전단변형이론 및 미분구적법을 이용한 곡선보의 면외 진동해석

Out-of-Plane Vibration Analysis of Curved Beams Considering Shear Deformation Using DQM

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요 지

곡선보(curved beam)의 회전관성(rotatory inertia) 및 전단변형(shear deformation)을 고려한 면외(out-of-plane) 자유진동을 해석하는데 미분구적법(DQM)을 이용하여 고정-고정 경계조건(boundary conditions)과 다양한 굽힘각(opening angles)에 따른 진동수(frequencies)를 계산하였다. DQM의 결과는 엄밀해(exact solution) 또는 다른 수치해석 결과와 비교하였으며, DQM은 적은 요소(grid points)를 사용하여 정확한 해석결과를 보여주었다.

핵심용어 : 곡선보, 미분구적법(DQM), 진동수, 수치해석, 전단변형, 진동

Abstract

The differential quadrature method(DQM) is applied to computation of eigenvalues of the equations of motion governing the free out-of-plane vibration for circular curved beams including the effects of rotatory inertia and transverse shearing deformation. Fundamental frequencies are calculated for the members with clamped-clamped end conditions and various opening angles. The results are compared with exact solutions or numerical solutions by other methods for cases in which they are available. The DQM provides good accuracy even when only a limited number of grid points is used.

Keywords : *curved beam, DQM, fundamental frequency, numerical solution, shear deformation, vibration*

1. Introduction

Curved beams are used frequently in highway bridge structures. Curved alignments of highway bridges and interchanges have been necessary for the smooth dissemination of traffic in large urban areas. The construction cost and time of curved beams associated with the substructure have been found to be significantly reduced by the use of curved beams. Furthermore, the construction time is a factor of immense importance in the selection of a suitable structural system where the construction site needs to be used for other operations during the construction period described by Kang and Yoo(1994). Owing to their importance in many fields of technology and engineering, the vibration behavior of elastic curved beams has been the subject of a large number of investigations. Despite of a number of advantages, a curved member behaves in an extremely complex manner as compared to a straight member, and practicing engineers have often been discouraged by the complexity because of the initial curvature. However, the mathematical difficulties associated with curved members have been largely overcome with the application of digital computers and the development of numerical methods.

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The early investigators into the in-plane vibration of rings were Hoppe(1871) and Love(1944). Love(1944) improved on Hoppe's theory by allowing for stretching of the ring. Lamb(1888) investigated the statics of a curved bar with various boundary conditions and the dynamics of an incomplete free-free ring of small curvature. Den Hartog(1928) used the Rayleigh-Ritz method for finding the lowest natural frequency of circular arcs with clamped ends, and his work was extended by Volterra and Morell(1961) for the vibration of arches having center lines in the form of cycloids, catenaries, or parabolas. Archer(1960) carried out for a mathematical study of the in-plane inextensional vibrations of an incomplete circular ring of small cross section with the basic equations of motion as given in Love(1944) and gave a prescribed time-dependent displacement at the other end for the case of clamped ends. Out-of-plane vibrations of complete and incomplete rings have been the subject of interest for several research workers. Ojalvo(1962) obtained the equations governing three-dimensional linear motions of elastic rings and the results for the generalized loadings and viscous damping using classical beam theory assumptions. The natural frequencies of elastic beams were usually calculated by the classical beam theory in which the effects of rotatory inertia and transverse shearing deformation were not considered. However, the numerical results obtained by the theory can not present accurate values for the beams of considerably thick cross sections. Irie et al.(1982) have analyzed the out-of-plane vibration of circular beams based on Bresse-Timoshenko beam theory in which the effects of rotatory inertia and transverse shearing deformation are taken into account.

Recently, Lee and Oh(1996) have developed an approximate method to obtain the natural frequencies of the out of plane vibration of circular curved beams using the Runge-Kutta and Regula-Falsi methods, and Kim and Park(2006) have proposed a new efficient 2-noded hybrid-mixed element for curved beam vibrations having the uniform and non-uniform cross sections.

A rather efficient alternate procedure for the solution of partial differential equations is the method of differential quadrature which was introduced by Bellman and Casti(1971). This simple direct technique can be applied to a large number of cases to circumvent the difficulties of programming complex algorithms for the computer, as well as excessive use of storage. This method is used in the present work to analyze the out-of-plane vibration of curved beams based on the classical beam theory and Bresse-Timoshenko beam theory. The lowest frequency parameters are calculated for the member of square and circular cross sections under clamped-clamped boundary conditions with various opening angles. The DQM results are compared with exact solutions by Ojalvo(1962) or transfer matrix solutions by Irie et al.(1982).

2. Governing Differential Equations

The uniform curved beam considered is shown in Fig. 1. A point on the centroidal axis is defined by the angle θ, measured from the left support. The tangential and radial displacements of the arch axis are v and w, respectively. Here, u is the displacement at right angles to the plane of the arch, R is the radius of the centroidal axis, and β is the angular rotation of a cross section of the principal axes about the tangential axis. These displacements are considered to be positive in the directions indicated. A mathematical study of the out-of-plane vibration of the curved beam of a small cross section is carried out starting with the basic equations of motion as given by Ojalvo(1962).

If the effects of rotatory inertia and transverse shearing deformation are neglected, the differential equation governing the coupled twist-bending vibration of a thin curved beam can be written as

$$\frac{\partial^4 u}{\partial \theta^4} - R\frac{\partial^2 \beta}{\partial \theta^2} - k\left(\frac{\partial^2 u}{\partial \theta^2} + R\frac{\partial^2 \beta}{\partial \theta^2}\right) = \frac{mR^4}{EI_z}\frac{\partial^2 u}{\partial t^2} \tag{1}$$

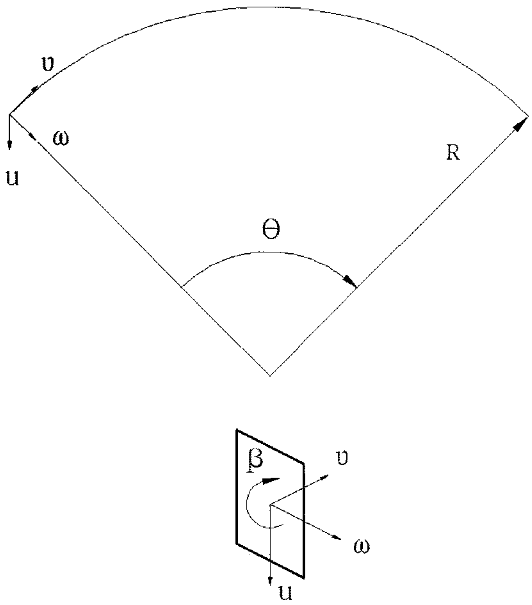

Fig. 1 Coordinate system for curved beam

$$\frac{\partial^2 u}{\partial \theta^2} - R\beta + k\left(R\frac{\partial^2 \beta}{\partial \theta^2} + \frac{\partial^2 u}{\partial \theta^2}\right) = 0 \tag{2}$$

or

$$\frac{\beta^{iv}}{\theta_0^4} + 2\frac{\beta''}{\theta_0^2} + \beta = -\lambda^2 \frac{1+k}{R} u \tag{3}$$

$$\frac{u''}{\theta_0^2} = \frac{R}{1+k}\left(\beta - k\frac{\beta''}{\theta_0^2}\right) \tag{4}$$

in which each prime denotes one differentiation with respect to the dimensionless distance coordinate, X, defined as

$$X = \frac{\theta}{\theta_0} \tag{5}$$

Here, k is the stiffness parameter GJ/EI_z, G is the shear modulus, J is the torsion constant of the cross section, I_z is the area moment of inertia of the cross section, E is the Young's modulus of elasticity for the material of the beam, m is the mass per unit length, θ_0 is the opening angle, and λ is the dimensionless parameter, related to the circular frequency ω. The moment-displacement relation can be expressed as

$$\lambda^2 = \frac{mR^4\omega^2}{GJ}, \quad M = \frac{EI_z}{R^2}(R\beta - u'') \tag{6}$$

If the beam is clamped at $\theta = 0$ and $\theta = \theta_0$, then the boundary conditions take the form

$$\beta(0) = u(0) = u'(0) = \beta(\theta_0) = u(\theta_0) = u'(\theta_0) = 0 \tag{7}$$

The differential equations governing the out-of-plane vibration of a circular beam based on the Bresse-Timoshenko beam theory, in which both the rotatory inertia and the shear deformation are taken into account, were given by Irie et al.(1982) as

$$\varkappa \frac{G}{E} s_z^2 \frac{u''}{R\theta_0^2} + p^2 \frac{u}{R} + \varkappa \frac{G}{E} s_z^2 \frac{\psi'}{\theta_0} = 0 \tag{8}$$

$$-\varkappa \frac{G}{E} s_z^2 \frac{u'}{R\theta_0} + (1+k)\frac{\beta'}{\theta_0} + \frac{\psi''}{\theta_0^2} - \left(k + \varkappa \frac{G}{E} s_z^2 - p^2 \frac{1}{s_z^2}\right)\psi = 0 \tag{9}$$

$$k\frac{\beta''}{\theta_0^2} - \left[1 - p^2\left(\frac{1}{s_z^2} + \frac{1}{s_x^2}\right)\right]\beta - (1+k)\frac{\psi'}{\theta_0} = 0 \tag{10}$$

Here, $\varkappa$ is the shear correction factor depending on the shape of the cross section, ν is the Poisson's ratio of the beam, and ψ is the slope of the displacement curve due to pure bending. For simplicity of the analysis, the following dimensionless variables have been introduced:

$$s_z^2 = \frac{AR^2}{I_z}, \quad s_x^2 = \frac{AR^2}{I_x}, \quad p^2 = \frac{mR^4\omega^2}{EI_z} \tag{11}$$

where s_x and s_z are the slenderness ratios, A is the cross sectional area, and p is the dimensionless parameter, related to the circular frequency ω.

If the beam is clamped at $\theta = 0$ and $\theta = \theta_0$, then the boundary conditions take the form

$$u(0) = \psi(0) = \beta(0) = u(\theta_0) = \psi(\theta_0) = \beta(\theta_0) = 0 \tag{12}$$

3. Differential Quadrature Method(DQM)

In many cases, moderately accurate solutions which can be calculated rapidly are desired at a few points in the respective physical domains. These solutions have traditionally been obtained by the standard finite difference and finite element methods have to be computed based on a large number of points. The mentioned methods depend strongly on the nature and refinement of the discretization of the domain. However, in order to get results even with only a limited amount of accuracy at or near a point of interest for a complicated problem, solutions often have to be computed based on a large number of surrounding points since the accuracy and stability of the aforementioned classical methods depend strongly on the nature and refinement scheme adopted to discretize the domain. Consequently, computational efforts are often considerable for these standard methods. In order to overcome the aforementioned complexities, an efficient procedure called differential quadrature method was introduced by Bellman and Casti(1971). By formulating the quadrature rule for a derivative as an analogous extension of quadrature for integrals in their introductory paper, they proposed the differential quadrature method as a new technique for the numerical solution of initial value problems of ordinary and partial differential equations. It was applied for the first time to static analysis of structural components by Jang et al.(1989). Kukreti et al.(1992) calculated the fundamental frequencies of tapered plates, and Farsa et al.(1993) applied the method to calculate the fundamental frequencies of general anisotropic and laminated plates. In another development, the quadrature method was introduced in lubrication mechanics by Malik and Bert(1994). The versatility of the DQM to engineering analysis in general and to structural analysis in particular is becoming increasingly evident by the related publications of recent years. Recently, Kang and Han(1998) applied the method to the static analysis of circular curved beams using classical and shear deformable beam theories, and Kang and Kim(2002) studied the buckling and the extensional vibration analysis of curved beams using the DQM.

From a mathematical point of view, the application of the differential quadrature method to a partial differential equation can be expressed as follows:

$$L\{f(x)\}_i = \sum_{j=1}^{N} W_{ij} f(x_j) \quad \text{for} \quad i, j = 1, 2, \ldots, N \tag{13}$$

where L denotes a differential operator, x_j are the discrete points considered in the domain, $f(x_j)$ are the function values at these points, W_{ij} are the weighting coefficients attached to these function values, and N denotes the number of discrete points in the domain. This equation, thus, can be expressed as the derivatives of a function at a discrete point in terms of the function values at all discrete points in the variable domain.

The general form of the function $f(x)$ is taken as

$$f_k(x) = x^{k-1} \quad \text{for} \quad k = 1, 2, \ldots, N \tag{14}$$

If the differential operator L represents an n^{th} derivative, then

$$\sum_{j=1}^{N} W_{ij} x_j^{k-1} = (k-1)(k-2)\cdots(k-n) x_i^{k-n-1} \quad \text{for} \quad i, k = 1, 2, \ldots, N \tag{15}$$

This expression represents N sets of N linear algebraic equations, giving a unique solution for the weighting coefficients, W_{ij}, since the coefficient matrix is a Vandermonde matrix which always has an inverse as described by Hamming(1973).

4. Application

Applying the DQM to equations (3) and (4) gives

$$\frac{1}{\theta_0^4} \sum_{j=1}^{N} D_{ij}\beta_j + \frac{2}{\theta_0^2} \sum_{j=1}^{N} B_{ij}\beta_j + \beta_i = -\lambda^2 \frac{1+k}{R} u_i \tag{16}$$

$$\frac{1}{\theta_0^2} \sum_{j=1}^{N} B_{ij} u_j = \frac{R}{1+k}\left(\beta_i - k\frac{1}{\theta_0^2} \sum_{j=1}^{N} B_{ij}\beta_j\right) \tag{17}$$

where B_{ij} and D_{ij} are the weighting coefficients for the second- and fourth-order derivatives along the dimensionless axis, respectively.

The boundary conditions for clamped ends, given by equation (7), can be expressed in differential quadrature form as follows:

$$\beta_1 = 0 \text{ and } u_1 = 0 \qquad \text{at } X = 0 \tag{18}$$

$$\beta_N = 0 \text{ and } u_N = 0 \qquad \text{at } X = 1 \tag{19}$$

$$\sum_{j=1}^{N} A_{2j} u_j = 0 \qquad \text{at } X = 0 + \delta \tag{20}$$

$$\sum_{j=1}^{N} A_{(N-1)j} u_j = 0 \qquad \text{at } X = 1 - \delta \tag{21}$$

where A_{ij} are the weighting coefficients for the first-order derivatives along the dimensionless axis, and δ denotes a very small dimensionless distance measured from the boundary ends of the member.

An intriguing issue in the quadrature solutions is the implementation of the boundary conditions, particularly in boundary value problems described by systems of higher than second order. Before solving these equations, one invokes the boundary conditions replacing the boundary-point equations by the DQ analog equations of the boundary conditions. This happens to be a rather simple matter with first- or second-order differential equations irrespective of domain dimensions and whether the boundary conditions are the Dirichlet and/or Neumann type or possibly of the mixed type. However, with the higher-order differential equations, implementation of the boundary conditions is not straightforward and needs careful consideration.

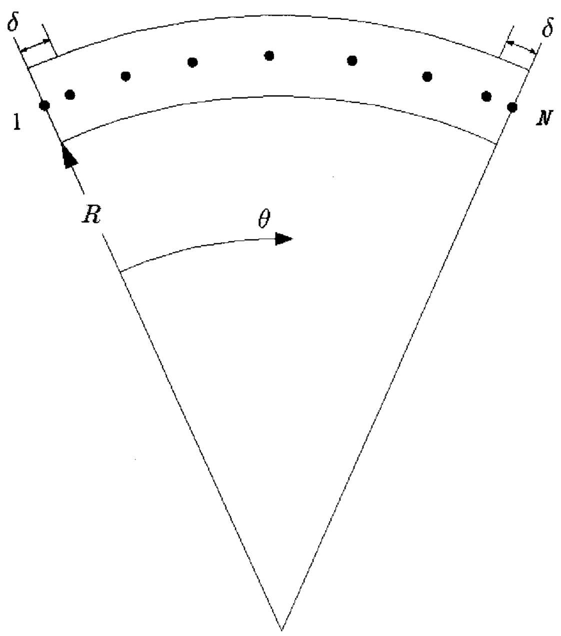

Fig. 2 Quadrature grid point for curved beam

The DQ analog of the two conditions at a boundary are written for the boundary points and their adjacent δ-points. The quadrature gird of a domain with the adjacent δ-points are shown in Fig. 2.

In the quadrature analog equations of the boundary conditions, the weighting coefficients should be the ones associated with the boundary points. Inasmuch as the boundary condition analog equations simply replace the quadrature analog of the governing equations at the boundary and adjacent points, the need for using the adjacent points sufficiently close to the boundary becomes somewhat ambiguous. The necessity of having the adjacent points close to the boundary points arises in problems where, in the process of the solution of the quadrature analog equations, the boundary-points values of the function are eliminated. This is actually the case with eigenvalue problems in which elimination of the function values even though the eigenvalue solution may have converged. The δ technique offers an adequate way for applying the double boundary conditions of beam problems and was applied quite successfully. However, an arbitrariness in the choice of the δ-value becomes apparent in the actual implementation of the boundary conditions. Thus, while the δ-value can not be large for an acceptable solution accuracy(possibly not greater than 0.001 in dimensionless value), with too small δ-value, the solution begins to oscillate.

Applying the DQM to equations (8), (9), and (10) gives

$$\varkappa \frac{G}{E} s_z^2 \frac{1}{R\theta_0^2} \sum_{j=1}^{N} B_{ij} u_j + p^2 \frac{1}{R} u_i + \varkappa \frac{G}{E} s_z^2 \frac{1}{\theta_0} \sum_{j=1}^{N} A_{ij} \phi_j = 0 \tag{22}$$

$$-\varkappa \frac{G}{E} s_z^2 \frac{1}{R\theta_0} \sum_{j=1}^{N} A_{ij} u_j + (1+k) \frac{1}{\theta_0} \sum_{j=1}^{N} A_{ij} \beta_j + \frac{1}{\theta_0^2} \sum_{j=1}^{N} B_{ij} \phi_j - \left(k + \varkappa \frac{G}{E} s_z^2 - p^2 \frac{1}{s_z^2}\right) \phi_i = 0 \tag{23}$$

$$k \frac{1}{\theta_0^2} \sum_{j=1}^{N} B_{ij} \beta_j - \left[1 - p^2 \left(\frac{1}{s_z^2} + \frac{1}{s_x^2}\right)\right] \beta_i$$

Table 1 Fundamental frequency parameters, $p=(mR^4\omega^2/EI_z)^{1/2}$, for out-of-plane vibration of clamped-clamped beams with square cross sections using Bresse-Timoshenko beam theory including a range of grid point N; ν = 0.3 and $\delta=1\times10^{-5}$

$s_x=s_z$	θ_0 (degrees)	Irie et al. (1982)	N				
			7	9	11	13	15
20	60	16.74	16.82	16.74	16.74	16.74	16.74
	120	4.282	4.332	4.280	4.283	4.283	4.283
	180	1.776	1.817	1.774	1.777	1.777	1.777
100	60	19.40	19.56	19.40	19.40	19.40	19.40
	120	4.451	4.449	4.449	4.452	4.452	4.453
	180	1.804	1.847	1.802	1.805	1.805	1.806

Table 2 Fundamental frequency parameters, $p=(mR^4\omega^2/EI_z)^{1/2}$, for out-of-plane vibration of clamped-clamped beams with circular cross sections using Bresse-Timoshenko beam theory including a range of δ; $\nu=0.3$, $\theta_0=180^0$, and $N=13$

$s_x=s_z$	Irie et al. (1982)	δ				
		0.08333	1×10^{-2}	1×10^{-4}	1×10^{-5}	1×10^{-6}
20	1.791	1.791	1.791	1.791	1.791	1.791
100	1.818	1.818	1.818	1.818	1.818	1.818

Table 3 Fundamental frequency parameters, $\lambda=(mR^4\omega^2/GJ)^{1/2}$, for out-of-plane vibration of thin curved beams with clamped ends using classical beam theory including a range of δ; $\nu=0.3$, $\theta_0=180^0$, and $N=13$

$k=GJ/EI_z$	Ojalvo(1962)	δ				
		0.08333	1×10^{-2}	1×10^{-4}	1×10^{-5}	1×10^{-6}
1.0	1.837	2.914	1.915	1.838	1.837	1.837

$$-(1+k)\frac{1}{\theta_0}\sum_{j=1}^{N}A_{ij}\phi_j=0 \tag{24}$$

The boundary conditions for clamped ends, given by equation (12), can be expressed in differential quadrature form as follows:

$$u_1=\phi_1=\beta_1=0 \quad \text{at } X=0 \tag{25}$$

$$u_N=\phi_N=\beta_N=0 \quad \text{at } X=1 \tag{26}$$

This set of equations together with the appropriate boundary conditions can be solved to obtain the fundamental natural frequency for the out-of-plane vibration of a curved beam.

5. Numerical Results and Comparisons

The fundamental frequency parameters of curved beams are calculated by the differential quadrature method(DQM), and the results are compared with the solutions by other methods including the effects of the rotatory inertia and shear deformation.

Tables 1~3 present the results of convergence studies relative to the number of grid point N and the parameter δ, respectively. Table 1 shows that the accuracy of the numerical solution increases with increasing N and passes through a maximum. Then, numerical instabilities arise if N becomes too large. The optimal value for N is found to be 11 to 13 using $\delta=1\times10^{-5}$. Tables 2 and 3 show the sensitivity of the numerical solution to the choice of δ using 13 grid points. From Table 2, the adjacent δ-points are not necessary for this case. Equally space grid points give the good accuracy because the implementation of the boundary conditions is straightforward(the one boundary condition at a boundary point). However, Table 3 shows that the solution accuracy decreases due to numerical instabilities if δ becomes too big because the implementation of the boundary conditions is not straightforward. The two conditions at a boundary point should be applied to the boundary point and the adjacent δ-point, respectively, as mentioned earlier. The optimal value for δ is found to be 1×10^{-5} to 1×10^{-6}, which is obtained from trial-and-error calculations. Therefore, all results are calculated using 13 grid points and $\delta=1\times0^{-5}$.

Fundamental frequency parameters, $\lambda=(mR^4\omega^2/GJ)^{1/2}$, for the out-of-plane vibration of thin curved beams with clamped ends neglecting the effects of the rotatory inertia and shear deformation are summarized and compared with the results by Ojalvo(1962) in Table 4. Engineering bounds on the stiffness parameter k are established in detail by Ojalvo(1962).

The values, $p=(mR^4\omega^2/EI_z)^{1/2}$, corresponding to the lowest natural frequencies are evaluated for the square and circular cross sections under clamped-clamped end conditions including the effects of the rotatory inertia and shear deformation, and numerical results are compared with transfer matrix solutions by Irie et al.(1982). The shear correction factor κ is taken to be 0.85 for the square cross section and 0.89 for the circular cross section, and the Poisson's ratio ν is 0.3. The results are summarized in Tables 5 and 6. As it can be seen from Tables 4~6, the numerical results show excellent agreement with the solutions by Ojalvo(1962) and those by Irie et al.(1982). From Tables 5 and 6, the frequency parameters of square cross section beams are generally smaller than those of circular cross section beams, and the difference between them is very small. The higher values of s_x and θ_0 have little effect on the fundamental natural frequency parameters. However, the lower values of s_x and θ_0 have a significant effect on the frequencies.

Table 4 Fundamental frequency parameters, $\lambda=(mR^4\omega^2/GJ)^{1/2}$, for out-of-plane vibration of thin curved beams with clamped ends using classical beam theory

θ_0 (degrees)	$k=GJ/EI_z$	$\lambda=(mR^4\omega^2/GJ)^{1/2}$	
		Ojalvo(1962)	DQM
180^0	0.005	6.860	6.860
	0.2	3.655	3.655
	0.5	2.517	2.517
	1.0	1.837	1.837
	1.625	1.461	1.460
270^0	0.005	1.817	1.818
	0.2	1.283	1.283
	0.5	0.9772	0.9772
	1.0	0.7603	0.7603
	1.625	0.6277	0.6277
360^0	0.005	0.6738	0.6738
	0.2	0.5788	0.5788
	0.5	0.5030	0.5030
	1.0	0.4382	0.4382
	1.625	0.3912	0.3912

Table 5 Fundamental frequency parameters, $p=(mR^4\omega^2/EI_z)^{1/2}$, for out-of-plane vibration of clamped-clamped beams with circular cross sections using Bresse-Timoshenko beam theory; $\nu=0.3$

$s_x=s_z$	θ_0(degrees)	Irie et al. (1982)	DQM
20	60	16.88	16.89
	120	4.309	4.309
	180	1.791	1.791
100	60	19.45	19.45
	120	4.473	4.473
	180	1.818	1.818

Table 6 Fundamental frequency parameters, $p=(mR^4\omega^2/EI_z)^{1/2}$, for out-of-plane vibration of clamped-clamped beams with square cross sections using Bresse-Timoshenko beam theory; $\nu=0.3$

$s_x=s_z$	θ_0(degrees)	Irie et al. (1982)	DQM
20	60	16.74	16.74
	120	4.282	4.283
	180	1.776	1.777
100	60	19.40	19.40
	120	4.451	4.452
	180	1.804	1.805

Table 7 Fundamental frequency parameters, $p=(mR^4\omega^2/EI_z)^{1/2}$, for out-of-plane vibration of clamped-clamped beams with square cross sections using both classical beam theory and Bresse-Timoshenko beam theory; $\nu=0.3$ and $k=1$

θ_0 (degrees)	Classical beam theory(DQM)	Bresse-Timoshenko beam theory (DQM)	
		s_x = 20	s_x = 100
180^0	1.837	1.777	1.805

Table 7 shows fundamental frequency parameters, $p=(mR^4\omega^2/EI_z)^{1/2}$, obtained by the DQM for clamped-clamped ends with square cross section beams neglecting both rotatory inertia and shear deformation(classical beam theory) or including both rotatory inertia and shear deformation(Bresse-Timoshenko beam theory). In general, as the slenderness ratios of beam cross sections become smaller, the frequencies become more significant.

6. Conclusions

The differential quadrature method(DQM) was used to compute the eigenvalues of the equations of motion governing the free out-of-plane vibration of curved beams based on the classical beam theory and Bresse-Timoshenko beam theory. The lowest frequency parameters were calculated for the member of square and circular cross sections under clamped-clamped boundary conditions and various opening angles. The present method gives results which agree very well with the solutions by other methods for the cases treated while requiring only a limited number of grid points.

Nomenclature

The following symbols are used in this paper:

A beam cross-sectional area

A_{ij} weighting coefficients for the first derivatives

B_{ij} weighting coefficients for the second derivatives

D_{ij} weighting coefficients for the fourth derivatives

E modulus of elasticity

$f(x)$ general function

$f(x_j)$ function value at point x_j

G shear modulus

I_x, I_z area moment of inertia about x-axis and z-axis, respectively

J torsion constant

k stiffness parameter

L differential operator

M bending moment

m mass per unit length

N number of discrete points

p fundamental frequency parameters, $(mR^4\omega^2/EI_z)^{1/2}$

R radius of centroidal axis

s_x, s_z slenderness ratios $\frac{AR^2}{I_x}$ and $\frac{AR^2}{I_z}$, respectively

u radial displacement in x-direction

v displacements in y-direction

X dimensionless position coordinate

x_j discrete point in domain

x, y, z coordinate axes, respectively

W_{ij} weighting coefficients

w tangential displacement in z-direction

β angular rotation

δ small dimensionless distance measured from boundary ends of member

θ angle from left support to generic point

θ_0 opening angle of member

λ^2 fundamental frequency parameters, $mR^4\omega^2/GJ$

$\varkappa$ shear correction factor

ν Poisson's ratio

ψ slope of displacement curve due to pure bending

ω circular frequency (rad/s)

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