

# Analysis on SC-2 Diversity Systems for the Reception of $M$ -ary Signals over Rayleigh Fading Channels

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## Abstract

When the  $M$ -ary signal experiences the Rayleigh fading, the diversity schemes can reduce the effects of fading since the probability that all the signals components will fade simultaneously are reduced considerably. The symbol error probabilities for various  $M$ -ary signals, such as MDPSK, MPSK and MQAM, are mathematically derived for the SC-2 (Selection Combining 2) demodulation system, whereby the two signals with the two largest amplitudes are coherently combined among the  $L$  branches. On the other hand, maximum ratio combining(MRC) requires the individual signals from each path to be time-aligned, cophased, optimally weighted by their own fading amplitude, and then summed. The propagation model used in this paper is the frequency-nonselective slow Rayleigh fading channel corrupted by the Additive White Gaussian Noise(AWGN). The numerical results presented in this paper are expected to provide information for the design of radio system using  $M$ -ary modulation method for above mentioned channel environment.

**Key words** : SC-2, Rayleigh Fading, MDPSK, MPSK, MQAM.

## I. Introduction

The statistics for various fading channel models and the resulting communication evaluation have been considerably studied as summarized in [1]. The statistical properties of mobile radio environments can be often specified by the following propagation effects: 1) long-term fading, 2) short-term fading<sup>[2]</sup>. In long-term fading, the change of effective height for mobile communication antenna exists due to the nature of the terrain. Its statistics follow the log-normal distribution. But the Rician model can be obtained from the direct wave and its scattering components, and both waves carry information<sup>[3]</sup>. On the other hand, in short-term fading, the scattering mechanism only results in numerous reflected components<sup>[4]</sup>. The Rayleigh model is used to characterize this fading in small geographical areas and sometimes does not account for large scale effects like shadowing by building and hills. When the fading index in the Rician model,  $K$ , goes to 0, the error performances lead to those of Rayleigh fading model.

In [5], the general formula for evaluating the error performance for either BPSK or DPSK signals with SC-2 under a Rayleigh fading channel was presented. If one of  $M > 2$  possible values is available, it is referred as the  $M$ -ary signaling. In  $M$ -ary signaling schemes, we may send one of  $M$  possible signals during each signaling interval of duration  $T$ . For almost all applications, the number of possible signals  $M = 2^n$ , where  $n$  is an

integer. The symbol duration  $T$  is  $nT_b$ , where  $T_b$  is the bit duration. In this paper we can represent the average symbol error rate(SER) by these SC-2 systems in receiving MDPSK, MPSK, and MQAM signals on Rayleigh fading channels. Especially we evaluate the error performance of coherent MPSK signals over the slow and flat fading channels when AWGN is present, using the approximation, an upper bound on the symbol error probability for large values of  $M$  signal waveforms. Next we compare the performance of SC-2 diversity reception of MDPSK, MPSK, and MQAM signals in slowly frequency-nonselective Rayleigh fading channels with an AWGN. The analytical results of these performance evaluations presented in this paper are expected to provide designers with important informations in designing  $M$ -ary modulation systems under the Rayleigh fading channel.

The remainder of this paper is organized as follows. The next section presents a system model with SC-2 diversity reception in a Rayleigh fading channel. The analytical results for the performance of the  $M$ -ary signals are explained in Section 3. Numerical results will be shown in Section 4. Finally, we summarize some known results in Section 5.

## II. System Model with SC-2 Diversity Reception

The optimal combination of the received signals is obtained by using MRC which calls for the increased

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complexity with respect to other diversity schemes. Anyway, it is frequently considered since its performance can be assumed as the upper bound to compare sub-optimal combining rules. On the other hand, among the suboptimal techniques, SC is attractive due to implementation simplicity and low cost.

The interest for SC application has been recently increased for the high-capacity mobile radio system. With reference to SC, applied here are the order statistics to select  $\mu$  branches from the branch with the largest amplitude, or to choose  $\mu$  branches from the branch with the largest signal-to-noise(SNR) at the  $L$  diversity branches, for a data recovery while assuming that the noise power is constant across all branches when the probability density function(PDF) for this combining is analyzed to evaluate the error rate performance of  $M$ -ary signals on Rayleigh fading channels<sup>[6],[7]</sup>.

Then the statistics of an instantaneous SNR  $\gamma$  are as follows:

$$\gamma = \gamma_L + \gamma_{L-1} + \gamma_{L-2} + \dots + \gamma_{L-\mu+1}, \quad L \geq \mu. \quad (1)$$

It is assumed that the statistical characteristics of diversity branches are independent each other over the Rayleigh fading channels.

Now, to derive the PDF for the instantaneous SNR, it is worthwhile to note that we introduce the following joint PDF of the statistics  $\gamma_L, \gamma_{L-1}, \dots,$  and  $\gamma_1$ , where  $0 \leq \gamma_1 \leq \gamma_2 \leq \dots \leq \gamma_L$ <sup>[6],[7]</sup>:

$$f(\gamma_L, \gamma_{L-1}, \dots, \gamma_1) = f(\gamma_L)f(\gamma_{L-1}) \dots f(\gamma_1). \quad (2)$$

Given each integration interval for the unnecessary random variables(rvs)  $\gamma_{L-\mu}, \gamma_{L-\mu-1}, \dots,$  and  $\gamma_1$ , we can perform the integration of those statistics and yield the marginal joint PDF of  $\gamma_L, \gamma_{L-1}, \dots,$  and  $\gamma_{L-\mu+1}$ .

Next, we can write, after the transform of rvs, the PDF of  $\gamma, \gamma_{L-1}, \dots, \gamma_{L-\mu+2},$  and  $\gamma_{L-\mu+1}$  as

$$f(\gamma_L, \dots, \gamma_{L-\mu+2}, \gamma_{L-\mu+1}) \frac{1}{|J|} = f(\gamma, \gamma_{L-1}, \dots, \gamma_{L-\mu+2}, \gamma_{L-\mu+1}) \quad (3)$$

where the Jacobian of the transformation,  $|J| = 1$  and  $0 \leq \gamma_{L-\mu+1} \leq \gamma_{L-\mu+2} \leq \dots \leq \gamma$ .

It follows that upon performing the integrations with respect to  $\gamma_{L-1}, \dots, \gamma_{L-\mu+2},$  and  $\gamma_{L-\mu+1}$ , we can obtain the PDF of  $\gamma$ .

In a traditional SC-2 to select the two branch signals with the two largest amplitudes from the original diversity branches, it can be shown that  $\mu = 2$ .

If each branch has equal fading parameter and average SNR  $\gamma_0$ , the conditional PDF of the received instantaneous SNR in a SC-2 diversity system on a Rayleigh fading channel is thus given by [5]~[8]

$$f(\gamma) = \int_0^{\gamma/2} f(\gamma_{L-1}, \gamma) d\gamma_{L-1} \equiv f_1(\gamma) + f_2(\gamma) \quad (4)$$

where

$$f_1(\gamma) = \frac{L(L-1)}{\gamma_0} e^{-\frac{\gamma}{\gamma_0}} \frac{\gamma}{2\gamma_0} \quad (5)$$

and

$$f_2(\gamma) = \frac{L(L-1)}{\gamma_0} e^{-\frac{\gamma}{\gamma_0}} \sum_{k=1}^{L-2} \binom{L-2}{k} \frac{(-1)^k}{k} (1 - e^{-\frac{k\gamma}{2\gamma_0}}) \quad (6)$$

### III. Performance Analysis

Once the statistics of the instantaneous SNR are determined as the function of the average SNR, the error performance in the Rayleigh fading channels can be evaluated by averaging the conditional probability of error over the PDF of the instantaneous SNR.

#### 3-1 Error Probability for MDPSK

When MDPSK signals experience no fading, the expression for the conditional probability of error is given by [9]

$$P_{s, MDPSK} = \frac{\sin \frac{\pi}{M}}{2\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\exp[-\gamma (1 - \cos \frac{\pi}{M} \cos \theta)]}{1 - \cos \frac{\pi}{M} \cos \theta} d\theta. \quad (7)$$

We can represent the average SER in receiving MDPSK signals with SC-2 diversity branches by averaging (7) over the PDF of an instantaneous SNR under the effect of Rayleigh fading channel as follows:

$$P_{e, MDPSK, SC2} = \int_0^\infty P_{s, MDPSK} f(\gamma) d\gamma \quad (8)$$

where  $P_{e, MDPSK, SC2}$  is the average SER of MDPSK signals under the Rayleigh fading model.

Consequently, we can find the symbol error probability under the Rayleigh fading model to be(See Appendix A.)

$$P_{e, MDPSK, SC2} = \frac{L(L-1)}{2\gamma_0^2} \frac{\sin \frac{\pi}{M}}{2\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{1}{1 - \cos \frac{\pi}{M} \cos \theta} \left( \frac{\gamma_0}{1 + \gamma_0 - \gamma_0 \cos \frac{\pi}{M} \cos \theta} \right)^2 d\theta + \frac{L(L-1)}{\gamma_0} \frac{\sin \frac{\pi}{M}}{2\pi} \sum_{k=1}^{L-2} \binom{L-2}{k} \frac{(-1)^k}{k} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{1}{1 - \cos \frac{\pi}{M} \cos \theta} \left[ \frac{\gamma_0}{1 + \gamma_0 - \gamma_0 \cos \frac{\pi}{M} \cos \theta} - \frac{2\gamma_0}{k+2+2\gamma_0(1 - \cos \frac{\pi}{M} \cos \theta)} \right] d\theta \quad (9)$$

which can be written in the integral-form, not in the closed-form.

We can observe that the result of (9) for  $L=2$  is equivalent to the result of [10, Eq. (5.2.13)] for Nakagami fading index  $m=1$ .

### 3-2 Error Probability for MPSK

The exact SER of coherent MPSK under a nonfading channel can be represented as [9]

$$P_{s, exact, MPSK} = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2} - \frac{\pi}{M}} \exp \left[ -\gamma \sin^2 \left( \frac{\pi}{M} \right) \sec^2 \theta \right] d\theta. \quad (10)$$

On the other hand, the approximation of coherent MPSK signals on the probability of symbol error for larger  $M$  may be represented as follows<sup>[11]</sup>:

$$P_{s, MPSK} = \text{erfc} \left( \sqrt{\gamma} \sin \frac{\pi}{M} \right) \quad (11)$$

where  $\text{erfc}(\cdot)$  is the error function defined as

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt. \quad (12)$$

Next, we can find the approximate performance of MPSK signals under the effect of SC-2 diversity in a Rayleigh fading channel to be (See Appendix B.)

$$\begin{aligned} P_{e, MPSK, SC2} &= \frac{3}{16} \frac{L(L-1)}{\gamma_0^2} \frac{1}{\sin^4 \frac{\pi}{M}} \cdot \\ & {}_2F_1 \left( 2, \frac{5}{2}; 3; -\frac{1}{\gamma_0 \sin^2 \frac{\pi}{M}} \right) \\ & + \frac{1}{2} \frac{L(L-1)}{\gamma_0} \frac{1}{\sin^2 \frac{\pi}{M}} \sum_{k=1}^{L-2} \binom{L-2}{k} \frac{(-1)^k}{k} \cdot \\ & \left[ {}_2F_1 \left( 1, \frac{3}{2}; 2; -\frac{1}{\gamma_0 \sin^2 \frac{\pi}{M}} \right) \right. \\ & \left. - {}_2F_1 \left( 1, \frac{3}{2}; 2; -\frac{2+k}{2\gamma_0} \frac{1}{\sin^2 \frac{\pi}{M}} \right) \right]. \quad (13) \end{aligned}$$

For the special case of  $K=0$ , we can thus find that the result of (13) for  $L=2$  corresponds to that of [12, Eq. (B.3)].

### 3-3 Error Probability for MQAM

Next, we analyze the performance of SC-2 diversity reception of MQAM signals in a Rayleigh fading channel. We can derive the integral-form performance exact for  $M=2^j$ , when  $j$  is even, by averaging the conditional probability of error over the PDF of an instantaneous SNR under the effect of Rayleigh fading channels.

MQAM is, in practice, frequently used technique which requires less average transmitted power to achieve

the same performance as MPSK signals. Thus, it may be still valuable to evaluate the performance of MQAM with SC-2 diversity receiver over Rayleigh fading channels.

Now, to derive the integral-form performance in a Rayleigh fading channel, given that  $j$  is even, we introduce the exact SER in the presence of AWGN channel, represented as [13]

$$\begin{aligned} P_{s, MQAM} &= \frac{4B}{\pi} \int_0^{\frac{\pi}{2}} \exp \left( -\frac{g\gamma}{\sin^2 \theta} \right) d\theta \\ & - \frac{4B^2}{\pi} \int_0^{\frac{\pi}{4}} \exp \left( -\frac{g\gamma}{\sin^2 \theta} \right) d\theta \quad (14) \end{aligned}$$

where  $g = \frac{3}{2(M-1)}$  and  $B = \frac{\sqrt{M-1}}{\sqrt{M}}$ .

Average SER for MQAM under the effect of SC-2 diversity can be shown to be given by

$$\begin{aligned} P_{e, MQAM, SC2} &= \int_0^{\infty} P_{s, MQAM} f(\gamma) d\gamma \\ & \equiv P_{e1, MQAM, SC2} + P_{e2, MQAM, SC2}. \quad (15) \end{aligned}$$

$P_{e1, MQAM, SC2}$  and  $P_{e2, MQAM, SC2}$  can be expressed as

$$\begin{aligned} P_{e1, MQAM, SC2} &= \int_0^{\infty} P_{s, MQAM} f_1(\gamma) d\gamma \\ & = \frac{L(L-1)}{\gamma_0^2} \frac{2B}{\pi} \left[ \int_0^{\frac{\pi}{2}} \left( \frac{\gamma_0 \sin^2 \theta}{\sin^2 \theta + g\gamma_0} \right)^2 d\theta \right. \\ & \left. - B \int_0^{\frac{\pi}{4}} \left( \frac{\gamma_0 \sin^2 \theta}{\sin^2 \theta + g\gamma_0} \right)^2 d\theta \right] \quad (16) \end{aligned}$$

and

$$\begin{aligned} P_{e2, MQAM, SC2} &= \int_0^{\infty} P_{s, MQAM} f_2(\gamma) d\gamma \\ & \equiv P_{e3, MQAM, SC2} - P_{e4, MQAM, SC2} \quad (17) \end{aligned}$$

where

$$\begin{aligned} P_{e3, MQAM, SC2} &= \frac{L(L-1)}{\gamma_0} \frac{4B}{\pi} \sum_{k=1}^{L-2} \binom{L-2}{k} \frac{(-1)^k}{k} \cdot \\ & \left[ \int_0^{\frac{\pi}{2}} \frac{\gamma_0 \sin^2 \theta}{\sin^2 \theta + g\gamma_0} d\theta - \int_0^{\frac{\pi}{2}} \frac{2\gamma_0 \sin^2 \theta}{(2+k) \sin^2 \theta + 2g\gamma_0} d\theta \right] \quad (18) \end{aligned}$$

and

$$\begin{aligned} P_{e4, MQAM, SC2} &= \frac{L(L-1)}{\gamma_0} \frac{4B^2}{\pi} \sum_{k=1}^{L-2} \binom{L-2}{k} \cdot \\ & \frac{(-1)^k}{k} \left[ \int_0^{\frac{\pi}{4}} \frac{\gamma_0 \sin^2 \theta}{\sin^2 \theta + g\gamma_0} d\theta \right. \\ & \left. - \int_0^{\frac{\pi}{4}} \frac{2\gamma_0 \sin^2 \theta}{(2+k) \sin^2 \theta + 2g\gamma_0} d\theta \right]. \quad (19) \end{aligned}$$

It is clear that given the number of diversity branches, the performance of MPSK for  $M=4$  is perfectly equivalent to that of MQAM in Fig. 2<sup>[3]</sup>.

### IV. Numerical Results

It is shown that in Fig. 1 given average SNR of the received signal, for smaller diversity branches, the distribution of the instantaneous SNR for the received signal become more shifted to the left and more decayed in a mobile radio system to choose 2 branches from the branch with the largest amplitude of the  $L$  diversity branches.

Let us assume that the performance of coherent MP-SK in the fading channels has, first of all, an approximation. Figs. 2, 3 illustrate the performances of SC-2 diversity system with  $L=3, 4$  in MDPSK, MPSK and MQAM signals for the SNR per symbol with  $M=4$  and  $M=16$ , respectively. These figures show that, by increasing the number of  $M$  for the given values of  $L$  in SC-2 diversity system, the substantial gain in performances decreases to achieve an equal SNR per symbol but the

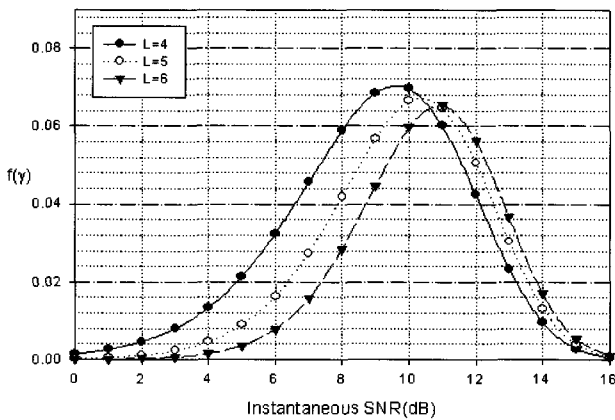


Fig. 1. The PDF of the instantaneous SNR for  $L=4, 5, 6$  and  $SNR=6$  dB.

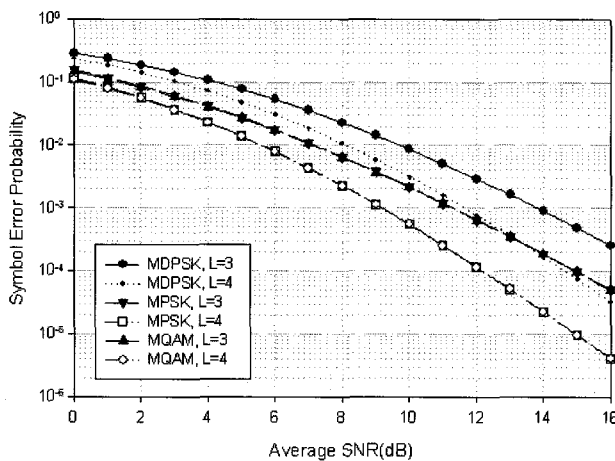


Fig. 2. Error performance comparisons of MDPSK, MPSK, and MQAM signals adopting SC-2 diversity technique for  $M=4$ .

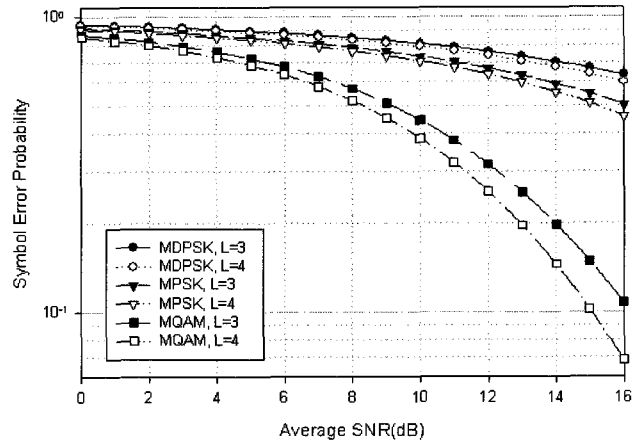


Fig. 3. Error performance comparisons of MDPSK, MPSK, and MQAM signals adopting SC-2 diversity technique for  $M=16$ .

actual evaluation does not have the important effect on the performance of  $M$ -ary signals. Furthermore, the performances of MDPSK signals with respect to that of MQAM signals are improved very restrictedly with increasing  $M$  for the given number of  $L$ . It is noted that the average SER of  $L=3$  comes closer to that of  $L=4$  by increasing the signal alphabet size  $M$  in each  $M$ -ary signals.

### V. Conclusion

The performances for MDPSK, MPSK and MQAM signals under the effect of SC-2 have been evaluated in the Rayleigh fading channel. The approximation to a symbol error probability for coherent MPSK in the fading channels has especially been presented.

It is expected result as the diversity branches  $L$  increase, the fading depth decreases. This result also shows that the restricted performance gain is achievable with increasing the number of the diversity branches,  $L$  in SC-2 diversity system.

The results of the present works are sufficiently general in offering a convenient method to evaluate the performance of several current  $M$ -ary modulation systems that operate on channels with a wide variety of fading conditions in wireless personal communications.

#### Appendix A: The Integral-form Derivation of (9)

In this Appendix, given that  $v$  is real number, substituting (4) and (7) into (8) and using the identity [14, p. 310, Eq. (3.351)]

$$\int_0^\infty x^n e^{-vx} dx = n! v^{-n-1}, \quad \text{Re } v > 0, \quad (A1)$$

we find the symbol error probability under the Ray-

leigh fading model to be

$$\begin{aligned}
 P_{e, \text{MDPSK, SC2}} &= \frac{L(L-1)}{2\gamma_0^2} \frac{\sin \frac{\pi}{M}}{2\pi} \cdot \\
 &\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{1}{1 - \cos \frac{\pi}{M} \cos \theta} \left( \frac{\gamma_0}{1 + \gamma_0 - \gamma_0 \cos \frac{\pi}{M} \cos \theta} \right)^2 d\theta \\
 &+ \frac{L(L-1)}{\gamma_0} \frac{\sin \frac{\pi}{M}}{2\pi} \sum_{k=1}^{L-2} \binom{L-2}{k} \frac{(-1)^k}{k} \cdot \\
 &\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{1}{1 - \cos \frac{\pi}{M} \cos \theta} \left[ \frac{\gamma_0}{1 + \gamma_0 - \gamma_0 \cos \frac{\pi}{M} \cos \theta} \right. \\
 &\left. - \frac{2\gamma_0}{k+2+2\gamma_0(1 - \cos \frac{\pi}{M} \cos \theta)} \right] d\theta. \quad (\text{A2})
 \end{aligned}$$

which can be written in the integral-form, not in the closed-form.

### Appendix B: The Closed-form Derivation of (13)

In this Appendix, given that  $\mu$ ,  $\nu$ , and  $\beta$  are real numbers, using the identity [14, p. 649, Eq. (6.286.1)]

$$\begin{aligned}
 &\int_0^\infty \text{erfc}(\beta x) e^{\mu^2 x^2} x^{\nu-1} dx \\
 &= \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu\beta^\nu}} {}_2F_1\left(\frac{\nu}{2}, \frac{\nu+1}{2}; \frac{\nu}{2}+1; \frac{\mu^2}{\beta^2}\right), \\
 &\quad \text{Re } \beta^2 > \text{Re } \mu^2, \text{Re } \nu > 0, \quad (\text{B1})
 \end{aligned}$$

where

$${}_2F_1(a, b, c, z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{z^n}{n!}, \quad (\text{B2})$$

we can find the approximate performance of MPSK signals under the effect of SC-2 diversity in a Rayleigh fading channel to be

$$\begin{aligned}
 P_{e, \text{MPSK, SC2}} &= \int_0^\infty P_{s, \text{MPSK}} f(\gamma) d\gamma \\
 &= \frac{3}{16} \frac{L(L-1)}{\gamma_0^2} \frac{1}{\sin^4 \frac{\pi}{M}} \cdot \\
 &{}_2F_1\left(2, \frac{5}{2}; 3; -\frac{1}{\gamma_0 \sin^2 \frac{\pi}{M}}\right) + \frac{1}{2} \frac{L(L-1)}{\gamma_0} \cdot \\
 &\frac{1}{\sin^2 \frac{\pi}{M}} \sum_{k=1}^{L-2} \binom{L-2}{k} \frac{(-1)^k}{k} \cdot \\
 &\left[ {}_2F_1\left(1, \frac{3}{2}; 2; -\frac{1}{\gamma_0 \sin^2 \frac{\pi}{M}}\right) \right. \\
 &\left. - {}_2F_1\left(1, \frac{3}{2}; 2; -\frac{2+k}{2\gamma_0} \frac{1}{\sin^2 \frac{\pi}{M}}\right) \right]. \quad (\text{B3})
 \end{aligned}$$

### Appendix C: The Finite-series Representation of

#### ${}_2F_1(\cdot)$ in (13)

Given that  $n$  is real number more than  $1/2$ , we assume

that  $G(z)$  is given by [15]

$$\begin{aligned}
 G(z) &= z^n {}_2F_1\left(n, n + \frac{1}{2}; n+1; -z\right) \\
 &= \sum_{i=0}^{\infty} \frac{n}{n+i} \frac{(2n-1+2i)!!}{2^i(2n-1)!!} \frac{(-1)^i}{i!} z^{n+i}, \quad \text{Re } n > \frac{1}{2} \quad (\text{C1})
 \end{aligned}$$

where

$$(2n-1)!! = (2n-1)(2n-3)\cdots 3 \cdot 1. \quad (\text{C2})$$

We note that since  $n$  is more than  $1/2$ ,  $G(0)$  is 0.

Then, (C1) can be represented as follows:

$$G(z) = n \int_0^z x^{n-1} \left[ \sum_{i=0}^{\infty} \frac{(2n-1+2i)!!}{2^i(2n-1)!!} \frac{(-x)^i}{i!} \right] dx + G(0). \quad (\text{C3})$$

The infinite-series formula of (C3) becomes

$$\sum_{i=0}^{\infty} \frac{(2n-1+2i)!!}{2^i(2n-1)!!} \frac{(-x)^i}{i!} = (x+1)^{-n-\frac{1}{2}}. \quad (\text{C4})$$

Hence, (C3) can be expressed as

$$G(z) = \int_0^z n x^{n-1} (x+1)^{-\frac{2n+1}{2}} dx. \quad (\text{C5})$$

Consequently, (C5) may be presented as follows [14, p. 73, Eq. (2.221)]:

$$\begin{aligned}
 G(z) &= -n \sum_{i=1}^{n-1} \frac{(-1)^i}{i+\frac{1}{2}} \binom{n-1}{i} \left[ (z+1)^{-i-\frac{1}{2}} - 1 \right] \\
 &= -\sum_{i=0}^{n-1} \frac{(-1)^i}{i+\frac{1}{2}} \frac{\Gamma(n+1)}{\Gamma(i+1)\Gamma(n-i)} \left[ (z+1)^{-i-\frac{1}{2}} - 1 \right]. \quad (\text{C6})
 \end{aligned}$$

Finally,  ${}_2F_1(\cdot)$  in (C.1) can be written as

$$\begin{aligned}
 &{}_2F_1\left(n, n + \frac{1}{2}; n+1; -z\right) = \\
 &-z^{-n} \sum_{i=0}^{n-1} \frac{(-1)^i}{i+\frac{1}{2}} \frac{\Gamma(n+1)}{\Gamma(i+1)\Gamma(n-i)} \left[ (z+1)^{-i-\frac{1}{2}} - 1 \right]. \quad (\text{C7})
 \end{aligned}$$

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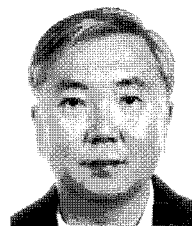
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