Korea-Australia Rheology Journal
Vol. 19, No. 3, November 2007 pp. 99-107

Effect of particle migration on the heat transfer of nanofluid

Hyun Uk Kang, Wun-gwi Kim and Sung Hyun Kim*
Department of Chemical and Biological Engineering, Korea University, Seoul 136-701, Korea
(Received March 26, 2007; final revision received August 15, 2007)

Abstract

A nanofluid is a mixture of solid nanoparticles and a common base fluid. Nanofluids have shown great potential in improving the heat transfer properties of liquids. However, previous studies on the characteristics of nanofluids did not adequately explain the enhancement of heat transfer. This study examined the distribution of particles in a fluid and compared the mechanism for the enhancement of heat transfer in a nanofluid with that in a general microparticle suspension. A theoretical model was formulated with shear-induced particle migration, viscosity-induced particle migration, particle migration by Brownian motion, as well as the inertial migration of particles. The results of the simulation showed that there was no significant particle migration, with no change in particle concentration in the radial direction. A uniform particle concentration is very important in the heat transfer of a nanofluid. As the particle concentration and effective thermal conductivity at the wall region is lower than that of the bulk fluid, due to particle migration to the center of a microfluid, the addition of microparticles in a fluid does not affect the heat transfer properties of that fluid. However, in a nanofluid, particle migration to the center occurs quite slowly, and the particle migration flux is very small. Therefore, the effective thermal conductivity at the wall region increases with increasing addition of nanoparticles. This may be one reason why a nanofluid shows a good convective heat transfer performance.

Keywords : particle migration, nanofluid, convective heat transfer

1. Introduction

A nanofluid is consists of suspended nanoparticles and a base fluid, and has a very high thermal conductivity compared with that predicted by classical theories. However, in previous studies, a nanofluid was considered to be a homogeneous liquid, and its material properties were assumed to be constant in all positions of the system. These assumptions are not realistic, and might cause misunderstandings in the heat transfer mechanism of a nanofluid. Even if nanoparticles are in a stationary system, they can exhibit Brownian motion on account of their small size and mass. Therefore, an examination of the motion of nanoparticles in a nanofluid is essential for examining nanofluids as a heat transfer medium in heat exchanger or heat transfer equipment. Some studies have examined the average migration of particles to the radial direction in a tube. However, a study on the motion of each particle in a fluid is very difficult because it demands considerable computational resources. There is no general mechanism to explain the thermal behavior of nanofluid, although many possible reasons have been proposed, such as Brownian motion, liquid layering, ballistic conduction of phonon. It is difficult to establish the model to predict the heat transfer properties of nanofluids. Some previous researchers report that Brownian motion is one important factor on the heat transfer of nanofluids. (Koo and Kleinstreuer, 2004; Jang and Choi, 2004; Prasher, Bhattacharya and Phelan, 2005; Koo and Kleinstreuer, 2005). Brownian motion will have an effect on migration of particles in fluid and thermal transport in the fluid by collision between particles and liquid molecules. Effect of Brownian motion on thermal conductivity has been studied by the previously mentioned researchers, but there are few studies on relation between particle migration and heat transfer of nanofluids.

The migration of solid particles in flows has attracted considerable interest by many researchers (Abbot *et al.*, 1991; Acrivos *et al.*, 1992; Batchelor, 1977; Brady, 1993; Koh and Leal, 1994; Nott and Brady, 1994). Generally, the Brownian effect, inertial forces, and shear forces are major factors for particle migration in a flow. However, Brownian motion is not important in a microparticle suspension. Therefore, previous studies on the migration of particles in a tube have examined inertial migration in homogeneous shear flow or migration by shear forces in non-homogeneous shear flow.

Segrè and Silberberg studied the migration of dilute sus-

*Corresponding author: kimsh@korea.ac.kr

pensions of neutrally buoyant spheres in a tube flows and reported that the particles migrate away from both the wall and the center, and accumulate at a relative radial position of approximately 0.6 (Segrè and Silberberg, 1962). The phenomenon of radial migration driven by inertia was termed the tubular pinch effect in order to indicate that the uniform distribution of microparticles over a pipe cross-section converges to a narrow annulus as the suspension moves downstream. This remarkable effect has been verified by many studies (Oliver, 1962; Jeffrey and Pearson, 1965; Tachibana, 1973), which showed that the equilibrium position of particles was shifted towards the center when the particles were lagging the flow, and towards the wall when they were leading it. The equilibrium position lies closer to the wall in the presence of increased inertia with a large Reynolds number. Han *et al.* confirmed that it was a robust phenomenon that could be observed for volume fractions up to 0.2 (Han *et al.*, 1999).

Ho and Leal reported that migration in quadratic-bounded shear flow was caused by the inertia due to the presence of particles, and showed that the variation in the shear rate combined with the presence of a wall acting to create repulsion resulted in an equilibrium position at r = 0.6R (Ho and Leal, 1974).

A phenomenological model for particle migration in shear-induced flow proposed by Leighton and Acrivos attributed migration to irreversible interactions (Leighton and Acrivos, 1987). They derived an expression to describe the diffusion of the particle flux in simple shear flow. Phillips *et al.* modified the expression into a diffusion equation to describe the evolution of particles with respect to time (Phillips *et al.*, 1992). This model assumed that Brownian motion makes a negligible contribution.

Recently, Frank *et al.* examined particle migration in pressure-driven flow of a Brownian suspension (Frank *et al.*, 2003), and reported that the migration effect becomes more pronounced at higher flow rate and Peclet numbers.

Kim examined the migration of spherical particles in a polymer solution under Poiseuille or torsional flows and reported that a slight change in the rheological property of the dispersing medium can cause radical changes in the flow behavior and the migration of particles, particularly in dilute suspensions (Kim, 2001). They also examined the migration of non-colloidal, spherical particles in a Newtonian fluid under Poiseuille flow by combining the inertial migration and the shear-induced migration in a concentrated suspension, and suggested that in order to understand the particle migration in the tube flow of a suspension, however, both inertia and particle-particle interactions should be properly taken into account (Kim, 2004).

Ding and Wen examined the migration of nanoparticles in pressure-driven laminar pipe flows of dilute suspensions numerically. In their study, Brownian motion was considered because of its importance in nanoparticles but they neglected the effect of inertia (Ding and Wen, 2005). However, there have been no experimental or theoretical studies on the nanoparticle migration in Poiseuille flows because of the difficulty in detection and the requirement of considerable computational resources.

This migration of particles effects on the thermal, conductive, and rheological properties of the fluid. For the heat transfer, the solid particle has large thermal conductivity compared to the liquid and the concentration of solid particles may enhance the thermal conductivity and heat transfer coefficient. In this study, the migration of particles in fulid was investigated using simple balance equation to find the tendency of the particle concentration with particle size and the relation between these particle migrations and the heat transfer enhancement of nanofluid.

2. Theory

The two main factors in particle migration, the shear-induced migration and flow-induced migration, coexist in all systems. It has been suggested that shear-induced migration is proportional to the product of the shear rate (velocity gradient) and the square of the particle diameter. On the other hand, flow-induced migration is proportional to the product of the flow velocity and the particle diameter at high Peclet number. However in the previous studies, only one component was examined at any one time. When studying shear-induced particle migration, particle dispersion due to bulk flow has been neglected because the particle Reynolds number is low. However, the product of the shear rate and the particle size may not have a higher order of magnitude than the bulk flow velocity in a nanofluid because of the very small size of the nanoparticles. Therefore, the neglect of the particle dispersion by bulk flow (flow-induced migration) is not valid in a nanofluid. In order to understand the migration of nanoparticles in the tube flow, all possible mechanisms need to be considered at the same time.

Initially, migration in non-uniform shear flow was taken into account. Three dispersion mechanisms have been proposed in non-uniform shear flows: shear-induced migration, whereby a particle moves due to a difference in the shear rate; viscosity gradient-induced migration, whereby the particles migrate due to a viscosity difference; and Brownian self-diffusion, whereby the particles move as a result of a concentration gradient. The particle migration flux according to the described mechanisms can be written as follows:

$$J_\mu = -K_\mu \dot{\gamma}\Phi^2\left(\frac{D_p^2}{\mu}\right)\frac{d\mu}{d\Phi}\nabla\Phi\,, \tag{1}$$

$$J_c = -K_c d_p^2(\Phi\nabla\dot{\gamma} + \Phi\gamma\nabla\Phi)\,, \tag{2}$$

$$J_b = -D_b\nabla\Phi = -\frac{k_B T}{3\pi\mu d_p}\nabla\Phi\,, \tag{3}$$

where J_μ, J_c and J_b are the particle fluxes by viscosity gradient, by non-uniform shear rate, and by Brownian motion, respectively. K_c, K_μ are constants, d_p is the particle diameter, Φ is the volume fraction of the particles, and K_B is the Boltzmann constant ($=1.38\times10^{-23}$ J/K).

A simplified approach was used for the motion of particles. Since the suspended particles are carried by the fluid, the distribution of particles in the flow was determined instead of examining the motion of the particles. For the particles in flow field, the total mass balance is given by

$$\frac{\partial\Phi}{\partial t}+\nabla\cdot u_p\Phi+\nabla\cdot J_D=0\,, \tag{4}$$

where u_p is the particle velocity and J_D is the particle dispersion flux, which is induced by the above three dispersion mechanisms.

The total particle flux is the sum of the convective flux and the dispersion flux, *i.e.*

$$J_p=u_p\Phi+J_D\,. \tag{5}$$

Therefore, the total mass balance can be written as

$$\frac{\partial\Phi}{\partial t}+\nabla\cdot J_p=0\,. \tag{6}$$

In the steady-state, it can be simplified as

$$\nabla\cdot J_p=0\,. \tag{7}$$

The above equation and the concentration distribution of particles in tube flow can be solved if the particle velocity is known. If the flow is in a steady-state and is fully developed, the mass balance of the particle phase over the control volume is as follows:

$$J_p+r\frac{dJ_p}{dr}=0\,. \tag{8}$$

Integration of above equation, and utilization of the symmetrical boundary condition of $J=0$ at $r=0$ yields:

$$J_p=J_{IM}+J_\mu+J_c+J_b=0 \tag{9}$$

or

$$J_{IM}-\left[K_\mu\dot{\gamma}\Phi^2\frac{d_p^2 d\mu}{\mu\,dr}+K_c d_p^2\Phi^2\frac{d\dot{\gamma}}{dr}+K_c d_p^2\Phi\dot{\gamma}\frac{d\Phi}{dr}+\frac{k_B T}{3\pi\mu d_p}\frac{d\Phi}{dr}\right]=0\,, \tag{10}$$

where J_{IM} is the particle flux by inertial migration.

The flux due to inertial migration can be written as:

$$J_{IM}=\Phi u_{IM}\,. \tag{11}$$

The inertial migration velocity u_{IM} was given by Ho and Leal as follows:

$$u_{IM}=\frac{\rho V_m^2 d_p^3}{6\pi\mu_0 d^2}G(s)=\frac{Re_p V_m d_p^2}{6\pi d^2}G(s)\,, \tag{12}$$

where V_m is the maximum velocity at the center, and s is

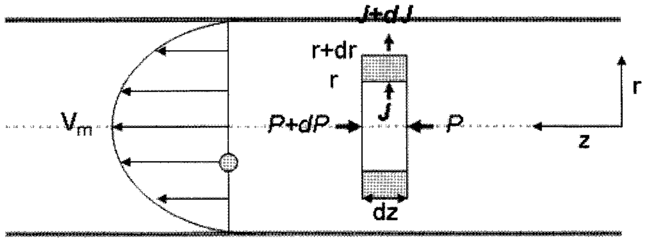


Fig. 1. Geometry of the system for two dimensional Poiseuille flow.

dimensionless coordinate given in Fig. 1. The value of function $G(s)$ was reported by Ho and Leal. The particle Reynolds number is defined as follows:

$$Re_p=\frac{\rho V_m d_p}{\mu_0}\,. \tag{13}$$

Equation (12) can be changed into the cylindrical coordinates.

$$u_{IM}=\frac{Re_p V_m d_p^2}{6\pi d^2}G(s)=\frac{Re_p V_m d_p^2}{24\pi R^2}G(s)\,. \tag{14}$$

The total mass balance by insertion of equation (11) and (14) can be expressed as equation (10).

$$\frac{Re_p V_m d_p^2\Phi}{24\pi R^2}G(s)$$
$$-\left[K_\mu\dot{\gamma}\Phi^2\frac{d_p^2 d\mu}{\mu\,dr}+K_c d_p^2\Phi^2\frac{d\dot{\gamma}}{dr}+K_c d_p^2\Phi\dot{\gamma}\frac{d\Phi}{dr}+\frac{k_B T}{3\pi\mu d_p}\frac{d\Phi}{dr}\right]=0\,. \tag{15}$$

In equation (15), the shear rate ($\dot{\gamma}$) and gradient of the particle volume fraction have been replaced by the corresponding one-dimensional forms. In order to solve the above equation, the shear rate and viscosity must be changed into the forms related to the particle volume fraction.

A steady-state laminar flow of a nanofluid through a horizontal tube was considered. It was assumed that the flow was one-dimensional, and pressure gradient in the radial direction was negligible. The momentum balance of the nanofluid over the control volume is as follows:

$$\frac{1}{r}\frac{d(r\tau)}{dr}=-\frac{dP}{dz}\,, \tag{16}$$

where P is the pressure, and τ is the shear stress. The integration of equation (16) gives the following equation with the boundary condition of $\tau=0$ at $r=0$.

$$\tau=-\frac{r}{2}\left(\frac{dP}{dz}\right)\,. \tag{17}$$

Because a nanofluid is generally a dilute suspension of nanoparticles, it was assumed to be a Newtonian fluid of which the shear stress was linearly dependent on the shear rate.

$$\tau = -\mu\dot{\gamma}. \tag{18}$$

A combination of the above two equation yields:

$$\dot{\gamma} = \frac{1}{2\mu}\left(\frac{dP}{dz}\right)r = \frac{du}{dr}, \tag{19}$$

where u is the axial velocity of the nanofluid.

The viscosity of the nanofluid is needed to solve this equation and is given as follows:

$$\mu_r = 1 + a\Phi, \tag{20}$$

where μ_r is the relative viscosity of a fluid, and a is the Einstein constant. For a dilute suspension, the value of a is 2.5. However, this value is higher in a nanofluid.

Equation (15) can be simplified by some mathematical operations. Dividing equation (15) by $\frac{V_m d_p^2}{R^2}$, results in:

$$\frac{Re_p\Phi}{24\pi}G(s)$$

$$-\left[\frac{K_\mu\dot{\gamma}\Phi^2}{V_m}\frac{R^2}{\mu}\frac{d\mu}{dr} + \frac{K_c R^2\Phi^2}{V_m}\frac{d\dot{\gamma}}{dr} + \frac{K_c R^2\Phi\dot{\gamma}}{V_m}\frac{d\Phi}{dr} + \frac{k_B T}{3\pi\mu d_p^3}\frac{R^2}{V_m}\frac{d\Phi}{dr}\right] = 0. \tag{21}$$

The following dimensionless terms will be used.

$$\bar{\mu} = \frac{\mu}{\mu_{BF}}, \tag{22}$$

$$\bar{\dot{\gamma}} = -\dot{\gamma}\frac{2\mu_{BF}}{(dP/dz)R} = \frac{\dot{\gamma}R}{V_m}, \tag{23}$$

$$\bar{r} = \frac{r}{R}, \tag{24}$$

$$\mathrm{Pe} = \frac{3\pi d_p^3(-dP/dz)R}{2k_B T}, \tag{25}$$

where Pe is the Peclet number which indicates physically the ratio of particle migration due to convection to that due to Brownian motion.

Equation (21) can be changed into a more simple form as follows:

$$\frac{Re_p\Phi}{24\pi}G(s)$$

$$-K_\mu\Phi^2\bar{\dot{\gamma}}\left[\frac{1}{\bar{\mu}}\frac{d\bar{\mu}}{d\bar{r}} + \frac{K_c}{K_\mu}\frac{1}{\bar{\dot{\gamma}}}\frac{d\bar{\dot{\gamma}}}{d\bar{r}} + \frac{K_c}{K_\mu}\frac{1}{\Phi}\frac{d\Phi}{d\bar{r}} - \frac{1}{K_\mu \mathrm{Pe}\Phi^2}\frac{1}{\bar{r}}\frac{d\Phi}{d\bar{r}}\right] = 0, \tag{26}$$

and the dimensionless shear rate and viscosity can be given by:

$$\bar{\dot{\gamma}} = -\frac{\bar{r}}{\bar{\mu}}, \tag{27}$$

$$\bar{\mu} = 1 + a\Phi. \tag{28}$$

The boundary condition of the above problem can be written as follows:

Table 1. Values of G($\bar{r}$)

r	G($\bar{r}$)	r	G($\bar{r}$)
0.10	−5.1939	0.60	3.6336
0.20	−3.8870	0.70	8.1197
0.30	−8.0237	0.80	7.6708
0.40	−10.6215	0.90	37.7181
0.50	−5.6609	1.00	207.4000

$$\int_0^1 2\bar{r}\Phi d\bar{r} = \Phi_0, \tag{29}$$

where Φ_0 is the initial particle concentration.

The solution to equation (26) gives the particle concentration profile in the radial direction. The effect of Brownian motion is unimportant for particles with a size greater than a few microns, and the last term in equation (26) can be neglected:

$$\frac{\mathrm{Re}_p\Phi}{24\pi}G(s)$$

$$-K_\mu\Phi^2\bar{\dot{\gamma}}\left[\frac{1}{\bar{\mu}}\frac{d\bar{\mu}}{d\bar{r}} + \frac{K_c}{K_\mu}\frac{1}{\bar{\dot{\gamma}}}\frac{d\bar{\dot{\gamma}}}{d\bar{r}} + \frac{K_c}{K_\mu}\frac{1}{\Phi}\frac{d\Phi}{d\bar{r}}\right] = 0. \tag{30}$$

This equation is the same as that obtained by Kim for micro-sized particles (Kim, 2004). If the effect of flow-induced migration is neglected, Equation 30 can be simplified to the following form:

$$\frac{1}{\bar{\mu}}\frac{d\bar{\mu}}{d\bar{r}} + \frac{K_c}{K_\mu}\frac{1}{\bar{\dot{\gamma}}}\frac{d\bar{\dot{\gamma}}}{d\bar{r}} + \frac{K_c}{K_\mu}\frac{1}{\Phi}\frac{d\Phi}{d\bar{r}} = 0. \tag{31}$$

This is the same as the equation derived by Phillips *et al.* for a micro-sized particle suspension in shear-induced flow (Phillips *et al.*, 1992). The function $G(s)$ was calculated using the data reported by Ho and Leal, which is listed in Table 1. The particle concentration was calculated numerically as a function of the initial concentration, particle size d_p, K_μ, K_c, viscosity μ and constant a.

3. Results and Discussion

There are many factors that affect the change in the distribution of particles. Numerical analysis has some problems because there is little information that can be used to solve these equations for nanofluids (30). For example, the constant K_μ and K_c was not estimated in the experiment. In addition, the Peclet number (Pe) and particle Reynolds number (Re_p) are dependent on the system parameter. The viscosity constant a is related to the particle-particle interaction in the liquid, which must be estimated in this experiment. Therefore, the precise distribution of particles cannot be calculated by numerical analysis. For this reason, only the qualitative results for the nanofluid that flows in a horizontal circular tube were proposed.

Only one boundary condition was used because equation (31) is first order. The boundary condition for concentration is given in equation (29).

Equation (30) contains the factors related to Brownian motion (Pe) and inertial motion ($Re_pG(\bar{r})$)., Equation 30 was simplified for a microfluid before the solving it for a nanofluid. The Brownian motion was either neglected or $\frac{1}{Pe}=0$ was used.

$$\frac{d\Phi}{d\bar{r}}=\frac{\frac{Re_pG(\bar{r})}{24\pi}\times\frac{\bar{\mu}}{\bar{r}K_\mu\Phi}+\frac{K_c\bar{\mu}}{K_\mu\bar{r}}}{\frac{a}{\bar{r}}\left(-\frac{K_c}{K_\mu}-1\right)-\frac{K_c}{K_\mu}\frac{1}{\Phi}}. \tag{32}$$

In a microfluid, the values K_c and K_μ are approximately 0.4 and 0.6, respectively (Chen *et al.*, 2004). Hence, the denominator term always has a negative sign. The sign of the equation is dependent on the sign of the numerator. The first term of the numerator, $G(\bar{r})$ has a negative sign in the range of r=0 to r=0.6, but has a positive sign in the range of r=0.6 to r=1. If the second term of the numerator is too large to offset the negative value of the first term in the range of r=0 to r=0.6, the numerator will have a positive sign, and $\frac{d\Phi}{d\bar{r}}$ will always have a negative sign. This means that the particles will migrate towards the center. If the particle concentration is small, then the first term of the numerator has a large value and the numerator may have a negative value in the range of r=0 to r=0.6. In that region, the slope of the particle concentration is >0, and the particle concentration increases with increasing radius. This means that the particles move to the wall side, which has been reported by Kim (Kim, 2004).

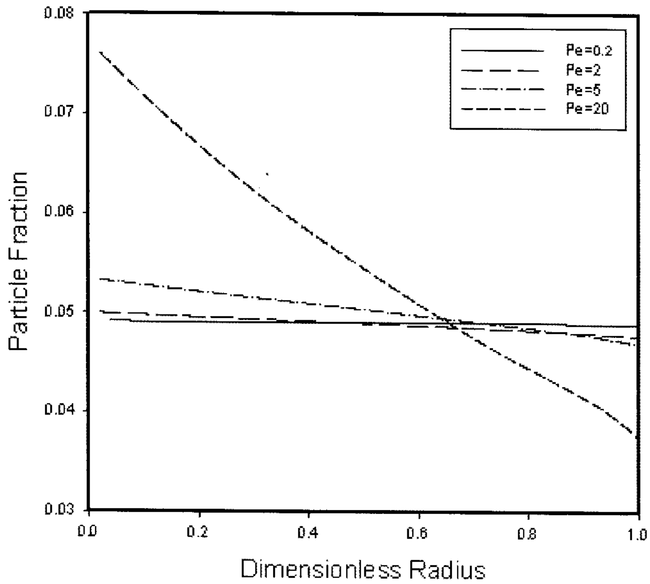


Fig. 2. Influence of the Peclet number on the concentration distribution.
(Φ_0=0.05, Re_p=0.01, a=2.5, K_c=0.4, K_μ=0.6).

Fig. 2 shows the effect of the Peclet number, which was defined in equation (25), on the particle concentration distribution for a mean particle volume fraction of 0.05. As shown in Fig. 2, a uniform distribution of particles was observed at Peclet numbers <5. On the other hand, a non-uniform distribution of particles was observed at a Peclet number of 20. The non-uniformity observed with high Peclet numbers is mainly due to the stronger contributions of shear-induced and viscosity-induced migration of particles. Brownian motion is important at Peclet numbers <10. The net effect of Brownian motion is a redistribution of suspended particles from higher concentration regions to lower concentration regions. The effect of the Peclet number is associated with a particle size under given flow condition due to the third power dependence in equation (25). The distribution of particles in the transverse plane becomes more uniform with decreasing particle size. This particle size dependency is similar to previous reports. If the particles are very small, the migration of particles to the center line due to the shear-induced migration proceeds very slowly, and the particle concentration distribution becomes uniform in all transverse planes. Under the laminar flow condition, simple calculation by equation (25) shows that values of Peclet number of 100 and 0.1 are equivalent to a particle size of 100~1000 nm and 10~100 nm, respectively.

It should be noted that the migration of particles to the center line may affect the heat transfer efficiency of the fluid due to the low particle concentration in the wall side. In previous studies on microfluids, the heat transfer efficiency of that fluid did not change despite the addition of solid particles with a high thermal conductivity. This phenomenon may be caused by the migration of particles toward the center line. Hence, the heat transfer resistance between two pipes does not change significantly, and the heat transfer efficiency does not increase. In a nanofluid, the movement of particles toward the center by shear-induced migration is small and the movement of particles by Brownian motion is very large, the heat transfer resistance between two pipes in heat exchanger decreases by the addition of nanoparticles because of the relatively high concentration of particles in the wall side compared to the microfluid.

Fig. 3 shows the influence of the mean particle concentration for $K_c=0.4$, $K_\mu=0.6$, $Re_p=0.01$, $a=2.5$ and Pe=2. As discussed in the previous section, in a very narrow particle size, the mean particle concentration will change the sign of the differential equation. Therefore, the solution of the differential equation may diverge to a physically impossible point. Accordingly, the simulation was performed in a restricted manner.

As the fraction of nanoparticles used in a nanofluid is generally very small, the maximum fraction was chosen as the value of 0.05. It can be seen that the average particle

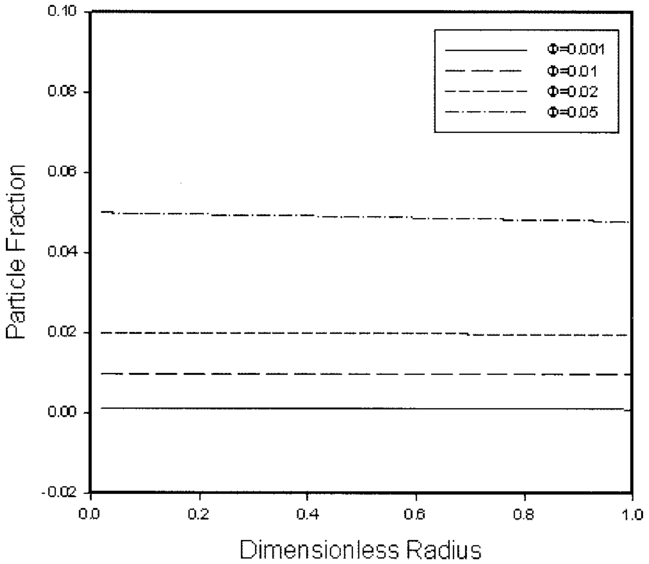
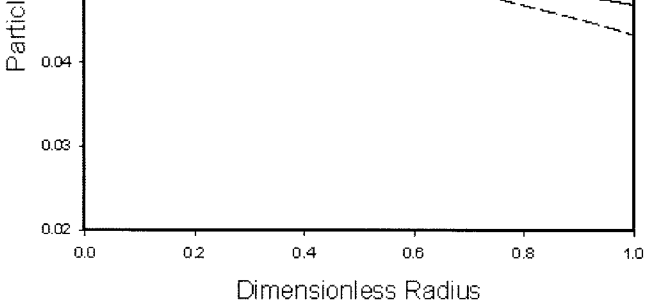


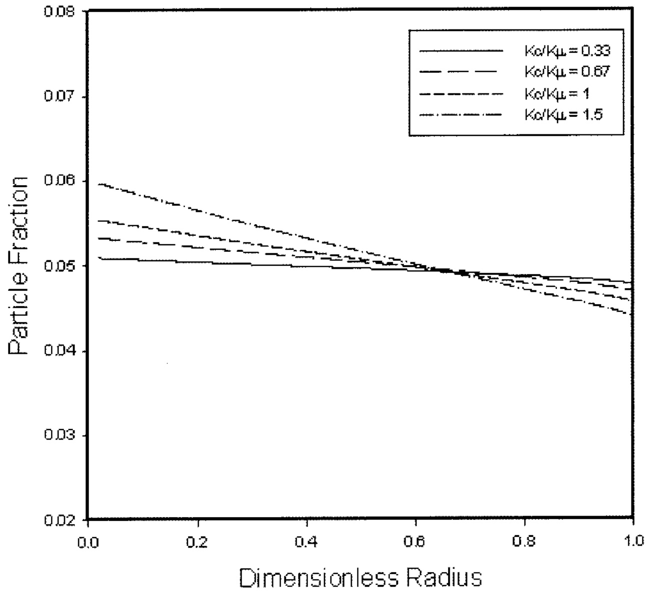
Fig. 3. Influence of the mean particle concentration on the concentration distribution.
(Pe=2, Re_p=0.01, a=2.5, K_c=0.4, K_μ=0.6).

concentration has no significant effect on the particle concentration distribution. The results shown in Fig. 3 suggest there is a relatively small mean particle concentration in a nanofluid due to the relatively homogeneous concentration distribution. This tendency was predicted from equation (30). The reason is associated with $1/\Phi^2$ in the last term of the denominator of equation (30). The last term of the denominator increases with decreasing average particle concentration. The absolute value of the slope $\frac{d\Phi}{d\bar{r}}$ becomes small, and the particle concentration becomes uniform.



(a) effect of the value of K_c and K_μ

Fig. 4. Influence of the constants K_c and K_μ on the concentration distribution. (Pe=5, Re_p=0.01, a=2.5, K_c/K_μ=0.67, Φ=0.05).



(b) effect of the ratio of K_c to K_μ

Fig. 4. Influence of the constants K_c and K_μ on the concentration distribution. (Pe=5, Re_p=0.01, a=2.5, K_μ=0.6, Φ=0.05).

The phenomenological constants K_c and K_μ will be estimated by the experimental study. However, the physical dependence of the particle distribution on the constants K_c and K_μ in a nanofluid was not investigated because measuring K_c and K_μ is difficult for a nanofluid due to the small particle size. Therefore, parametric analysis was carried out, and the results are shown in Fig. 4 for ϕ=0.05, Re_p=0.01, Pe=5 and a=2.5.

There was a non-uniform particle concentration at both the magnitude of K_c and K_μ (Fig. 4(a)) and a ratio of $\frac{K_c}{K_\mu}$ (Fig. 4(b)). At a small K_c and K_μ, the particles are distributed uniformly, which indicate that Brownian motion is dominant. As both two constants increase, the shear-induced migration toward the center line also increases. Hence, the particle concentration distribution will be non-uniform. This phenomenon can also be seen in Fig. 4(b). The shear-induced migration increases with increasing K_c to K_μ ratio. Although there was a non-uniform concentration distribution of particles, the degree of non-uniformity in the nanofluid was smaller than that in a microfluid due to the Brownian motion.

From equation (30) the inertial migration term, $Re_pG(\bar{r})$ is in the numerator. The particle Reynolds number is the only parameter that effects the inertial migration in equation (32). This inertial migration is important in studies of particle migration in a microfluid. The influence of inertial migration decreases with decreasing particle size. In a nanofluid, the effect of inertial migration was neglected, as shown in Fig. 5. In Fig. 5, the fluid of Re_p=1 shows a max-

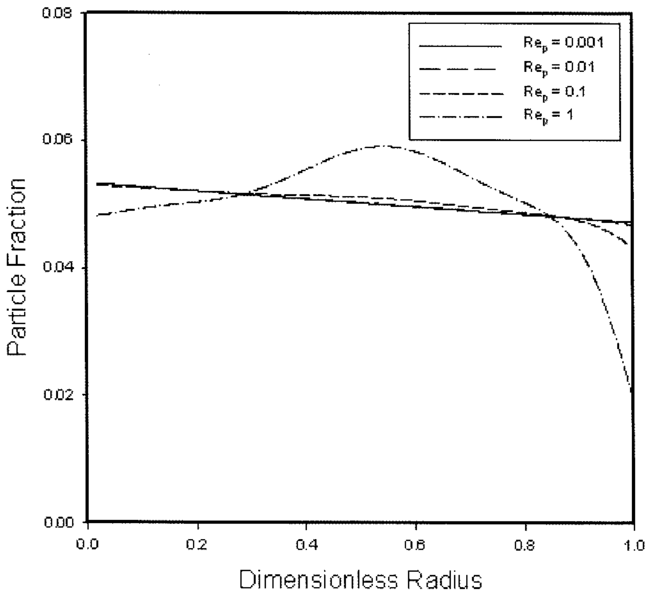


Fig. 5. Influence of Re_p on the concentration distribution. ($\mathrm{Pe}=5$, $\Phi=0.05$, $a=2.5$, $K_c=0.4$, $K_\mu=0.6$).

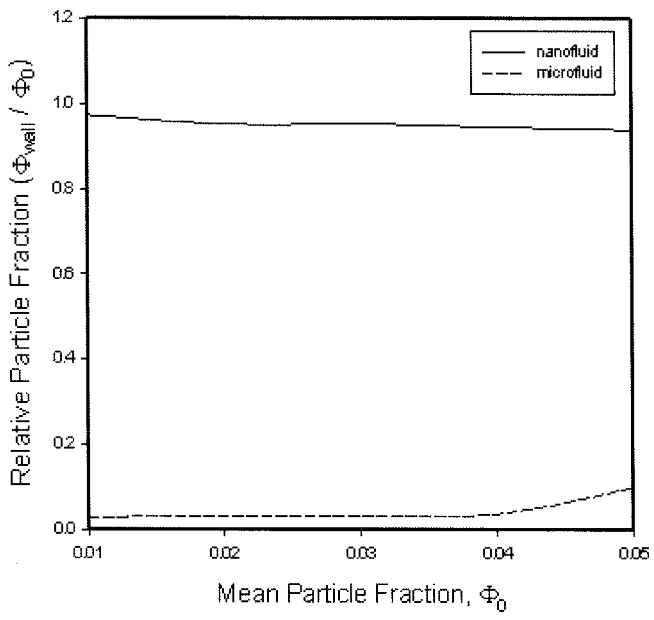


Fig. 6. Relative particle fraction as a function of the mean particle fraction.
($Pe=20$, $\mathrm{Re}_p=1$, $a=2.5$, $K_c/K_\mu=0.67$ for microfluid, $Pe=2$, $\mathrm{Re}_p=0.01$, $a=2.5$, $K_c/K_\mu=0.67$ for nanofluid).

imum concentration at $\bar{r}=0.6$, and the concentration near the wall decreases to 0.02. This result was observed in the microfluid of which the particle size was generally larger than sub-micron size. On the other hand, the fluid of $\mathrm{Re}_p=0.001$ shows a very uniform distribution, and the particle concentration near the wall was larger than that of the fluid of $\mathrm{Re}_p=1$. As discussed in the previous section, this high concentration near the wall side can lead to a decrease in heat transfer resistance of the system due to the high thermal conductivity of the solid nanoparticles. Therefore, the heat transfer efficiency can increase.

Particle migration causes a change in the particle concentration in the transverse plane, as shown in Fig. 6. These results were calculated under the condition of $Pe=20$, $\mathrm{Re}_p=1$, $a=2.5$, $K_c/K_c=0.67$ for microfluid, and $Pe=2$, $\mathrm{Re}_p=1$, $a=2.5$, $K_c/K_c=0.67$ for nanofluid. This is because the nanoparticles do not move toward the center line, and the relative particle fraction is almost 1. However, significant migration occurs with microparticles, and the particle fraction at the wall is much smaller than the average particle fraction. This particle distribution of nanofluid causes some changes of material properties such as thermal, rheological, electrical properties. In this study, the thermal property of nanofluid was calculated. Some researchers have reported that the heat transfer coefficients of nanofluid are higher than those of pure fluids, but the reasons are not explained clearly (Xuand and Li, 2003; Wen and Ding, 2004; Yang *et al.*, 2005). One possible mechanism of those enhancements of heat transfer may be the migration of nanoparticles to the wall side of tube by Brownian motion as mentioned above. Fig. 7 shows the calculated results of the relative heat transfer coefficient based on particle migration using the results shown in Fig. 6. The physical properties of fluid were calculated by equation (33), (34), (35) and (36). The enhancement of heat transfer coefficient of nanofluid calculated using Sieder-Tate equation (37) is much larger than that of microfluid (Sieder and Tate, 1936).

$$\mu=\mu_{BF}(1+2.5\Phi), \tag{33}$$

$$\rho=\rho_{BF}(1+\Phi)+\rho_{NP}\Phi, \tag{34}$$

$$C_p=C_{pBF}\frac{\rho_{BF}(1-\Phi)}{\rho_{BF}(a-\Phi)+\rho_{NP}\Phi}+C_{pNP}\frac{\rho_{NP}\Phi}{\rho_{BF}(a-\Phi)+\rho_{NP}\Phi}, \tag{35}$$

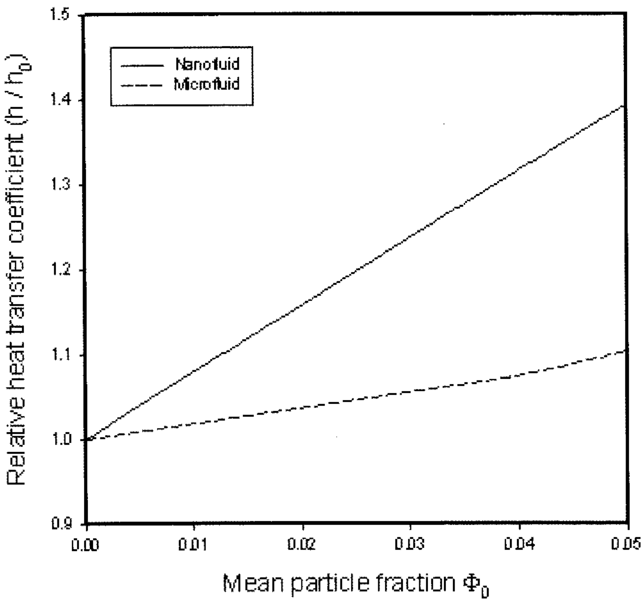


Fig. 7. Increase in the relative heat transfer coefficient as a function of mean particle fraction based on particle migration.

$$k_{eff} = k_{BF}\left(\frac{k_{NP}+2k_{BF}-2\Phi(k_{BF}-k_{NP})}{k_{NP}+2k_{BF}+\Phi(k_{BF}-k_{NP})}\right), \qquad (36)$$

$$h = 1.86\frac{k}{L}\left(\mathrm{RePr}\frac{D}{L}\right)^{1/3}\left(\frac{\mu_{BF}}{\mu_w}\right)^{0.14} \approx 1.86\left(\frac{D^2u}{L^4}\right)^{1/3}(\rho C_p k_{eff}^2)^{1/3}. \qquad (37)$$

In the above equations, C_p, k, D, L and h are specific heat, thermal conductivity, tube diameter, tube length, and heat transfer coefficient, respectively. Then, the effective thermal conductivity of a nanofluid increases with increasing particle volume fraction and it will be calculated using the particle fraction in the wall side. As shown in Fig. 7, the relative heat transfer coefficient of nanofluid may increase as 40% with nanoparticles of 5%. On the other hand, that of microfluid increases only about 5%. Although, this result was calculated with some assumptions and without the consideration for the kinds of nanofluids and the physical meanings of some variables, it showed the difference of heat transfer ability of nanofluid compared to that of microfluid clearly.

The particle migration may lead to a non-homogeneous thermal conductivity due to the non-homogeneous distribution of the particle concentration. In general, microparticles at the wall region tend to move toward the center direction, and the concentration at that region is much smaller than the average particle concentration. This decreasing particle concentration at the wall side can reduce the effect of the addition of solid particles in a fluid, and the thermal conductivity on the thermal resistance region between the bulk fluid and the wall of a pipe does not change sufficiently to enhance the heat transfer of a fluid. Therefore, the heat transfer coefficient will not change much despite the addition of solid particles with very high thermal conductivity. On the other hand, nanoparticles in a fluid do not move toward the center line, and the particle concentration is uniform in the transverse plane by Brownian motion, then the thermal conductivity on the particle depleted layer increases with the addition of solid nanoparticles. Consequently, the heat transfer coefficient might also increase with the addition of nanoparticles. Brownian motion itself can enhance energy transfer as shown in some previous studies. In those studies, Brownian motion effect on thermal conductivity of nanofluid was found to be more significant when compared to other possible reasons, such as liquid layering, phonon diffusion, formation of fractal structure, and so on. Therefore, the heat transfer efficiency of a nanofluid in a heat exchanger may increase by both thermal conductivity increasing of fluid and small thermal resistance in the wall region due to Brownian motion of nanoparticles.

4. Conclusion

This study examined the particle concentration distribution using a numerical simulation of particle migration. A theoretical model was formulated with shear-induced particle migration, viscosity-induced particle migration, particle migration by Brownian motion, as well as the inertial migration of particles. Particle migration due to shear-induced migration can result in significant non-uniformity in particle concentration over the cross-section, particularly for large particles. In the case of a nanofluid, Brownian motion of nanoparticles decreases the non-uniformity, and the inertial migration does not cause a significant change in the concentration distribution.

A uniform particle concentration is very important for the heat transfer of a nanofluid. The convective heat transfer coefficient is related to the thermal conductivity of a fluid in the thermal resistance region. As the particle concentration and the effective thermal conductivity at the wall region becomes lower than that of the bulk fluid due to particle migration to the center in a microparticle suspension, the addition of microparticles in a fluid does not affect the heat transfer properties of the fluid. However, in a nanofluid, particle migration to the center occurs quite slowly, and the effective thermal conductivity at the wall region increases. This may be one reason why a nanofluid has good convective heat transfer properties. Therefore, the heat transfer of nanofluids may be enhanced by increase of thermal conductivity and decrease of thermal resistance in the wallside region due to Brownian motion of nanoparticles.

Acknowledgements

This study was supported by research grants from the Korea Science and Engineering Foundation (KOSEF) through the Applied Rheology Center (ARC) at Korea University, Seoul, Korea.

Notations

a = viscosity constant [-]
C_p = Specific heat [J/kg]
D = Diameter of tube [m]
D_b = Brownian diffusion coefficient $= \frac{k_B T}{3\pi\mu d_p}$ [m²/s]
d_p = particle diameter [m]
h = heat transfer coefficient [W/m² K]
J_b = particle flux by Brownian motion [m³/m² s]
J_c = particle flux by non-uniform shear rate [m³/m² s]
J_D = particle dispersion flux $= J_b + J_c + J_\mu$ [m³/m² s]
J_{IM} = particle flux by inertial migration [m³/m² s]
J_p = total particle flux [m³/m² s]
J_μ = particle flux by viscosity gradient [m³/m² s]
k = thermal conductivity [W/m K]
k_B = Boltzmann constant [1.38×10^{-23} J/K]
K_c = phenomenological constant [-]
K_μ = phenomenological constant [-]
L = length of tube [m]
P = pressure [Pa]
Pe = Peclet number $= \frac{3\pi d_p^3(-dP/dz)R}{2k_B T}$ [-]
r = radial position [m]

$\bar{r}$ = dimensionless radial position [-]
R = pipe radius [m]
Re_p = particle Reynolds number $= \frac{\rho V_m d_p}{\mu}$ [-]
t = time [s]
T = temperature [K]
u = axial velocity of fluid [m/s]
$\bar{u}$ = dimensionless axial velocity of fluid [-]
u_{IM} = inertial migration velocity [m/s]
u_p = particle velocity [m/s]
V_m = maximum velocity at the center [m/s]

Greek symbol

$\dot{\gamma}$ = shear rate [1/s]
$\bar{\dot{\gamma}}$ = dimensionless shear rate [-]
Φ = volume fraction of particles in fluid [-]
μ = viscosity [Pa s]
$\bar{\mu}$ = dimensionless viscosity $= \frac{\mu}{\mu_{BF}}$ [-]
μ_{BF} = viscosity of base fluid [Pa s]
μ_r = relative viscosity $= \bar{\mu}$ [-]
ρ = density [Kg/m^3]
τ = shear stress [Pa]

Subscript

b = property by Brownian motion
B = Boltzmann
BF = base fluid
c = property by shear rate gradient
D = dispersion
eff = effective
f = base fluid
IM = inertial migration
m = maximum
p = particle
r = relative
w = wall
μ = property by viscosity gradient

References

Abbott J.R., N. Tetlow, A.L. Graham, S.A. Altobelli, E. Fukushima, L.A. Mondy and T.S. Stephens, 1991, Experimental observations of particle migration in concentrated suspensions: Couette flow, *J. Rheol.*, **35**, 773-795.

Acrivos A., G.K. Batchelor, E.J. Hinch, D.L. Koch and R. Mauri, 1992, Longitudinal shear-induced diffusion of spheres in a dilute suspension, *J. Fluid Mech.*, **240**, 651-657.

Batchelor G.K., 1977, The effect of Brownian motion on the bulk stress in a suspension of spherical particles, *J. Fluid Mech.*, **83**, 97-117.

Brady J. 1993, The rheological behavior of concentrated colloidal dispersions, *J. Chem. Phys.*, **99**, 567-581.

Chen, X, Y.C. Lam, Z.Y. Wang and K.W. Tan, 2004, Determination of phenomenological constants of shear-induced particle migration model, *Comp. Mater. Sci.*, **30**, 223-229.

Ding, Y. and D. Wen, 2005, Particle migration in a flow of nanoparticle suspensions, *Powder Tech.*, **149**, 84-92.

Frank, M., D. Anderson, E.R. Weeks and J.F. Morris, 2003, Particle migration in pressure-driven flow of a Brownian suspension, *J. Fluid Mech.*, **493**, 363-378.

Han, M.S., C. Kim, M. Kim, S. Lee, 1999, Particle migration in tube flow of suspensions, *J. Rheol.*, **43**, 1157-1174.

Ho, B.P. and L.G. Leal, 1974, Inertial migration of rigid spheres in two-dimensional unidirectional flows, *J. Fluid Mech.*, **65**, 365-400.

Jang, S.P. and S.U.S. Choi, 2004, Role of Brownian motion in the enhanced thermal conductivity of nanofluid, *Appl. Phys. Lett.*, **84**, 4316-4318.

Jeffrey, R.C. and J.R. Pearson, 1965, Particle motion in laminar vertical tube flow, *J. Fluid Mech.*, **22**, 721-735.

Kim, C., 2001, Migration in concentrated suspension of spherical particles dispersed in polymer solution, *Korea-Australia Rheol. J.*, **13**, 19-27.

Kim, C., 2004, Mathematical model of migration of spherical particles in tube flow under the influence of inertia and particle-particle interaction, *Korean J. Chem. Eng.*, **21**, 27-33.

Koh C., P. Hookham and L.G. Leal, 1994, An experimental investigation of concentrated suspension flows in a rectangular channel, *J. Fluid Mech.*, **266**, 1-32.

Koo, J. and C. Kleinstreuer, 2004, A new thermal conductivity model for nanofluids, *J. Nanoparticle Res.*, **6**, 577-588.

Koo, J. and C. Kleinstreuer, 2005, Impact anaylsis of nanoparticle motion mechanisms on nanofluid thermal conductivity, *Int. Comm. Heat Mass Transfer*, **32**, 1111-1118.

Leighton, D. and A. Acrivos, 1987, The shear-induced mirgation of particles in concentrated suspensions, *J. Fluid Mech.*, **181**, 415-439.

Nott P. and J.F. Brady, 1994, Pressure-driven flow of suspensions: simulation and theory, *J. Fluid Mech.*, **275**, 157-199.

Oliver, R, 1962, Influence of particle rotation on radial migration in the Poiseuille flow of suspensions, *Nature*, **194**, 1269-1271.

Prasher, R., P. Bhattacharya and P.E. Phelan, 2005, Thermal conductivity of nanoscale colloidal solutions (nanofluids), *Phys. Rev. Lett.*, **94**, 025901.

Phillips, R.J., R.C. Armstrong, R.A. Brown, A.L. Graham and J.R. Abbott, 1992, A constitutive equation for concentrated suspensions that accounts for shear-induced particle migration, *Phys. Fluids A, Fluid Dyn.*, **4**, 30-40.

Segrè, G. and A. Silberberg, 1962, Behavior of macroscopic rigid spheres in Poiseuille flow: Part 2. Experimental results and interpretation, *J. Fluid Mech.*, **14**, 136-157.

Sieder E.N. and G.E. Tate, 1936, Heat Transfer and Pressure Drop of Liquids in Tube, *Ind. Eng. Chem.*, **28(12)**, 1429–1435.

Tachibana, M., 1973, "On the behaviour of a sphere in the laminar tube flows", *Rheol. Acta*, **12**, 58-69.

Xuan .Y and Q. Li, 2003, Investigation on convective heat transfer and flow features of nanofluids, *J. Heat Trans.*, **125**, 151-155.

Wen D. and Y. Ding, 2004, Experimental investigation into convective heat transfer of naofluids at the entrance region under laminar flow conditions, *Int. J. Heat Mass Trans.*, **47**, 5181.

Yang Y., Z.G. Zhang, E.A. Grulke, W.B. Anderson and G. Wu, 2005, Heat transfer properties of nanoparticle-in-fluid dispersions (nanofluids) in laminar flow, *Int. J. Heat Mass Trans.*, **48**, 117-1116.