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Diagonal Tension Failure Model for RC Slender Beams without Shear Reinforcement Based on Kinematical Conditions (I) - Development

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ABSTRACT: *A mechanical model was developed to predict the behavior of point-loaded RC slender beams (a/d > 2.5) without stirrups. It is commonly accepted by most researchers that a diagonal tension crack plays a predominant role in the failure mode of these beams, but the failure mechanism of these members is still debatable. In this paper, it was assumed that diagonal tension failure was triggered by the concrete cover splitting due to the dowel action at the initial location of diagonal tension cracks, which propagate from flexural cracks. When concrete cover splitting occurred, the shape of a diagonal tension crack was simultaneously developed, which can be determined from the principal tensile stress trajectory. This fictitious crack rotates onto the crack tip with load increase. During the rotation, all forces acting on the crack (i.e, dowel force of longitudinal bars, vertical component of concrete tensile force, shear force by aggregate interlock, shear force in compression zone) were calculated by considering the kinematical conditions such as crack width or sliding. These forces except for the shear force in the compression zone were uncoupled with respect to crack width and sliding by the proposed constitutive relations for friction along the crack. Uncoupling the shear forces along the crack was aimed at distinguishing each force from the total shear force and clarifying the failure mechanism of RC slender beams without stirrups. In addition, a proposed method deriving the dowel force of longitudinal bars made it possible to predict the secondary shear failure. The proposed model can be used to predict not only the entire behavior of point-loaded RC slender shear beams, but also the ultimate shear strength. The experiments used to validate the proposed model are reported in a companion paper.*

1. Introduction

Many structural concrete members such as slabs, footings, joints and lightly stressed members are constructed without shear reinforcement. To evaluate the shear capacity of these kinds of structures, empirical formulas and mechanical models have been extensively studied and developed over the years. Research works in the past have been concentrated on finding the influencing parameters and on formulating design specifications based on test results. The major parameters which influence the shear behavior are known as follows : concrete tensile strength, longitudinal reinforcement ratio, shear span-to-depth ratio, axial force or amount of prestress, and the depth of the members which account for size effect. Whereas, in recent research, more efforts are given to develope rational models representing shear transfer mechanisms as well, which considers : shear stresses in the flexural compression zone, interface shear transfer between crack (aggregate interlock), dowel action, arch action and residual tensile stress transmitted directly across cracks.

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For the mechanical model, most of researchers focussed their efforts to the estimation of shear strength, which is essential for ultimate strength design. Examples of them are arch models or truss and strut-and-tie models. These static models used the distribution and magnitude of the internal forces and stresses based on the force-equilibrium at failure. Other shear static models are based on more fundamental theories, such as fracture mechanics or plasticity theory. These static models are adopted on the design specifications partly as an alternative of empirical formula. In the meanwhile, somewhat different mechanical models based on kinematical relation are proposed by Walther (1966), Broms (1969), Reineck (1990) and Fisher (1997). On the contrary to the static models, they adopted a kinematical failure mechanism and made it possible to estimate the shear strength as well as kinematical terms like deformation and strain. Recently some other aspect of strength design, such as ductility, gather more interest than before due to increase of demand for the safety of earthquake excitation. As a result, not only strength itself but also deformation can be important for structural designers. Moreover an assumed failure mechanism can be compared directly with experimental results through a kinematical model.

In the present study, a kinematical shear model was developed and used to evaluate the shear strengths as well as the kinematical conditions.

2. Mechanical Model

2.1 General

It is commonly agreed by most researchers that the shear failure of slender reinforced concrete beams (a/d > 2.5) without shear reinforcement is characterized by the occurrence of diagonal tension crack. However, considerable differences in opinion exist regarding the failure mechanism of diagonal tension failure. Most researchers believe that the diagonal tension crack is a simple extension of a previously developed flexural crack, which becomes inclined with increase of load because the sections are subjected to bending as well as shear. On the other hand, some researchers, such as Krefeld and Thurston (1966), Moody et al. (1955) and Kim and White (1991;1999), reported that diagonal tension crack was distinguished from ordinary flexural cracks. In addition to the initial location of diagonal tension crack which can strongly affects the distribution of internal forces, the horizontal cracking (sometimes called dowel or dowel-split cracking) along the longitudinal reinforcement is also important issue. This cracking is conventionally considered as a secondary crack that forms after the compression zone has failed. However, some other researchers, including Krefeld and Thurston (1966), Chana (1987), and Fischer (1997), stated that the horizontal cracking is a necessary and sufficient condition for diagonal tension failure.

It is intuitive that shear failure of reinforced concrete beams without shear reinforcement is caused by diagonal tension cracking that develops perpendicular to the principal tensile stresses. And this fundamental phenomenon was also mentioned by Mörsch (1902).

As a result, several assumptions were made in the present study to simplify the model based on the facts mentioned above. It is assumed that diagonal tension failure was triggered by dowel-split cracking and diagonal tension crack, which follows the principal tensile stress trajectories, was propagated from the flexural crack to simplify the model. Thus, the initial flexural cracks greatly influence the stress distribution that follows the development of the diagonal tension crack. That is a reason why the flexural analysis is involved in the present model. All of these assumptions and schemes are depicted in Fig. 1.

2.2 Geometries of diagonal tension crack

The location of critical flexural crack, x_s, was derived by some researchers, such as Krefeld and Thurston (1966), Kim and White (1991), Fischer (1997) and Nielsen (1998) depending on their own theories. Although they are based on different theories, all of them is a function of the shear span-to-depth ratio (a/d) and has a similar value. In the present study, proposed model adopted an equation proposed by Fischer (1997).

$$x_s = \frac{3}{4}\frac{a}{d}\sqrt[3]{\frac{a/d}{21.9}}d \tag{1}$$

The shape of diagonal tension crack is determined from the derivation of principal tensile stresses. The procedure is expressed as follows,

$$f_{1,2} = \frac{f_y}{2} \pm \sqrt{\frac{f_y}{2} + \tau_{xy}^2} \tag{2}$$

When considers a rectangular section

$$f_{1,2} = \frac{M(y)}{2I}\cdot x \pm \sqrt{\left\{\frac{M(y)}{2I}\cdot x\right\}^2 + \left\{\frac{V(y)}{2I}\cdot\left(\frac{h^2}{4} - x^2\right)\right\}^2} \tag{3}$$

where $M(y)$ and $V(y)$ are moment and shear according to arbitrary locations dependent on load conditions, and moment of inertial, I, height of section, h, are constant dependent on a considered section. After gathering same terms on each sides, by squaring on both sides we obtain

$$[f_{1,2} - \frac{M(y)}{2I}\cdot x]^2 = \left\{\frac{M(y)}{2I}\cdot x\right\}^2 + \left\{\frac{V(y)}{2I}\cdot\left(\frac{h^2}{4} - x^2\right)\right\}^2 \tag{4}$$

Equation (4) will be a general form Eq. (5)

$$(f_{1,2})^2 - \left\{\frac{V(y)}{2I}\cdot\left(\frac{h^2}{4} - x^2\right)\right\}^2 - \frac{M(y)}{I}\cdot x = 0 \tag{5}$$

For solving this equation, it is necessary to iterate variable, x, with any constant distance y, which means that $V(y)$ and $M(y)$ will be any constant value. Of course, this equation is dependent on external load condition. This equation gives a general solution for stress trajectories. With this general equations we can deal with various load conditions, which can be covered for the proposed model.

2.3 Kinematical conditions and Internal forces

When dowel force at the location of critical crack, D, reaches some cracking value, Dcr, depicted as a state II in Fig. 1, diagonal tension failure simultaneously occurs, depicted as a state III in Fig. 1.

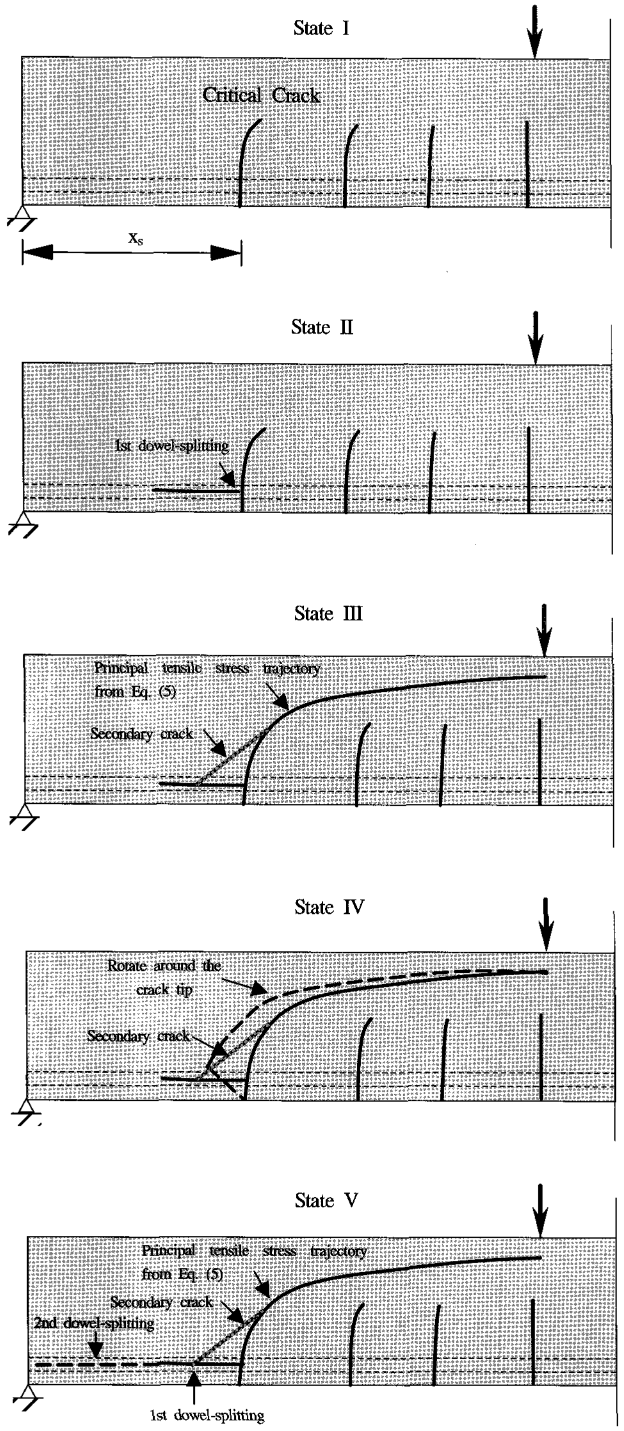

Fig. 1 Scheme of proposed model

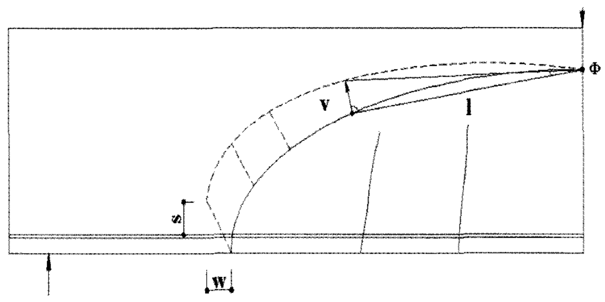

Fig. 2 Kinematical conditions at the state III

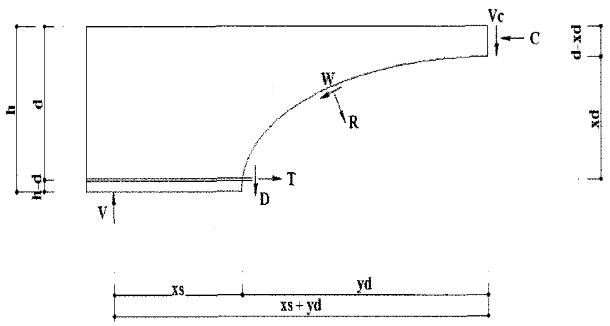

Fig. 3 Free body diagram at the state III

The shape of diagonal tension crack is derived from Eq. (5) with kinematical conditions shown in Fig. 2. Finally, shear capacity of beam can be estimated at this state by the satisfying the equilibrium condition as shown in Fig. 3. Internal forces acting along the diagonal crack are as follows; compressive force and shear force in the compression zone, C & Vc, friction force (aggregate interlock) and tensile force of concrete along the crack, W & R, tensile force and dowel force of longitudinal reinforcement T & D, shear capacity of beam, V.

2.4 Uncoupling kinematical conditions

To calculate the normal and tangential stresses, which will compose the normal and tangential forces (W & R in Fig. 3), it is necessary to know the crack opening, w, and crack sliding, δ, in the arbitrary points in the diagonal tension crack. These values can be determined form the deformation of a diagonal tension crack as shown in Fig. 2. The deformation at an arbitrary point, v, arises from Eq. (6).

$$v = l \cdot \tan(\Phi) \tag{6}$$

The relations between v and w, δ are shown in Fig. 4.

$$y'(x) = \tan(\beta) \tag{7}$$

$$\gamma = \arctan\left(\frac{y_d - y}{x_d - x}\right) - \beta \tag{8}$$

Thus, crack width and sliding at the arbitrary point can be uncoupled from geometrical condition.

$$w = v \cdot \cos(\gamma) \tag{9}$$
$$\delta = v \cdot \sin(\gamma) \tag{10}$$

3. Shear Transfer Mechanism

3.1 Dowel force

This force is the most important one among all forces acting on the diagonal tension crack due to its main role for triggering the diagonal tension failure. Figure 5 shows a detailed failure mechanism of splitting failure caused by dowel action. The bearing stress under longitudinal bars was derived from Timoshenko's (1958) beam theory on elastic foundation and given Eq. (11) ~ Eq. (14).

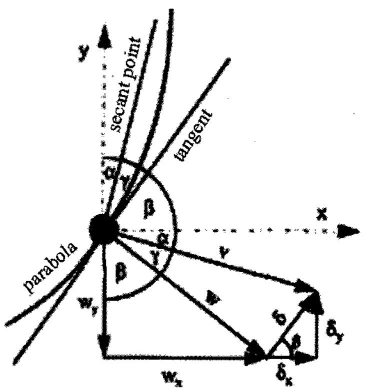

Fig. 4 Geometrical relations for crack opening and sliding

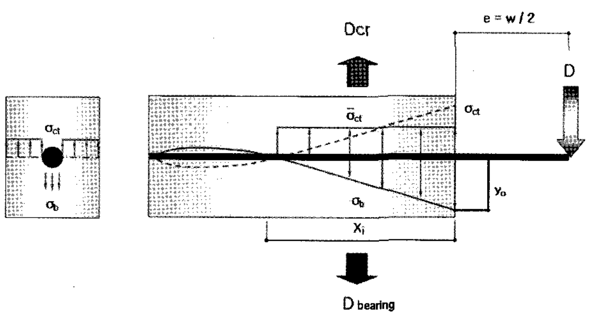

Fig. 5 Stress distribution in the concrete

$$y = e^{\beta x}(A\cos\beta x + B\sin\beta x) + e^{-\beta x}(C\cos\beta x + D\sin\beta x) \tag{11}$$

$$y = \frac{Pe^{-\beta x}}{4\beta^3 EI}[2\cos\beta x - \beta w(\cos\beta x - \sin\beta x)] \tag{12}$$

$$\sigma_b = \frac{kPe^{-\beta x}}{4\beta^3 EI}[2\cos\beta x - \beta w(\cos\beta x - \sin\beta x)] \tag{13}$$

$$\sigma_{b,x=0} = \frac{kP}{4\beta^3 EI}[2 - \beta w] \tag{14}$$

where P = dowel force (D) in Fig. 6.

When bearing stress at the crack surface reaches to tensile strength of concrete, that is $\sigma_{b,x=0} = \sigma_{ct}$, equilibrium condition is calculated. If the bearing force integrated from bearing stresses along the longitudinal bars, $D_{bearing}$, exceeds the tensile force, D_{cr}, concrete cover splitting will be occurred.

$$D_{cr} = (b - d_b)\,\overline{\sigma_{ct}}\,x_i \tag{15}$$

Another advantage using Timoshenko's theory is capability of a continuous prediction for secondary splitting failure that is a state V depicted in Fig. 1.

Dowel force, D in Fig. 6, existing at the critical flexural crack was derived from Reineck's (1990) tooth model shown in Fig. 7, where it is designated as V_d.

$$D = \frac{\tan\beta_{cr}}{z}\left[V\left\{x + \frac{d-c}{\tan\beta_{cr}}\left(1 + \frac{2}{3}\frac{c}{z}\right)\right\} - \epsilon_s\, E_s\, A_s\, z\right] \tag{16}$$

3.2 Tensile force of longitudinal bar

Diagonal tension crack is the last crack before the support. Behind this crack no new shear crack except web shear crack is created. This can be used to determine the tensile force in the reinforcement. The tensile force has to be smaller than the force which can be introduced along the bond length of the reinforcement.

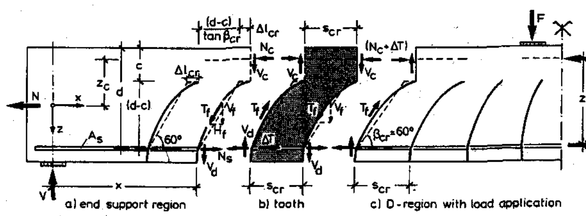

Fig. 6 Reineck's tooth model

The crack opening is defined by the assumed crack rotation shown in Fig. 2. The following formula of König and Fehling (1988) is applied to the crack opening in Fig.2.

$$w = \frac{d_s \cdot \sigma_s^2}{5 \cdot \tau_{sm} \cdot E_s} \quad (17)$$

From this formula, it is possible to calculate the tensile stress of longitudinal bar using crack width to tensile stress considering tensile stiffening effect. We used crack width to tensile stress relation proposed by CEB-FIP (1993). Finally, tensile force of longitudinal bar can be calculated.

3.3 Shear force by aggregated interlock (W) and tensile force of concrete tensile force

From stress-displacement relations, tensile stress, $\sigma(w)$, and shear stress, $\tau(\gamma)$, can be calculated at the arbitrary points. By integrating these values through the diagonal tension crack, tensile force, R, shear force, W, are obtained.

$$R = \int_A \sigma(w)\, dA = \int_0^{x_d} \sigma(w) \cdot \frac{b}{\cos\beta}\, dx \quad (18)$$

$$W = \int_A \tau(\delta)\, dA = \int_0^{x_d} \tau(\delta) \cdot \frac{b}{\cos\beta}\, dx \quad (19)$$

3.4 Compressive force and shear force at the compression zone

These forces can be calculated by forming the moment equilibrium around the point of intersection between the two force. After the loss of shear force by aggregate interlock by rotation of the diagonal shear crack, moment equilibrium can be resisted by uncracked compression zone. Since they further resist the shear force after the dowel splitting crack developed, we need the descending branch of stress-strain curve for expecting the behavior after the pack dowel capacity. For the descending part of stress-strain relation of concrete, we introduce CDZ model in the following chapter.

4. Material Laws

4.1 Concrete in tension

Tensile stress-crack opening relations proposed by Remmel (1994) was adopted in the proposed model. He suggested uniaxial σ-w relations as a exponential form with a restrained fracture energy G_F about d_{max} = 8 and 16 mm.

$$\sigma_{ct} = f_{t1} \cdot e^{-(w/w_1)^c} + f_{t2} \cdot (1 - w/w_2) \quad (20)$$

$$G_F = 65\, N/m \cdot \ln(1 + \frac{f_c}{10\, N/mm^2}) \quad (21)$$

4.2 Concrete in compression

For slender beams without shear reinforcement, a diagonal tension crack penetrates the compression zone, and causes a diagonal tension failure. Therefore, it is necessary to adopt a rational material model taking into account this phenomenon. Compressive damage zone (CDZ) model proposed by Markeset (1993) is a excellent material model to account for this failure mode as shown in Fig. 7. Figure 8 shows a unified stress-strain curve which was combined from all failure modes consisting CDZ model.

$$\epsilon_m = \epsilon + \epsilon^d \cdot L^d/L + \mathrm{w}/\mathrm{L} \quad (22)$$

4.3 Concrete in shear

The most popular shear stress-crack sliding model is Warlaven's (1981) one. He suggested a linear formula including crack width and sliding for normal and shear stress caused by aggregate interlock.

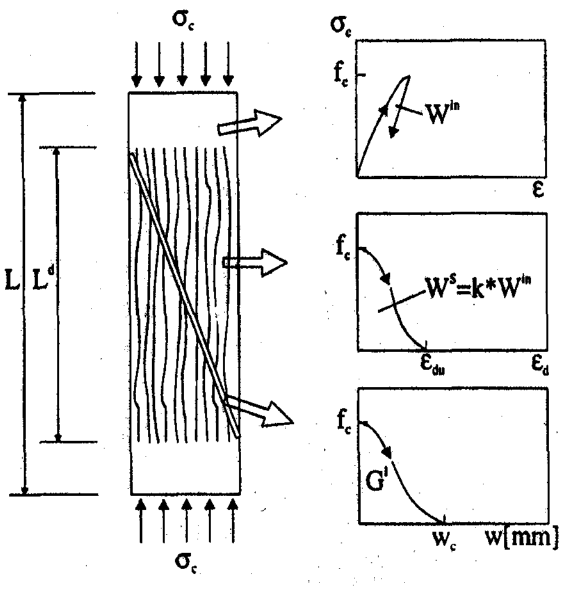

Fig. 7 Compression damage zone model

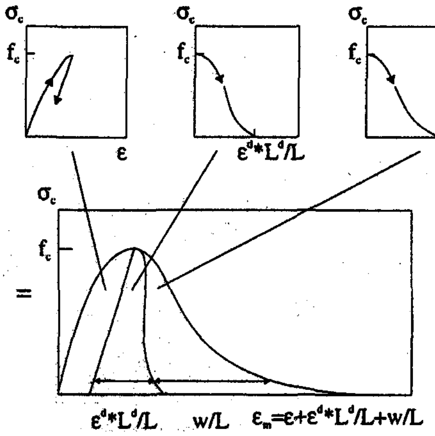

Fig. 8 Unified stress-strain relations for CDZ model

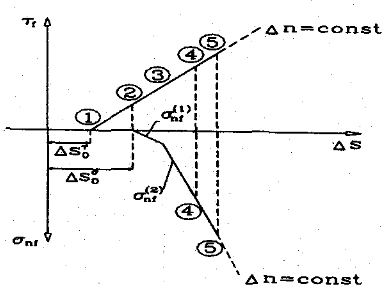

Fig. 9 Relations between stresses and displacements

In the present study, kinematical conditions according to the rotation of diagonal tension crack were uncoupled. Therefor, it is also necessary to uncouple these stresses. This could be achieved by the assumption that no normal stresses active on the crack face. In Fig. 9, state between ① and ② belongs this assumption. In this state, shear stress is as follows ;

$$\tau_f=\tau_{f0}\cdot\frac{\Delta s-\Delta s_0^{\tau}}{\Delta s_0^{\sigma}-\Delta s_0^{\tau}} \tag{23}$$

where τ_{f0} is a maximum shear stress not companying normal stress under some crack opening. Reineck (1990) proposed a formula to estimate this stress. However, his formula overestimated this stress compared with test results. Warlaven's (1981) formula also showed same tendency. Therefore, a rational formula was proposed to estimate τ_{f0} properly for getting better results in uncoupling process. The proposed formula showed a good agreement as shown in Fig. 10.

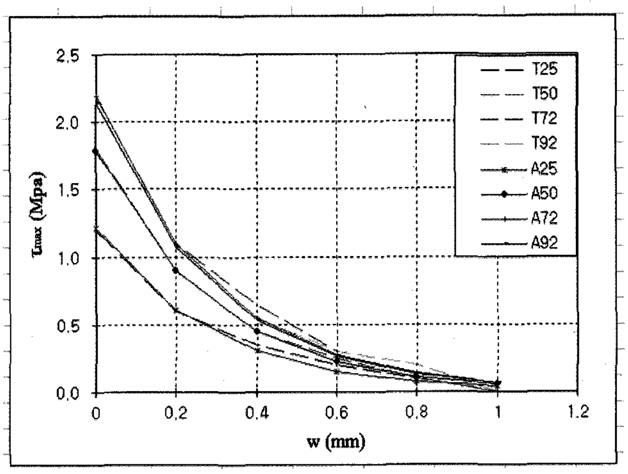

Fig. 10 Comparison between the Warlaven's test and proposed formula according to concrete strength

$$\tau_{f0}=0.2\cdot\left(\frac{1}{2}\right)^{\Delta n/0.2}\cdot f'^{\,0.56}_{cc} \tag{24}$$

5. Analytical Method

5.1 Basic procedure

As loads increase, diagonal tension crack rises after the formation of flexural crack. In this analysis, diagonal tension crack is triggered by flexural crack growth. Therefore, it is necessary to estimate an flexural crack width at the starting point of a diagonal tension crack. This is done by iteration of flexural analysis. This crack width is also an necessary and sufficient condition for forming a proposed model. After diagonal crack initiation, the shear capacity of slender shear beam is determined. An flow chart for this procedure is shown Fig. 11.

5.2 Flexural analysis

Flexural behavior can be analyzed by using equilibrium condition, compatibility condition and constitutive relations of materials. The average strains of concrete and steel can be calculated from the average curvature. After calculating the average strains, it is possible to calculate average crack spacing, s_{rm} , average crack width, w_m. Flow chart for flexural analysis is shown in Fig. 12.

5.3 Proposed model

Calculation procedure followed by main program is presented in Fig. 13. When secondary splitting failure occurs, this process keeps to iterate until equilibrium conditions is satisfied with increase of δ_0.

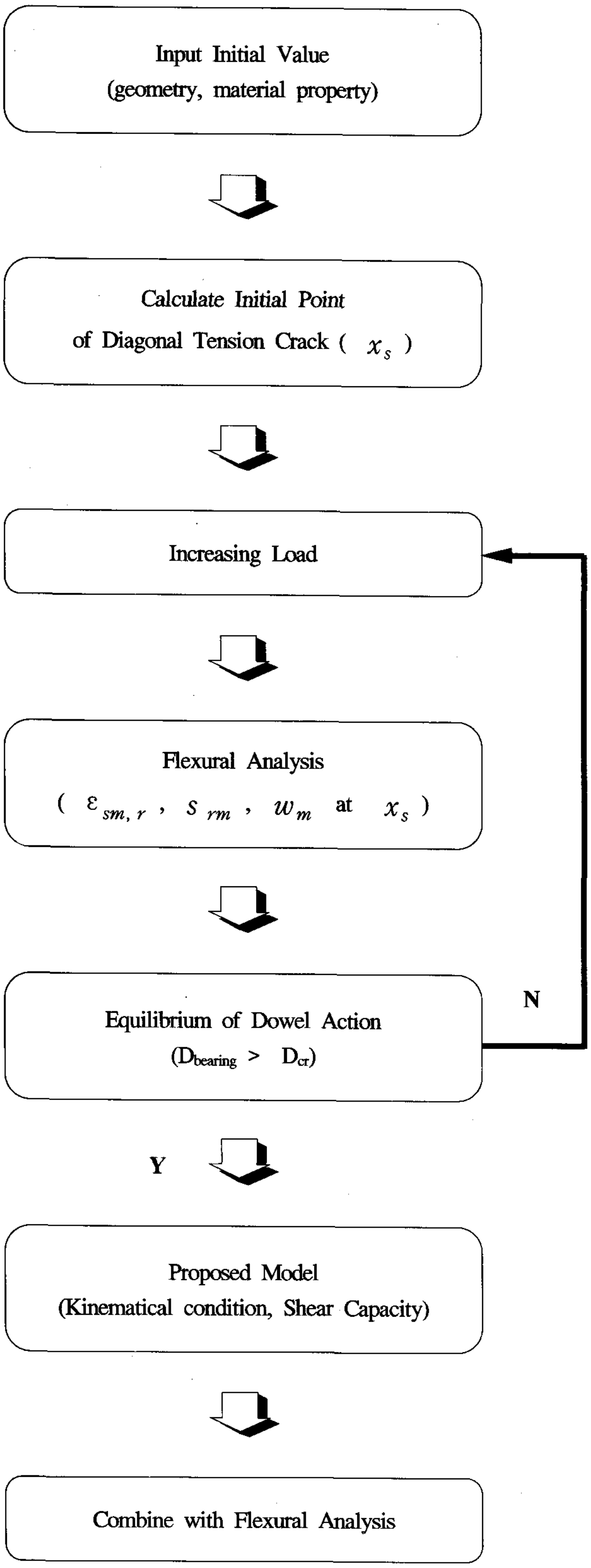

Fig. 11 Basic procedure

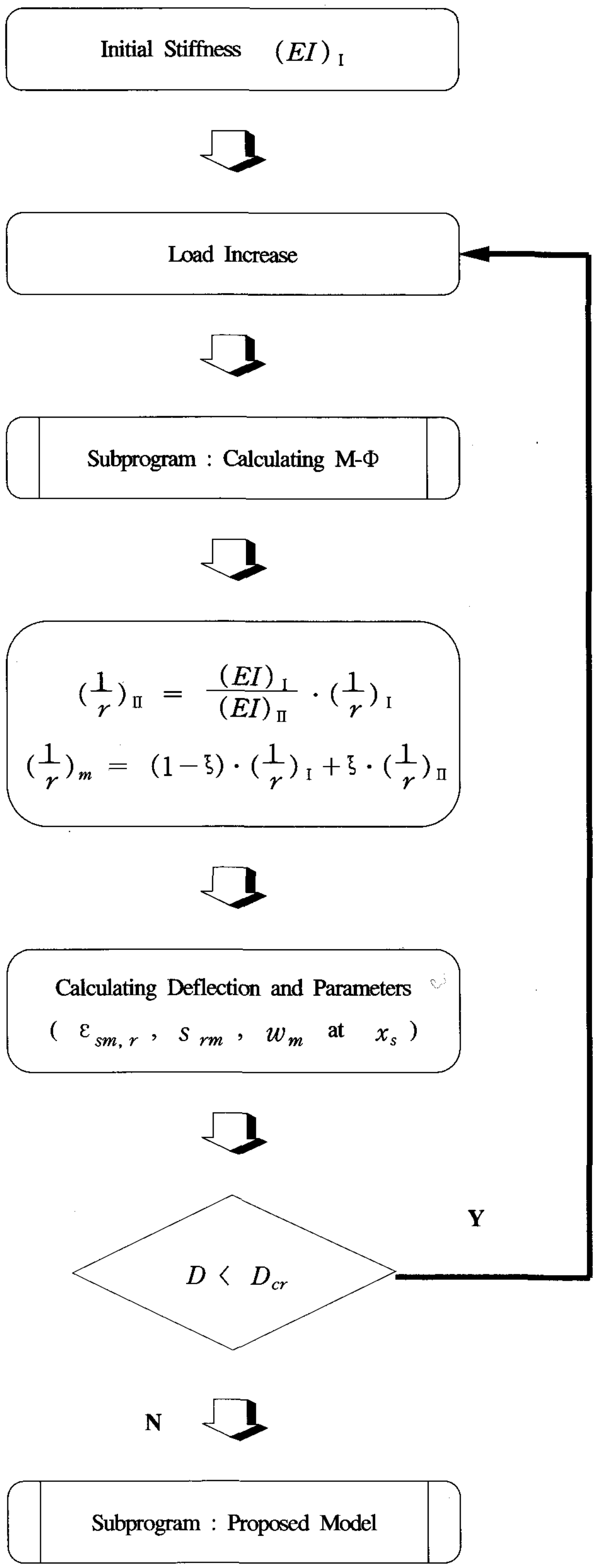

Fig. 12 Flexural analysis

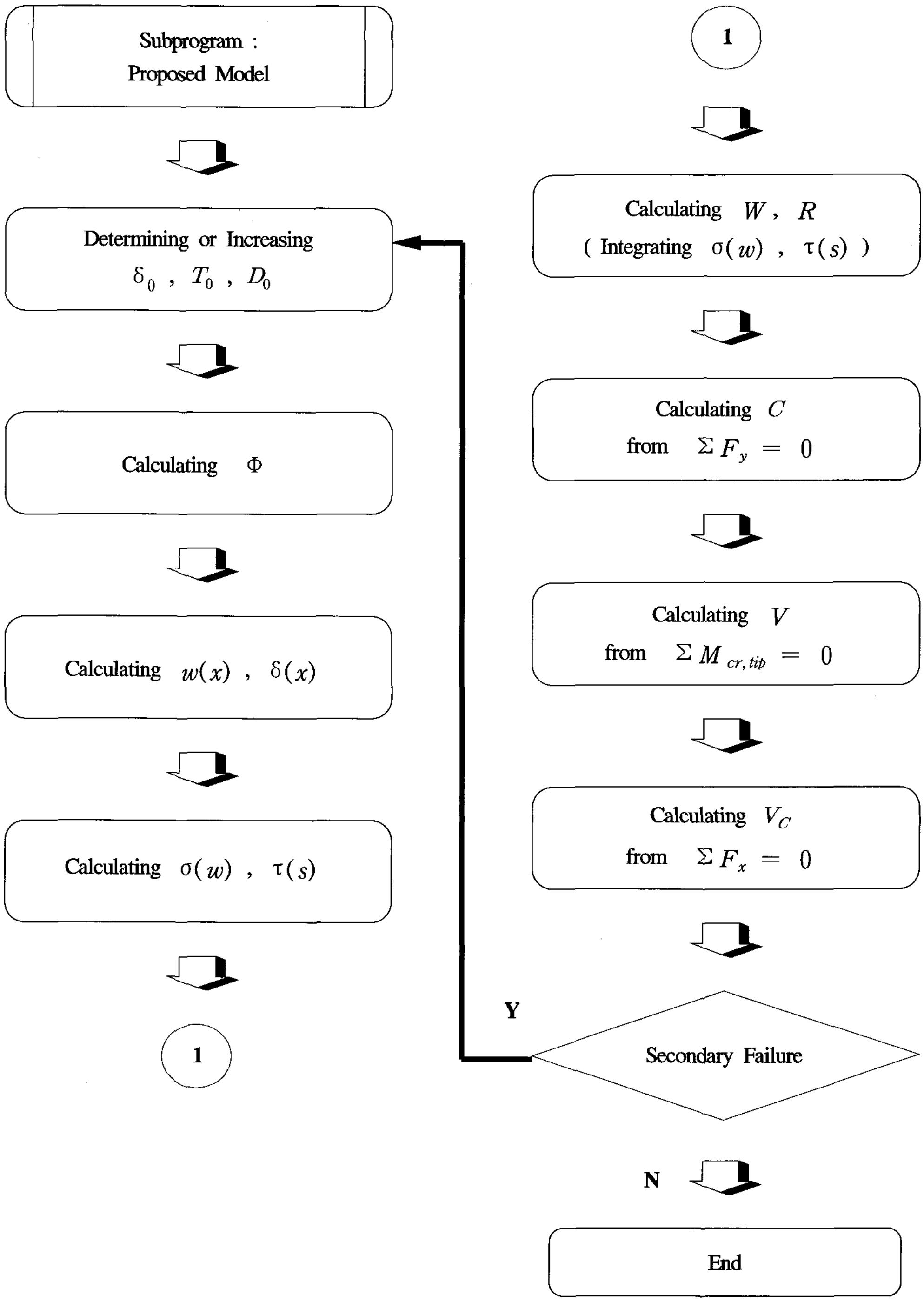

Fig. 13 Proposed model

6. Conclusions

In this paper, a mechanical model based on kinematical condition was proposed to simulate the shear behavior of reinforced concrete slender beams (*a/d* > 2.5) without shear reinforcement. Analysis was performed based on rational constitutive laws for concrete, compatibility conditions considering tension stiffening effect, and equilibrium conditions according to uncracked and cracked state.

Proposed model assumed the shape of diagonal tension crcak from the principal tensile stress trajectory. After development of this crack, a rotational condition was used to estimate the contribution of shear resisting forces, which are shear force in compression zone, vertical force in crack face and dowel force. These forces were derived from the constitutive relationship of each behavior model. It was assumed that shear failure was originated from concrete cover splitting by dowel action at the initial location of diagonal tension crack. Using this kinematical condition, it was possible to predict the shape and location of diagonal tension crack, crack width and sliding, each shear transfer mechanism, finally ultimate shear capacity. As a result, the proposed model provided a deeper insight for the contribution of each shear resisting force, and made a foundation to develop a more precise design equation considering the effect of shear deformation. Shear beam tests were performed to validate the various aspects of proposed model in a companion paper.

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