_1:

$$y-\sum_{mn}a_m\bar{q}_{mn}=0\text{,} \tag{B.1}$$

$$z_k-\sum_{mn}b_{km}t_{mn}\bar{q}_{mn}-Z_k(\bar{x}_k)=0\text{, }\forall k\text{,} \tag{B.2}$$

$$\frac{\partial L_1(\cdot)}{\partial q_{mn}}=p_{mn}+wt_{mn}-\bar{\pi}a_m-\sum_k\bar{\varphi}_k b_{km}t_{mn}-\bar{\gamma}_{mn}=0\text{, }\forall mn\text{,} \tag{B.3}$$

$$\frac{\partial L_1(\cdot)}{\partial x_{kj}}=(p_j+wt_j)-\bar{\varphi}_k\frac{\partial Z_k(\bar{x}_k)}{\partial x_{kj}}=0\text{, }\forall kj \tag{B.4}$$

$$\bar{\gamma}_{mn}\bar{q}_{mn}=0\text{, }\bar{q}_{mn}\geq 0\text{ and }\bar{\gamma}_{mn}\geq 0\text{, }\forall mn \tag{B.5}$$

Here, it should be noted that, if $\bar{x}_{k+0}$, contrary to A.1.3, Eq. (B.4) does not hold.

(2) Proof of Lemma 2: When only option mn is available, the Kuhn-Tucker conditions in (B.3)~(B.5) are modified as follows: $\bar{\gamma}_{mn}$=0 in (B.3) and (B.5), and $\bar{q}_{mn}>0$ in (B.5). These modified versions of (B.3) and (B.4) gives the expressions of π_{mn} and $\bar{\varphi}_k$ in the lemma, respectively.

(3) Proof of Propositions 1.1 and 1.2: Equation (B.4) implies that the implicit price $\bar{\varphi}_k$ in the case of multiple options being available is identical to the one in the case of only one option being available. Therefore the proof can be completed by estimating the implicit price $\bar{\pi}$.

The implicit price $\bar{\pi}$ is estimated under the condition that option mn is chosen; that is, $\bar{q}_{mn}>0$. It follows from (B.5) that $\bar{q}_{mn}>0$ implies $\bar{\gamma}$=0. Substituting these relationships into (B.3) gives

$$p_{mn}+wt_{mn}-\bar{\pi}a_m-\sum_k\bar{\varphi}_k b_{km}t_{mn}=\bar{\gamma}_{mn}=0 \tag{B.6}$$

Hence it follows that

$$\bar{\pi}=\pi_{mn}=\frac{1}{a_m}(p_{mn}+v_m t_{mn}) \tag{B.7}$$

Subsequently, suppose that $\bar{q}_{m'n'}>0$ for some $m'n'\neq mn$ as well as $\bar{q}_{mn}>0$. Then it follows that $\bar{\gamma}_{m'n'}=0$. Hence, it is certain by (B.5) that

$$\overline{\pi} = \pi_{mn} = \pi_{m'n'} \tag{B.8}$$

Finally suppose that $\bar{q}_{m'n'} = 0$ for every $mn \neq mn$. In this case, (B.5) implies that $\bar{\gamma}_{m'n'} > 0$. Substituting the condition $\bar{\gamma}_{m'n'} > 0$ into (B.3) gives

$$p_{m'n'} + wt_{m'n'} - \overline{\pi} a_{m'} - \sum_k \overline{\varphi}_k b_{km'} t_{m'n'} = \overline{\gamma}_{m'n'} \rangle 0,$$

$$\forall m'n' \neq mn \tag{B.9}$$

The above equation implies that

$$\overline{\pi} = \pi_{mn} \langle \pi_{m'n'}. \tag{B.10}$$

Hence, by (B.7), (B.8) and (B.9), it follows that

$$\overline{\pi} = \pi_{mn} = \min_{m'n'} \left\{ \pi_{m'n'} = \frac{1}{a_{m'}} (p_{m'n'} + v_{m'} t_{m'n'}) \right\} \tag{B.11}$$

(4) Proof of Proposition 1.3: Consider the first case when $\overline{\pi} = \pi_{mn} < \pi_{m'n'}, \forall_{m'n'} \neq mn$. Then, by (B.3) and (B.5), it holds that

$$\overline{\gamma}_{mn} = 0 \text{ and } \overline{q}_{mn} \rangle 0, \text{ but } \overline{\gamma}_{m'n'} \rangle 0 \text{ and } \overline{q}_{m'n'} = 0 \tag{B.12}$$

Hence, by (B.1), it follows that $a_m \bar{q}_{mn} = y$. Consider the second case when $\overline{\pi} < \pi_{mn}$. Then, by (B.3), it holds that $\bar{\gamma}_{mn} > 0$. Hence, by (B.5), it follows that $\bar{q}_{mn} = 0$. Finally consider the third case when $\overline{\pi} = \pi_{mn} = \pi_{m'n'}$ for every $m'n' \in I_{mn}$. In this case, by (B.3), it holds that

$$\overline{\gamma}_{m'n'} = 0, \text{ and } \overline{q}_{m'n'} \geq 0, \ \forall m'n' \in I_{mn} \tag{B.13}$$

Hence, by (B.1), it follows that

$$\sum_{m'n' \in I_{mn}} a_{m'} \overline{q}_{m'n'} = y. \text{ Q.E.D.} \tag{B.14}$$

【Appendix C: Proof of Lemma 3】

The details of the proof are as follows:

$$C(p,t;y,z) = \sum_{mn} (p_{mn} + wt_{mn}) \overline{q}_{mn} + \sum_{kj} (p_j + wt_j) \overline{x}_{kj} \tag{C.1}$$

$$= \sum_{mn} \left(\overline{\pi} a_m + \sum_k \overline{\varphi}_k b_{km} t_{mn} + \overline{\gamma}_{mn} \right) \overline{q}_{mn} + \sum_{kj} \overline{\varphi}_k \overline{x}_{kj} \frac{\partial Z_k(\overline{x}_k)}{\partial x_{kj}} \text{ by (B.3) and (B.4)} \tag{C.2}$$

$$= \sum_{mn} \overline{\pi} a_m \overline{q}_{mn} + \sum_k \overline{\varphi}_k \left(\sum_{mn} b_{km} t_{mn} \overline{q}_{mn} + Z_k(\overline{x}_k) \right) \text{ by (B.5) and A.1.3} \tag{C.3}$$

$$= \overline{\pi} y + \sum_k \overline{\varphi}_k z_k \text{ by A.1.1 and A.12.} \tag{C.4}$$

Here, (C.3) is derived by substituting the following equalities: $\bar{\gamma}_{mn} \bar{q}_{mn} = 0$ in (B.5), and $\sum_j \bar{x}_{kj} \partial Z_k(\overline{x_k}) / \partial x_{kj} = Z_k(\overline{x_k})$, which holds under Assumption 1.3. Q.E.D.

【Appendix D: Proof of Proposition 2】

(1) We first prove the following part of Proposition 2:

$$\overline{\pi}(p,t) = \pi_{mn}(p_{mn}, t_{mn}) \Leftrightarrow U(\overline{y}, \overline{z}) = U(\overline{y}_{mn}, \overline{z}_{mn}) \tag{D.1}$$

Here $\bar{\pi}$ and $U(\overline{y}, \bar{z})$ are the optimal solutions for the utility maximization problem L_0 in Eq. (1) such that

$$L_o(q,x,y,z,\lambda,\mu,\phi,\eta) = \max U(y,z) + \lambda \left(\sum_{mn} a_m q_{mn} - y \right)$$
$$+ \sum_k \mu_k \left(\sum_{mn} b_{km} t_{mn} q_{mn} + Z_k(x_k) - z_k \right) + \sum_{mn} \phi_{mn} q_{mn}$$

$$+\eta\left(\overline{M}-\sum_{mn}(p_{mn}+wt_{mn})\,q_{mn}-\sum_{kj}(p_j+wt_j)x_{kj}\right) \quad \text{(D.2)}$$

On the other hand, π_{mn} and $U(\check{y}_{mn}, \bar{z}_{mn})$ are the optimal solutions for the two sub-optimization problems L_1 in Eq. (2) and L_2 in Eq. (3), such that

$$L_1(q,x,\pi,\varphi,\gamma) = \min\left\{\sum_{mn}(p_{mn}+wt_{mn})\,q_{mn}+\sum_{kj}(p_j+wt_j)x_{kj}\right\}$$
$$+\pi\left(y-\sum_{mn}a_m q_{mn}\right)+\sum_k\varphi_k\left(z_k-\sum_{mn}b_{km}t_{mn}q_{mn}-Z_k(x_k)\right)$$
$$-\sum_{mn}\gamma_{mn}q_{mn} \quad \text{(D.3)}$$

$$L_2(y,z,\eta)=\max U(y,z)+\eta\left(\overline{M}-C(p,t;y,z)\right) \quad \text{(D.4)}$$

i) We first construct the optimality conditions for the two different utility maximization problems. The relevant parts of the Kuhn-Tucker conditions for the Lagrangian L_0 are as follows:

$$\sum_{mn}a_m\bar{q}_{mn}-\bar{y}=0 \quad \text{(D.5)}$$

$$\frac{\partial L_o(\cdot)}{\partial q_{mn}}=\bar{\lambda}a_m-\bar{\eta}\left(p_{mn}+v_m t_{mn}\right)+\bar{\phi}_{mn}=0,\ \forall\, mn, \quad \text{(D.6)}$$

$$\frac{\partial L_o(\cdot)}{\partial y}=\frac{\partial U(\bar{y},\bar{z})}{\partial y}-\bar{\lambda}=0, \quad \text{(D.7)}$$

$$\frac{\partial L_o(\cdot)}{\partial z_k}=\frac{\partial U(\bar{y},\bar{z})}{\partial z_k}-\bar{\eta}\bar{\varphi}_k=0 \quad \text{(D.8)}$$

$$\bar{\phi}_{mn}\bar{q}_{mn}=0,\ \bar{\phi}_{mn}\ge 0 \text{ and } \bar{q}_{mn}\ge 0. \quad \text{(D.9)}$$

where $v_m=w-\sum_k\bar{\varphi}_k b_{km}$. Here (D.6) is obtained by substituting the fist order condition of L_0 with respect to x_{kj} into the first order condition with respect to q_{mn}.

Subsequently, we construct the optimality conditions for the Lagrangian L_1 in (D.3) and the Lagrangian L_2 in (D.4). Firstly, the optimality conditions for the Lagrangian L_1 can be expressed as follows:

$$\bar{\pi}=\pi_{mn}=\min_{m'n'}\left\{\pi_{m'n'}=\frac{1}{a_{m'}}\left(p_{m'n'}+v_{m'}t_{m'n'}\right)\right\} \quad \text{(D.10)}$$

$$y=a_m\breve{q}_{mn}>0 \text{ and } \breve{q}_{m'n'}=0,\ \forall\, m'n'\ne mn \quad \text{(D.11)}$$

These two equations are no other than (B.11) and (B.5), respectively. Subsequently, the first-order conditions of the Lagrangian L_2 are as follows:

$$\frac{\partial L_2(\cdot)}{\partial y}=\frac{\partial U(\hat{y}_{mn},\hat{z}_{mn})}{\partial y}-\hat{\eta}\bar{\pi}=0 \quad \text{(D.12)}$$

$$\frac{\partial L_2(\cdot)}{\partial z_k}=\frac{\partial U(\hat{y}_{mn},\hat{z}_{mn})}{\partial z_k}-\hat{\eta}\bar{\varphi}_k=0 \quad \text{(D.13)}$$

Here note that the optimal solutions of the Lagrangians L_1 and L_2 are expressed by $\hat{q}_{mn}$, $\hat{y}_{mn}$, $\hat{z}_{mn}$, and so on.

ii) Prove that the left side of (D.1) implies the right. To this end, it suffices to prove that the Kuhn-Tucker conditions for L_0 in (D.5)~(D.9) are identical to the ones for L_1 in (D.10) and (D.11) and for L_2 in (D.12) and (D.13), under the condition of the left-side equality such that

$$\bar{\pi}=\pi_{mn}=\min_{m'n'}\left\{\pi_{m'n'}=\frac{1}{a_{m'}}\left(p_{m'n'}+v_{m'}t_{m'n'}\right)\right\} \quad \text{(D.14)}$$

The reason why it suffices to prove the assertion that the two sets of the kuhn-Tucker conditions are identical is as follows. By the convexity assumptions in A.2.1, the assertion implies that the two different sets of the optimality conditions yield the unique optimal solutions such that

$$\bar{y}=a_m\bar{q}_{mn}=\hat{y}=a_m\hat{q}_{mn},\ \text{and } \bar{z}_k=\hat{z}_k,\ \forall\, k \quad \text{(D.15)}$$

Then it follows that that the left side of (D.1) implies the right side, as claimed.

To prove the assertion, we first convert the optimality conditions of L_0 into those of L_2. To this end, we rearrange (D.6) as follows:

$$\frac{\bar{\lambda}}{\bar{\eta}} = \frac{1}{a_{m'}}\left((p_{m'n'} + v_{m'} t_{m'n'}) - \bar{\phi}_{m'n'}\right), \quad \forall m'n' \qquad \text{(D.16)}$$

Here, all the Lagrangian multipliers are positive or zero; $\bar{\lambda} > o$, $\bar{\eta} > 0$, $\overline{\varnothing}_{m'n'} > 0$. On the other hand, (D.9) implies the following:

$\bar{q}_{mn} > 0$ and $\overline{\varnothing}_{mn} = 0$, but $\bar{q}_{m'n'} = 0$ and $\overline{\varnothing}_{m'n'} > 0$, $\forall m'n' \neq mn$. (D.17)

Merging (D.14) and (D.17) into (D.16) gives

$$\frac{\bar{\lambda}}{\bar{\eta}} = \pi_{mn} = \min_{m'n'}\left\{\frac{1}{a_{m'}}(p_{m'n'} + v_{m'} t_{m'n'})\right\} = \bar{\pi} \qquad \text{(D.18)}$$

Subsequently, substituting (D.17) into (D.5) yields

$$\bar{y} = a_m \bar{q}_{mn}, \text{ and } \bar{q}_{m'n'} = 0, \quad \forall m'n' \neq mn \qquad \text{(D.19)}$$

Finally, substituting (D.18) into (D.7) gives

$$\frac{\partial U(\bar{y}, \bar{z})}{\partial y} = \bar{\lambda} = \bar{\eta}\pi_{mn} = \bar{\eta}\bar{\pi}. \qquad \text{(D.20)}$$

Using the above results, we can readily confirm that the two sets of Kuhn-Tucker conditions are identical; that is,

(D.10) = (D.18), (D.11) = (D.19),
(D.12) = (D.20), (D.13) = (D.8).

This implies the assertion.

iii) Prove that the right side of (D.1) implies the left. To this end, it suffices to prove the assertion that (D.15) implies (D.14) using the two sets of the Kuhn-Tucker conditions in (D.5)~(D.13). Its proof is worked in the following sequence. First, (D.15) implies that $(\bar{y}, \bar{z}) = (\hat{y}_{mn}, \hat{z}_{mn})$. Hence, by (D.7) and (D.12), it follows that

$$\frac{\partial U(\bar{y}, \bar{z})}{\partial y} = \bar{\lambda} = \frac{\partial U(\hat{y}_{mn}, \hat{z}_{mn})}{\partial y} = \hat{\eta}\pi_{mn} \qquad \text{(D.21)}$$

Second, substituting (D.15) into (D.11) yields (D.17). Third, substituting (D.17) into (D.6) yields gives

$$\frac{\bar{\lambda}}{\bar{\eta}} = \pi_{mn} = \min_{m'n'}\left\{\frac{1}{a_{m'}}(p_{m'n'} + v_{m'} t_{m'n'})\right\} \qquad \text{(D.22)}$$

Fourth, substituting (D.22) into (D.7) yields (D.20). Fifth, by (D.20) and (D.21), it follows that

$$\bar{\eta}\pi_{mn} = \hat{\eta}\pi_{mn} \text{ or } \bar{\eta} = \hat{\eta} \qquad \text{(D.23)}$$

Therefore, by (D.22) and (D.23), it follows the equality in (D.14).This implies the assertion.

(2) The proof of the following part of Proposition 2:

$$\pi_{mn} \langle \pi_{m'n'} \Leftrightarrow U(\bar{y}_{mn}, \bar{z}_{mn}) \rangle U(\bar{y}_{m'n'}, \bar{z}_{m'n'}), \qquad \text{(D.24)}$$

i) We first prove that the left-side inequality of (D.24) implies that of the right. To this end, we introduce the production possibility set s_{mn} defined by

$$S_{mn}(y, z; \pi_{mn}, \bar{\varphi}) = \left\{y, z \mid \bar{M} - \pi_{mn} y - \sum_k \bar{\varphi}_k z_k \geq 0, (y, z) \geq 0\right\},$$
$$\forall mn \qquad \text{(D.25)}$$

where $\overline{\varphi} \equiv (\overline{\varphi_1},...,\overline{\varphi}_k)$. The set s_{mn} is convex, since it is a polyhedron in RK+1. Also, as will be shown later, it holds that

$$\pi_{mn} \langle \pi_{m'n'} \Longrightarrow S_{mn}(\cdot) \supset S_{m'n'}(\cdot),\ \forall m'n' \neq mn \quad \text{(D.26)}$$

On the other hand, the optimal solution $(\overline{y}_{mn}, \overline{z}_{mn})$ in RK+1, $\forall_{mn}$, is the tangent point between the iso-indifference curve of $U(y,z) = U(\overline{y}_{mn}, \overline{z}_{mn})$ and the production possibility frontier of s_{mn}. Therefore, it follows that

$$S_{mn}(\cdot) \supset S_{m'n'}(\cdot) \Rightarrow U(\overline{y}_{mn}, \overline{z}_{mn}) \rangle U(\overline{y}_{m'n'}, \overline{z}_{m'n'}),$$
$$\forall m'n' \neq mn \quad \text{(D.27)}$$

since s_{mn} is convex, and $U(y,z)$ is increasing and concave in (y,z). Then, by (D.26) and (D.27), it is clear that the left inequality of (D.24) implies the right side, as claimed.

Therefore, we can complete the proof by showing (D.26). Let y_{mn} be the maximum value of y, such that $(y_{mn}, z \in S_{mn})$ for an arbitrarily given value of z. Then the condition in the left side of (D.24) implies that

$$y_{mn} \rangle y_{m'n'},\ \forall z \text{ and } \forall m'n' \neq mn \quad \text{(D.28)}$$

Hence, it follows that $S_{mn} \supset S_{m'n'}$, as claimed.

ii) Prove the converse that the right side of (D.24) implies that of the left side. By the principle of rationality in the revealed preference theorem, it follows that

$$U(\overline{y}_{mn}, \overline{z}_{mn}) \rangle U(\overline{y}_{m'n'}, \overline{z}_{m'n'}) \Longrightarrow$$
$$\pi_{mn} y_{mn} + \sum_k \overline{\varphi}_k \overline{z}_{kmn} \rangle \pi_{m'n'} y_{m'n'} + \sum_k \overline{\varphi}_k \overline{z}_{km'n'} \quad \text{(D.29)}$$

On the other hand,

$$\overline{M} - \pi_{mn} \overline{y}_{mn} - \sum_k \overline{\varphi}_k \overline{z}_{kmn} = 0,\ \forall mn \quad \text{(D.30)}$$

This implies that the optimal solution $(\overline{y}_{mn}, \overline{z}_{mn})$ is located on the frontier of the production possibility set s_{mn} defined (D.25). Therefore,

$$\pi_{mn} y_{mn} + \sum_k \overline{\varphi}_k \overline{z}_{kmn} \rangle \pi_{m'n'} y_{m'n'} + \sum_k \overline{\varphi}_k \overline{z}_{km'n'} \Longrightarrow$$
$$S_{mn}(\cdot) \supset S_{m'n'}(\cdot) \quad \text{(D.31)}$$

Moreover the right side of the above inequality and the consistency of the implicit price implies that, for every z, it holds that

$$\pi_{mn} y_{mn} + \sum_k \overline{\varphi}_k \overline{z}_k \rangle \pi_{m'n'} y_{m'n'} + \sum_k \overline{\varphi}_k \overline{z}_k \quad \text{(D.32)}$$

This implies that (D.28) holds. Therefore, the converse holds.

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