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Determination of Stress Intensity Factor K_I from Two Fringe Orders by Fringe Multiplication and Sharpening

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Abstract Stress intensity factor is one of the most important parameters in fracture mechanics. Both the stress field distribution and the crack propagation are closely related to these parameters. Due to the complexity of actual engineering problems, it is difficult to calculate the stress intensity factor by theoretical formulation, so photoelasticity method is a good choice. In this paper, modified two parameter method is employed to calculate stress intensity factor for opening mode by using data from more than one photoelastic fringe loop. For getting accurate experiment results, the initial fringes are doubled and sharpened by digital image programs from the fringe patterns obtained by a CCD camera. Photoelastic results are compared with those obtained by the use of empirical equation and FEM. Good agreement shows that the methods utilized in experiments are considerably reliable. The photoelastic experiment can be used for bench mark in theoretical study and other experiments.

Keywords: Stress Intensity Factor, Photoelasticity, Modified Two Parameter Method, Irwin's Method, Fracture Mechanics

1. Introduction

According to the theories of material strength, where there is a crack in the structure part, the stresses in the vicinity of the crack tip are always much higher than those far away from it and the structure failure often take place in those positions at the stress which is much lower than the ultimate stress. Even though the size of crack may be very small, it should be a concern in the design. In fact, many engineering accidents take place due to the crack occurrence and propagation and lead to terrible disasters. For the complexity of the engineering problems, it is hard to observe the cracks in the parts straightforwardly and more hard to evaluate the crack propagation.

Lots of theoretical and experimental studies show that the stress intensity factor is one of most important parameters for the stress field distribution and crack propagation (Liebowitz, 1971 and Murakami, 1987). In this paper, the modified two-parameter method (Bradley and Kobayashi, 1970) is employed to calculate stress intensity factor for opening-mode. This method is derived from Irwin's method (Wells and Post, 1958 and Irwin, 1958) and some modifications have been done as shown in Fig. 1 (a) and (b). Irwin's method develops a relation for the opening-mode stress intensity factor in terms of the geometric characteristics of the fringe loops near the crack tip and requires only a single data point on the fringe. But there still is a problem for Irwin method that it is very sensitive to the

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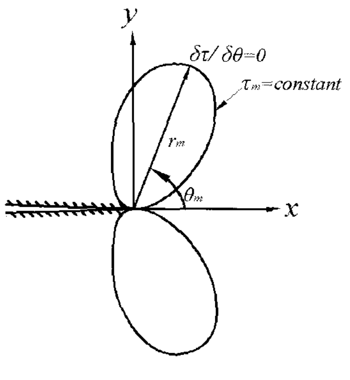
errors in measurement. Even small measurement errors of r_m and θ_m in Fig. 1 (a) will lead to big difference in determining stress intensity factor. For getting accurate experiment results, the initial photoelastic fringes are doubled and sharpened by digital image programs from the fringe patterns obtained by a CCD camera.

2. Theoretical Formulation

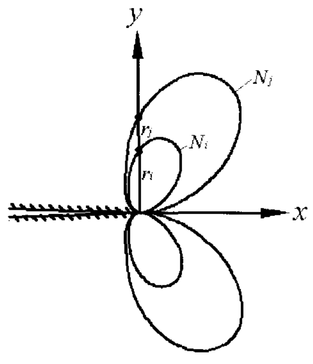
2.1 Basic Equations

The opening-mode stress intensity factor K_I in terms of the geometric characteristics of the fringe loops near the tip of a crack illustrated in Fig. 1 (a) (Irwin, 1958).

In a discussion of the photoelastic work done by Wells and Post (Irwin, 1958), Irwin points out that according to the Westergaard equations,



(a) Irwin's method



(b) two-parameter method

Fig. 1 Characteristic geometry of isochromatic fringe loops near the crack tip

the characteristic loops of the isochromatic fringes at a crack tip, will have their maximum radii normal to the crack line, thus θ_m shown as Fig. 1 will be 90°. However, in actual tests on a large plate with a central through-thickness crack loaded in uniaxial tension, the isochromatic fringes lean forward as above figure and θ_m less than 90°. To account for this leaning of the fringe loops, Irwin suggested the addition of a constant term, a so called nonsingular term σ_{ox}, to the σ_x as shown in eqn. (1a).

The stress field in the vicinity of the crack tip is given by

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)-\sigma_{0x} \quad (1a)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \quad (1b)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \quad (1c)$$

where the stress σ_{0x} was subtracted from the expression from σ_x to provide another degree of freedom in bringing the theoretical fringe loops in correspondence with the experimentally observed fringe loops. σ_x and σ_y are components of normal stress along x and y directions, τ_{xy} is the shear stress.

The maximum shear stress τ_m is expressed in terms of the Cartesian stress components as

$$(2\tau_m)^2 = (\sigma_y-\sigma_x)^2+(2\tau_{xy})^2 \quad (2)$$

Substituting eqns. (1) into eqn. (2), we can get

$$(2\tau_m)^2 = \frac{K_I^2}{2\pi r}\sin^2\theta+\frac{2\sigma_{0x}K_I}{\sqrt{2\pi r}}\sin\theta\sin\frac{3\theta}{2}+\sigma_{0x}^2 \quad (3)$$

where r is the length of the line determined by the crack tip and the point of fringe loop shown as Fig. 1 (a), θ is the angle between this line and crack direction.

On the line for $\theta = 90^{o}$, eqn. (3) can be simplified and the stress intensity factor K_I is expressed as eqn. (4). K_I has two roots as $K_I = \sqrt{\pi r}\,\lfloor \pm\sqrt{8\tau_m^2 - \sigma_{0x}^2} - \sigma_{0x} \rfloor$. But for the actual engineering problems, the stress intensity factor can not be a minus value. Thus, only one root was employed as shown by eqn. (4).

$$K_I = \sqrt{\pi r}\left\lfloor \sqrt{8\tau_m^2 - \sigma_{0x}^2} - \sigma_{0x} \right\rfloor \quad (4)$$

According to "modified Westergaard equation", σ_{0x} is one of the coefficients of higher order terms which versus to $\sqrt{r}$. For the region we are interested in, it is always in the vicinity of crack tip where "r" is a small value and the stress is much higher than those far away from the crack tip. Thus, σ_{0x}^2 often can be neglected since σ_{0x}^2 is small relative to $8\tau^2{}_m$. Then eqn. (4) can be simplified as $K_I = \sqrt{\pi r}\,\lfloor 2\sqrt{2}\tau_m - \sigma_{0x} \rfloor$. For a fringe loop, τ_m and r can be determined. But K_I and σ_{0x} cannot be obtained by only one equation, so we employed two equations for different fringe loops to uncouple K_I and σ_{0x}. Therefore, the stress intensity factor K_I is given by

$$K_I = \sqrt{2\pi r_i}\,\frac{(2\tau_m)_i - (2\tau_m)_j}{1 - \sqrt{r_i / r_j}} \quad (5)$$

where r_i and r_j are the distance from the points on i^{th} and j^{th} fringe loops to the crack tip.

According to the stress-optic law, the maximum shear stress τ_m can be determined from the isochromatic data and given by (Dally and Riley, 1991)

$$\tau_m = \frac{Nf_\sigma}{2t} \quad (6)$$

where N is the isochromatic fringe order, f_σ is the material fringe constant and t is the thickness of the specimen. By substituting eqn. (6) into eqn. (5), one obtains the basic relationship between isochromatic fringe order and the in-plane stress components

$$K_I = \frac{\sqrt{2\pi r_i}\, f_\sigma[(N)_i - (N)_j]}{t\left(1 - \sqrt{r_i / r_j}\right)} \quad (7)$$

2.2 Doubling and Sharpening Techniques for Isochromatic Images

The techniques of fringe doubling and sharpening were employed in order to obtain accurate isochromatic fringe patterns. For fringe doubling technique, two images are used as below (Baek et al., 2000 and Baek and Burger, 1991) :

$$I_R = |I_L - I_D| = A|\cos(2\pi N)| \quad (8)$$

where 'A' is a proportionality constant, I_L and I_D are the light intensities of the light field and dark-field isochromatic fringe patterns, respectively. In order for I_R of eqn. (8) to be zero, $\cos(2\pi N)$ should be zero. In a regular polariscope arrangement, dark and light fringes appear as a half-order interval alternately (N = 0, 1/2, 1, 3/2, 2, ...). However, after fringe multiplication, dark and light fringes, whose fringe orders are N = 0, 1/4, 2/4, 3/4, 1, 5/4, etc., appear as a quarter-order interval alternately. As a result, fringe patterns processed by eqn. (8) are twice-multiplied images.

The sharpening technique described here comes from the proportions of the gradient vector (Baek et al., 2000 and Baek and Burger, 1991). To sharpen photoelastic fringes, measured changes in the gradient direction through out an area are used. The operator, T, which is used for sharpening fringes, is given by eqn. (9)

$$T = A\left\{1 - \frac{|\Sigma \nabla_x| + |\Sigma \nabla_y|}{\Sigma|\nabla_x| + \Sigma|\nabla_y|}\right\} \quad (9)$$

where 'A' is a proportionality constant, ∇_x and ∇_y are x and y components of the photoelastic fringe gradient vector, respectively.

3. Experiment and Analysis

3.1 Photoelasticity Experiment

In this experiment, a PSM-1 (Budynas, 1999) plate shown as Fig. 2 is subjected to the uni-axial tensile stress $\sigma = 4.207 MPa$.

The material properties and geometric dimensions of the specimen are as shown in Table 1.

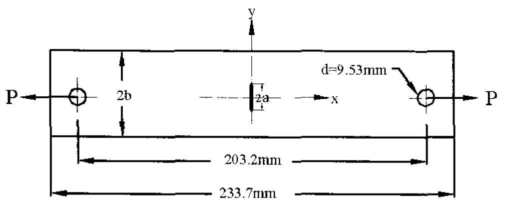


Fig. 2 Finite width plate with a center crack applied to uni-axial tension

Table 1 Material properties and geometric dimensions of specimen

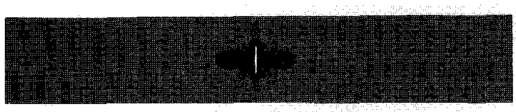
Descriptions	Symbol	Magnitude
Young's elastic modulus	E	2482 MPa
Poisson's ratio	v	0.38
Material fringe constant	f_σ	7005 N/m
Initial crack length	$2a$	12.7 mm
Width of specimen	b	38.1 mm
Thickness of specimen	t	3.175 mm

3.2 FEM Analysis

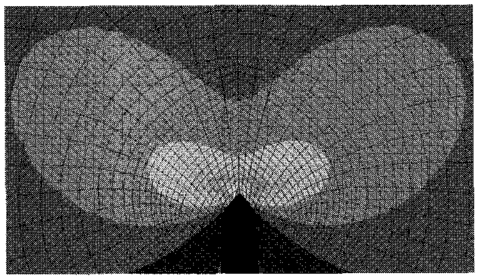
In this paper, a common FEM software ABAQUS® is used to discretize the specimen into the CPS4R element, a kind of bilinear plane stress quadrilateral element. The finite element model is shown as Fig. 3.

3.3 Experiment Results Analysis

In this experiment, the isochromatic fringe patterns are obtained in dark field and light field. The initial fringe pattern images are shown as Fig. 4:

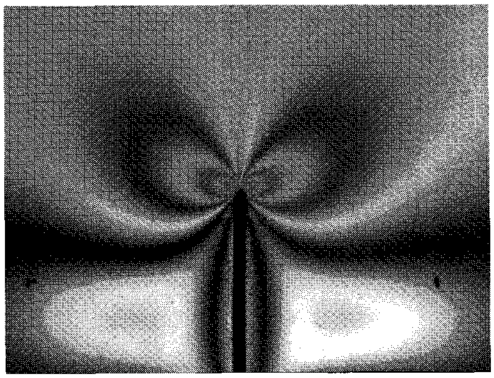


(a)

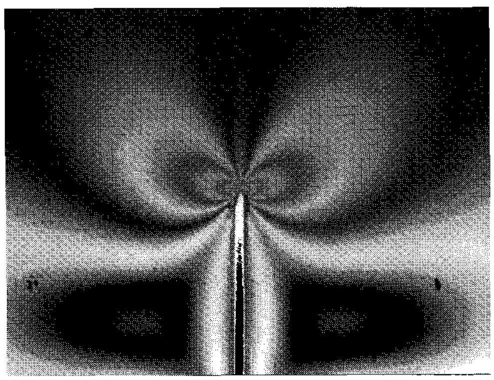


(b)

Fig. 3 (a) FEM model created in ABAQUS for specimen, (b) the detailed stress distribution in the vicinity of crack tip



(a)



(b)

Fig. 4 (a) dark field fringe pattern, (b) light field fringe pattern

In order to obtain accurate fringe data, fringes are twice multiplied and sharpened using the algorithms of eqns. (8) and (9). Figure 5 shows the twice-multiplied fringe pattern and sharpened fringe pattern of specimen. During loading process, fringe lines whose fringe order is 1, 1.5, 2, 2.5, 3 are easy to distinguish, thus, from the theory of doubling and sharpening techniques introduced above, it is possible to identify the fringe lines whose fringe order are 1, 1.25, 1.5, 1.75, …, respectively.

From processed fringe patterns, we can draw a line crossing crack tip and perpendicular with crack direction. The fringe orders N and distance of r are obtained for different fringe loops from the fringe pattern. But it should be noticed that the distance of r must be converted to actual length in specimen instead of measured value from images. By eqn. (7), the stress intensity factor K_I is calculated and shown as Table 2. For some fringe loops are very near with other fringe loops, we choose fringe loops as far from each other as we can to improve the precision of experiment results.

Three groups of results, obtained by photoelasticity, empirical equation and FEM, respectively were compared with each other in Table 2.

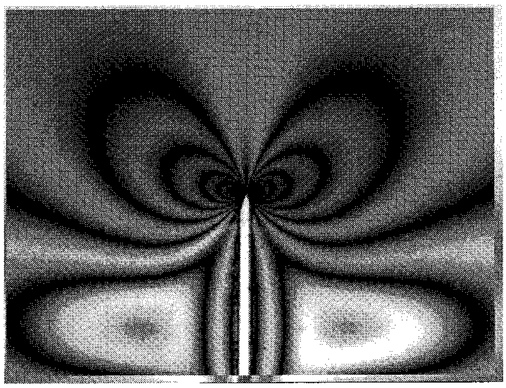
The empirical equation of stress intensity factor is shown as (Liebowitz, 1971 and Anderson, 1995)

$$K_I = F\sigma\sqrt{\pi a} \tag{10}$$

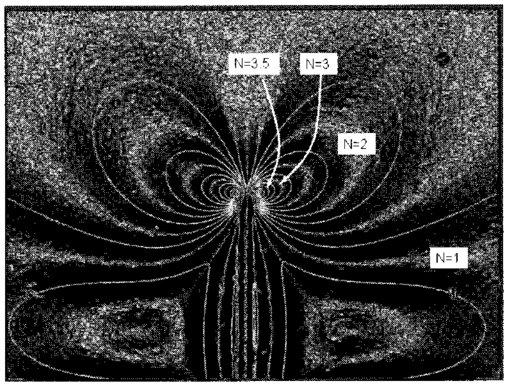
where F is the coefficient of geometry of specimen and expressed in terms of b, half of specimen width and a, half of crack length by

$$F = \frac{1-0.5(a/b)+0.326(a/b)^2}{\sqrt{1-(a/b)}} \tag{11}$$

From Table 2, it is reasonable to say three groups of results agree well with each other, even there are some errors along the different methods. It is verified that measurement by photoelasticity method can be applied to calculate the stress intensity factor for center crack.



(a)



(b)

Fig. 5 (a) twice multiplied fringe pattern, (b) sharpened fringe pattern

Table 2 Normalized mode I stress intensity K_I obtained by two-parameter method

i^{th} Fringe Order	j^{th} Fringe Order	$K_I/\sigma_0\sqrt{\pi a}$		
		Photoelasticity	Empirical Equation	FEM
2.75	2.25	1.105	1.065	1.059
	3.25	1.044		
	3.5	1.049		
3	2.25	1.138		
	2.5	1.075		
	3.5	0.984		
3.25	2.25	1.073		
	2.5	1.017		
	2.75	1.044		
3.5	2.5	1.027		
	2.75	1.049		
	3	0.984		

4. Discussions and Conclusions

In this study, the stress intensity factor K_I for opening-mode is calculated by using different two fringe loops. To get more accurate results, the photoelastic images are twice multiplied and sharpened. From the processed fringe patterns, we can see that there are more fringe loops in the vicinity of crack tip than those in the place far away from it. When two or more fringe loops occurs, the two parameter method can provide more accurate result with good measurement data. The present use of fringe multiplication and sharpening overcomes the difficulties associated with the measurement of the coordinate of the crack tip and fringe data.

Comparing the stress intensity factor K_I calculated by three different methods, photoelasticity, empirical equation and FEM, we can see that they are nearly the same with each other. Good agreement of the results indicates that the method utilized in this paper is properly reliable and the results can be used as bench mark in the theoretical studies and other experiments. Another important advantage for this method is that it can be conveniently applied in actual engineering problems with the help of photoelastic coating and has a potential future.

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