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The Numerical Analysis of Jeju Harbor Flow Considering Effect of Seasonal Wind

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Abstract : *The effect of seasonal wind on the tidal circulation in Jeju harbor was examined by using a numerical shallow water model. A finite element for analyzing shallow water flow is presented. The Galerkin method is employed for spatial discretization. Two step explicit finite element scheme is used to discretize the time function, which has advantage in problems treating large numbers of elements and unsteady state. The numerical simulation is compared with three cases; Case 1 does not consider the effect of wind, Case 2 and Case 3 consider the effect of summer and winter seasonal wind, respectively. According to result considering effect of seasonal wind, velocity of current vector shows slightly stronger than that of case 1 in the flow field. It can be concluded that the present method is a useful and effective tool in tidal current analysis.*

Key words : *Finite element method, Seasonal wind, Galerkin method, Tidal current*

1. Introduction

Horizontal flow of the seawater induced by the movement of celestial bodies, which are moon, sun etc, is called the tidal current. When tidal current approaches the shallow water zone, the tidal current shows complicated motion induced by Coriolis forces, shearing stresses, surface winds and frictional forces. It is, therefore, very important in costal engineering to investigate the motion of seawater in the shallow water zone. And a flow in the seawater plays an importance role in environment of shallow water zone.

A number of finite element methods have been applied to the analysis of the shallow water zone, application fields include lake circulation, drift, tidal currents in coastal zone, ocean circulation, tsunami wave propagation and pollution diffusion etc. (Jung and Kim, 1992; Kawahara, 1976; Kawahara, 1977; Kawahara and Umetsu, 1986, 1988; Pearson and Winter, 1977; Young and Liggett, 1977). The flow at the shallow water zone is analyzed by using finite element method with linear triangle element. Assuming that tidal current is periodic, the Galerkin finite element method is employed for the numerical integration in space. Two step explicit scheme is used for the discretization of time(Kim, 1995).

In this study, to examine the effect of seasonal wind on flow of the seawater in Jeju harbor, the flow of the seawater is calculated with effect of summer seasonal wind, winter seasonal wind and without effect of seasonal wind, respectively. And then computation results are compared with flow of seawater in the shallow water zone.

2. Basic Equation

We shall consider an area of sea with the rectangular coordinates X, Y, Z as illustrated in Fig. 1. Let the surface of the mean sea level designate the reference plane $Z=0$. Let $h(X, Y)$ be the depth measured from the mean sea level to the sea bed. The elevation from the mean sea level to the temporary sea surface is called the tidal elevation, and it is denoted as $\eta(X, Y, t)$ with the time variable $t(\mathrm{sec})$. The total depth of the sea is therefore given by $H=h+\eta$. It is assumed that the density of the seawater $\rho(=kg/m^3)$ is constant. With the velocity components u and v of the current on the $X-$direction and $Y-$direction respectively, the U and V of the vertically averaged velocity are expressed by

$$U=\frac{1}{H}\int_{-h}^{\eta} udz, \quad V=\frac{1}{H}\int_{-h}^{\eta} vdz \tag{1}$$

The equation of continuity can be written as follow,

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$$z+\frac{\partial(UH)}{\partial x}+\frac{\partial(VH)}{\partial y}=0 \tag{2}$$

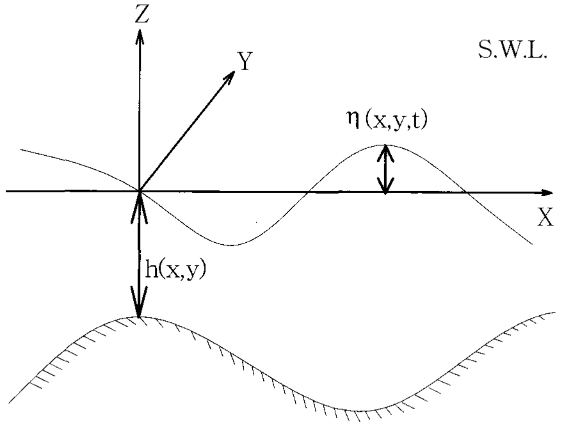


Fig. 1 Vertical cross section of the sea

Suppose that the acceleration in vertical direction is negligibly small and assume that the pressure is hydrostatic, the X-component and Y-component of the momentum equation considering effect of seasonal wind can be given approximately by Pinder and Gray(1977) as follows.

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}+g\frac{\partial \eta}{\partial x}-fV$$
$$=\frac{1}{\rho H}\left\{\frac{\partial}{\partial x}\left(\mu H\frac{\partial U}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu H\frac{\partial U}{\partial y}\right)\right\}$$
$$+\frac{KW^2}{H}cos\psi-\frac{gU\sqrt{U^2+V^2}}{HC^2} \tag{3}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}+g\frac{\partial \eta}{\partial y}+fU$$
$$=\frac{1}{\rho H}\left\{\frac{\partial}{\partial x}\left(\mu H\frac{\partial V}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu H\frac{\partial V}{\partial y}\right)\right\}$$
$$+\frac{KW^2}{H}sin\psi-\frac{gV\sqrt{U^2+V^2}}{HC^2} \tag{4}$$

where f is the Coriolis forces, given by $f=2wsin\phi$ with the angular velocity of the terrestrial rotation $w=7.292\times10^{-5}(rad/sec)$ and the latitude $\phi(rad)$, g is the acceleration of gravity$(=9.81m/s^2)$, μ is the eddy viscosity, and the second term on the right hand side in each of these momentum equations account for the wind stress the X-component and Y-component, respectively. An effect of wind on the flow in shallow water zone is considered based on proposition by Pinder and Gray(1977).

$$K=\begin{cases}1.0\times10^{-3} & (W\leqq 5m)\\ 1.5\times10^{-3} & (5m\leqq W\leqq 15m)\\ 2.0\times10^{-3} & (15m\leqq W\leqq 20m)\end{cases} \tag{5}$$

where K is a dimensionless coefficient of a superficial force that is a function of wind speed due to winds blowing on the surface, $W(m/s)$ is the wind speed 10 meter high above the sea surface, ψ is the angle of the wind direction to the X-direction, and C is the Chezy's friction coefficient $(=1/nh^{1/6}$, n: *Manning's Roughness Coefficient*$)$ on the sea bed.

In this study, to examine the effect of seasonal wind on flow of seawater in Jeju harbor, wind blows from NW direction is predominate in winter season and wind blows from SE direction is predominate in summer season according to measured data from 1961 year to 1990 year by MOMAF(Ministry of Maritime Affairs and Fisheries). And Table 2 shows summer and winter seasonal wind direction and speed.

3. Finite Element Formulation

Let Ω be a flow domain. the continuity equation (2) and the momentum equations (3) and (4) are employed to the flow model in shallow water using Galerkin method for the discretization of the spatial unknown variables.

For the computation of continuity and momentum equations (2), (3) and (4) are employed. multiplying both sides of basic equations by the weighting functions and integrating over the domain Ω, the finite element solution equations can be derived.

$$\int_\Omega \eta^*\frac{\partial U}{\partial t}d\Omega+\int_\Omega \eta^*\frac{\partial(UH)}{\partial x}d\Omega$$
$$+\int_\Omega \eta^* v\frac{\partial(VH)}{\partial y}d\Omega=0 \tag{6}$$

$$\int_\Omega V^*\frac{\partial V}{\partial t}d\Omega+\int_\Omega V^* U\frac{\partial V}{\partial x}d\Omega+\int_\Omega V^* V\frac{\partial V}{\partial y}d\Omega$$
$$+g\int_\Omega V^*\frac{\partial \eta}{\partial y}d\Omega+\int_\Omega V^* fUd\Omega+$$
$$\int_\Omega V^*\frac{gV\sqrt{U^2+V^2}}{HC^2}d\Omega-\int_\Omega V^*\frac{KW^2}{H}sin\psi d\Omega$$
$$-\int_\Omega V^* v_l\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right)d\Omega=0 \tag{7}$$

$$\int_{\Omega} U^* \frac{\partial U}{\partial t} d\Omega + \int_{\Omega} U^* U \frac{\partial U}{\partial x} d\Omega + \int_{\Omega} U^* V \frac{\partial U}{\partial y} d\Omega \tag{8}$$

$$+ g \int_{\Omega} U^* \frac{\partial \eta}{\partial x} d\Omega - \int_{\Omega} U^* f V d\Omega +$$

$$\int_{\Omega} U^* \frac{gU\sqrt{U^2+V^2}}{HC^2} d\Omega - \int_{\Omega} U^* \frac{KW^2}{H} cos\psi d\Omega$$

$$- \int_{\Omega} U^* v_l \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) d\Omega = 0$$

where, v_l is $\frac{\mu}{\rho}$.

The flow domain is subdivided into triangles. Inside each triangle e with its three verticals as the element nodes 1, 2, 3. The approximate equations of both weighting function of velocity and water elevation are linearly interpolated by the interpolations as follows.

$$\left.\begin{array}{ll} U = \sum_{\alpha=1}^{3} \phi_\alpha u_\alpha, & U^* = \sum_{\alpha=1}^{3} \phi_\alpha u_\alpha^* \\ V = \sum_{\alpha=1}^{3} \phi_\alpha v_\alpha, & V^* = \sum_{\alpha=1}^{3} \phi_\alpha v_\alpha{}^* \\ \eta = \sum_{\alpha=1}^{3} \phi_\alpha \eta_\alpha, & \eta^* = \sum_{\alpha=1}^{3} \phi_\alpha \eta_\alpha^* \end{array}\right\} \tag{9}$$

where $\phi_\alpha(\alpha = 1,2,3)$ are linear interpolation functions, given by

$$\phi_\alpha = \frac{1}{2\Delta^e}(a_\alpha + b_\alpha x + c_\alpha y) \quad (\alpha = 1,2,3) \tag{10}$$

with the area Δ^e of the triangle, and weighting functions $(u_\alpha^*, v_\alpha^*, \eta_\alpha^*)$ are nodal values of the corresponding unknowns.

The above interpolation functions are substituted into Equations (6) and (7). From the continuity in the interpolation of η, U and V and the arbitrariness of η^*, U^* and V^*, the element equations is obtained. And then, assembling all the element equations over an analytical domain, the total equations is obtained.

$$[Z]\{\dot{D}\} + [T]\{D\} = 0 \tag{11}$$

where, $[Z] = \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix}$, $\dot{D} = \begin{Bmatrix} \dot{\eta} \\ \dot{U} \\ \dot{V} \end{Bmatrix}$,

$$[T] = \begin{bmatrix} XH & YH & 0 \\ XU+I+N & YU-L & A \\ XV+L & YV+I+N & B \end{bmatrix},$$

$$\{D\} = \begin{Bmatrix} U \\ V \\ \eta \end{Bmatrix}, \; M = \frac{\Delta}{12}(1+\delta), \; A = g\frac{b}{6}, \; B = g\frac{c}{6},$$

$$X = \frac{b}{24}(1+\delta), \; Y = \frac{c}{24}(1+\delta),$$

$$L = f\frac{\Delta}{12}(1+\delta), \; I = \frac{g}{Hc^2}\frac{\Delta}{12}(1+\delta)\sqrt{u^2+v^2}$$

The differential terms with regard to time are included in Equation (11), and they are discretized using the two step explicit scheme(Kawahara and Umetsu, [1988]).

$$[\bar{Z}]\{k^{n+1/2}\} = [\tilde{Z}]\{k^n\} - \frac{\Delta t}{2}[T]\{k^n\} \tag{12}$$

$$[\bar{Z}]\{k^{n+1}\} = [\tilde{Z}]\{k^n\} - \Delta t[T]\{k^{n+1/2}\} \tag{13}$$

where, $[\bar{Z}] = \begin{bmatrix} \bar{M} & 0 & 0 \\ 0 & \bar{M} & 0 \\ 0 & 0 & \bar{M} \end{bmatrix}$, $\bar{M} = \frac{\Delta}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Δ is the area of a triangle. the mass matrix $[Z]$ is replaced on the righthand side with the selectively lumped mass matrix $[\bar{Z}]$. This technique is used to reduce the numerical damping effect and to adjust the numerical stability.

$$[\tilde{Z}] = \zeta[\bar{Z}] + (1-\zeta)[Z] \tag{14}$$

where ζ is referred to as the selectively lumping parameter, which is used to control the numerical damping and the numerical stability. The parameter ζ is precisely investigated by Kawahara et al.(1982). and chosen to be 0.90 in this study. From Equations (12) and (13), the unknown variables η, U and V can be solved simultaneously.

4. Numerical computation

The tidal current of Jeju harbor is estimated based on equations (12) and (13). The flow of computations are carried out three cases; Case 1 does not consider seasonal wind, Case 2 and Case 3 consider seasonal wind.

Fig. 2 shows a triangle mesh idealization for the present finite element method, which consists of 1,437 nodes and 2,582 elements. The water depth of computation domain is shown in Fig. 3. Moreover, the velocity of incoming water from Sanji River by field survey is taken as $20cm/s$.

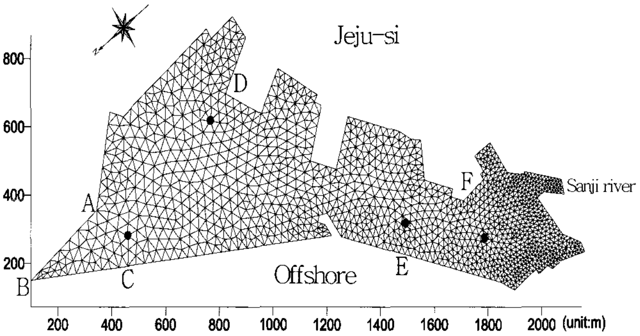


Fig. 2 Finite element idealization of Jeju harbor

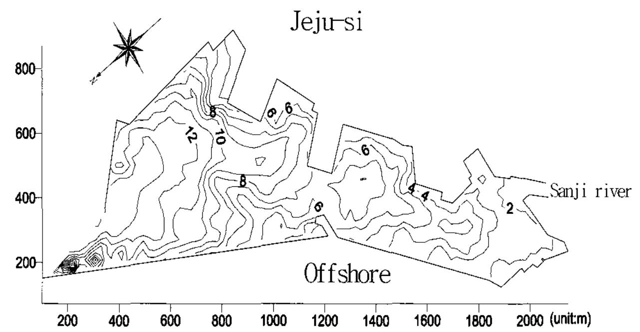


Fig. 3 Water depth of Jeju harbor

4.1 Verification of numerical model

The initial condition in a actual coastal zone is difficult to know, the computation starts with the calm sea domain is assumed as the initial boundary condition.

$$\eta = U = V = 0 \quad at \quad t = 0 \tag{15}$$

It is assumed that still water level is without the fluctuation of water level and the normal velocity to the coastline is zero.

Table 1 shows the harmonic analysis of M_2, S_2, K_1 and O_1 in the Jeju harbor. Along the entrance $A-B$, the water elevation of incident waves is assumed to be given by:

$$\eta = \sum_{m=1}^{4} a_m \sin\left(\frac{2\pi}{T_m}t - k_m\right) \tag{16}$$

where a_m is the amplitude, k_m is the phase delay, T_m is the period of M_2, S_2, K_1 and Q_1 constituent and t is the time increment in computation, respectively. The time increment is set as $\triangle t = 0.2\mathrm{sec}$ according to the following stability condition by Kawahara et al.(1982).

$$\frac{\Delta t}{\Delta x} \leqq \frac{\sqrt{2}-\frac{e}{\sqrt{3}}}{3} \cdot \frac{1}{\sqrt{gh}} \tag{17}$$

where e is Lumping parameter.

The tidal current of Jeju harbor is computed and compared with calculated data and measured data by MOMAF(2003) for verification of model as shown in Fig. 4.

Fig. 4 shows comparison with three tidal currents and tidal curve; one is estimated tidal current (●) based on considered 4 constituents in this study and the other is estimated tidal current(■) based on just considered M_2 constituent, and tidal curve measured by MOMAF(2003). According to Fig. 4, present work, which considered 4 constituents, is closer agreement with measured tidal curve than that of estimated based on just considered M_2 constituent only. In this work the eddy viscosity and Manning's roughness coefficient are $1.0m^2/s$ and 0.03, respectively.

Table 1 The results of harmonic analysis for tide at Jeju harbor(OHA, 1993)

m	Constituent	Amplitude $a_m(cm)$	Period T_m (hour)	Phase delay k_m(degree)
1	M_2	69.7	12.4206	303.5
2	S_2	29.5	12.0000	324.1
3	K_1	23	25.8193	210.2
4	O_1	16.8	23.9345	189.2

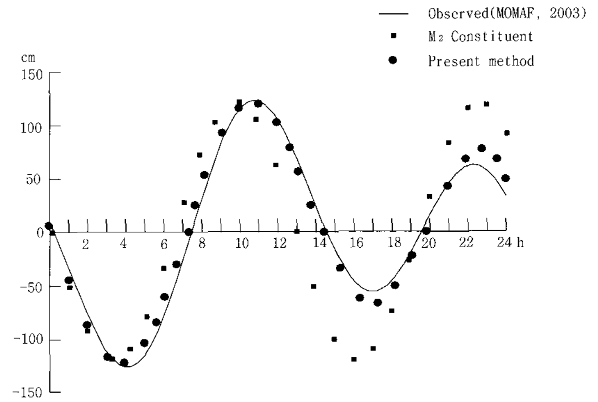


Fig. 4 Tidal curve comparison between observed and present method

4.2 Tidal computation of Jeju harbor

Fig. 5 shows the calculated velocity without effect of seasonal wind at the high water, flood tide, low water and ebb tide using Eq. (15) as the incident wave condition, respectively.

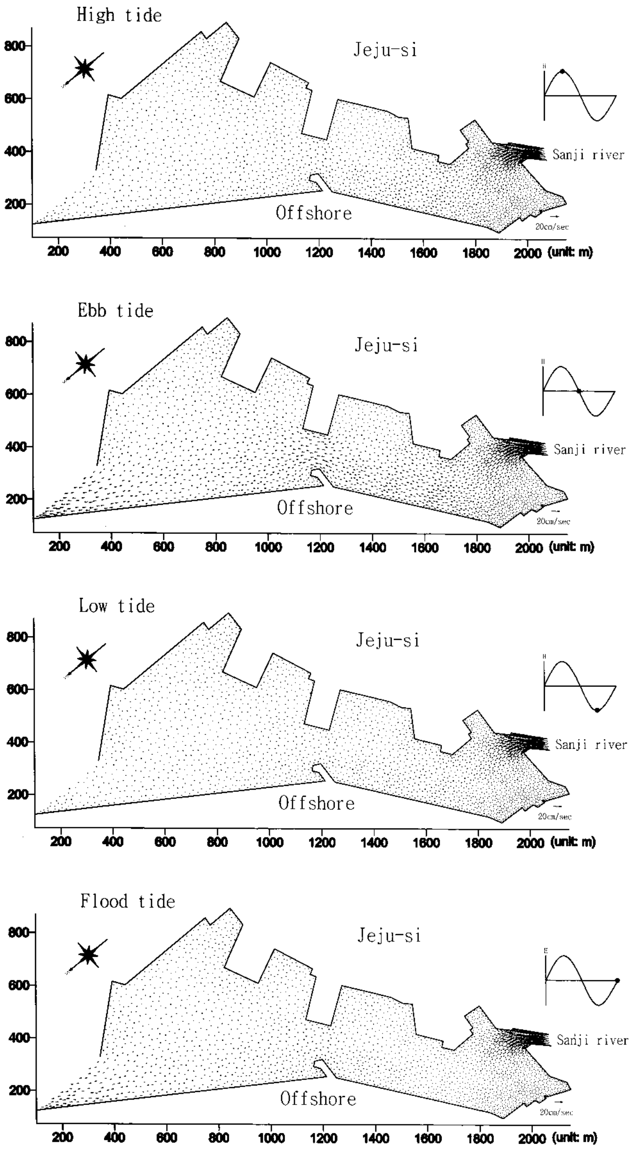


Fig. 5 Velocity vectors in Jeju harbor without seasonal wind effect

Fig. 6 and Fig. 7 are onsidered the effect of seasonal wind, these show result of numerical analysis with seasonal wind as shown in Table 2, respectively (MOMAF, 1998).

The velocity of flow vectors as shown in Fig. 6 with the effect of winter seasonal wind that predominately blows from NW direction increases generally than its of Fig. 5 that do not consider effect of seasonal wind. Fig. 6 shows that the counter clockwise vortex of tidal current is weakly formed on Jeju harbor entrance portion in flood water, and counter clockwise vortex of tidal current is little strongly formed on Jeju harbor inside. The low water of Fig. 6 shows that as tidal zone occurs on Jeju harbor inside, vortex of counter clockwise is formed and vortex of counter clockwise is formed to Jeju harbor entrance department and vortex of clockwise is formed on Jeju harbor center portion. Specially, the calculated velocity vetors at the flood tide and ebb tide show that vortex of strong counter clockwise and clockwise vortex are formed on Jeju harbor inside. Also, tendency that microscopic vortex is formed partially can be known.

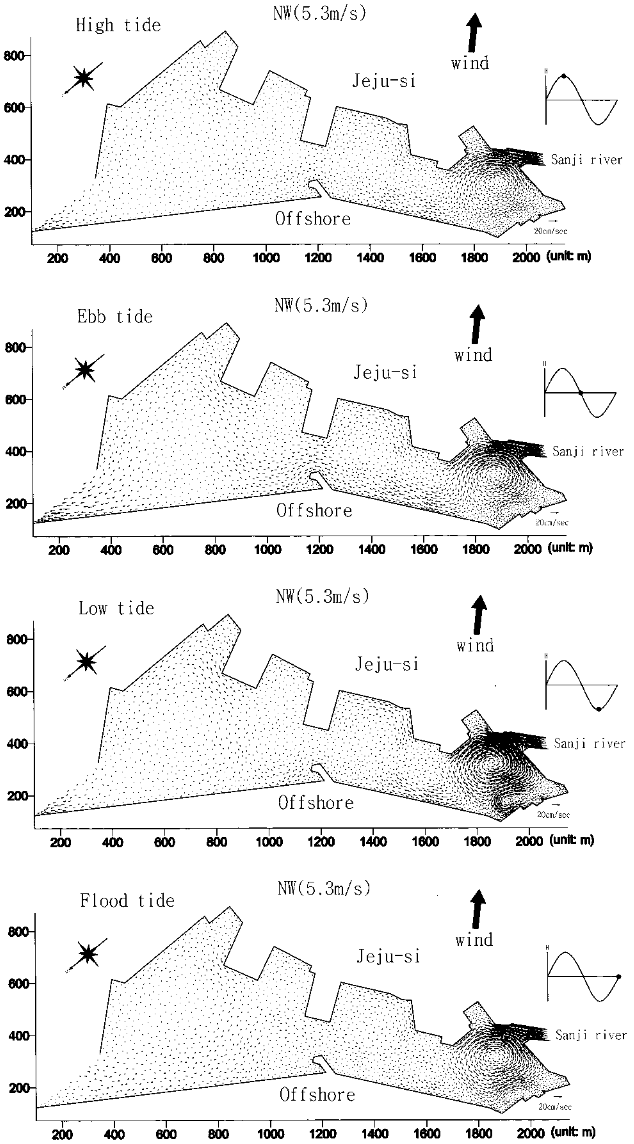


Fig. 6 Velocity vectors in Jeju harbor with seasonal wind effect in winter(NW, 5.3m/s)

Fig. 7 shows the velocity of flow vectors considering the effect of summer seasonal wind that predominately blows from SE direction. It shows that the vortex of counter clockwise is formed inside of Jeju harbor and the velocity of flow vectors is a little weak, because the summer seasonal wind blows from SE direction and is smaller average of velocity than that of winter seasonal wind

Table 2 shows that velocity vectors of three Case; Case 1 unconsidering seasonal wind, Case 2 (summer season) and Case 3 (winter season) considering seasonal wind stress at nodal point C, D, E and F in Fig. 2, respectively. Table 2 shows that velocity vectors of Case 2 and Case 3 considering seasonal wind stress are bigger than that of Case 1 unconsidering seasonal wind stress.

Moreover, velocity vector of Case 3 is bigger than that of Case 2.

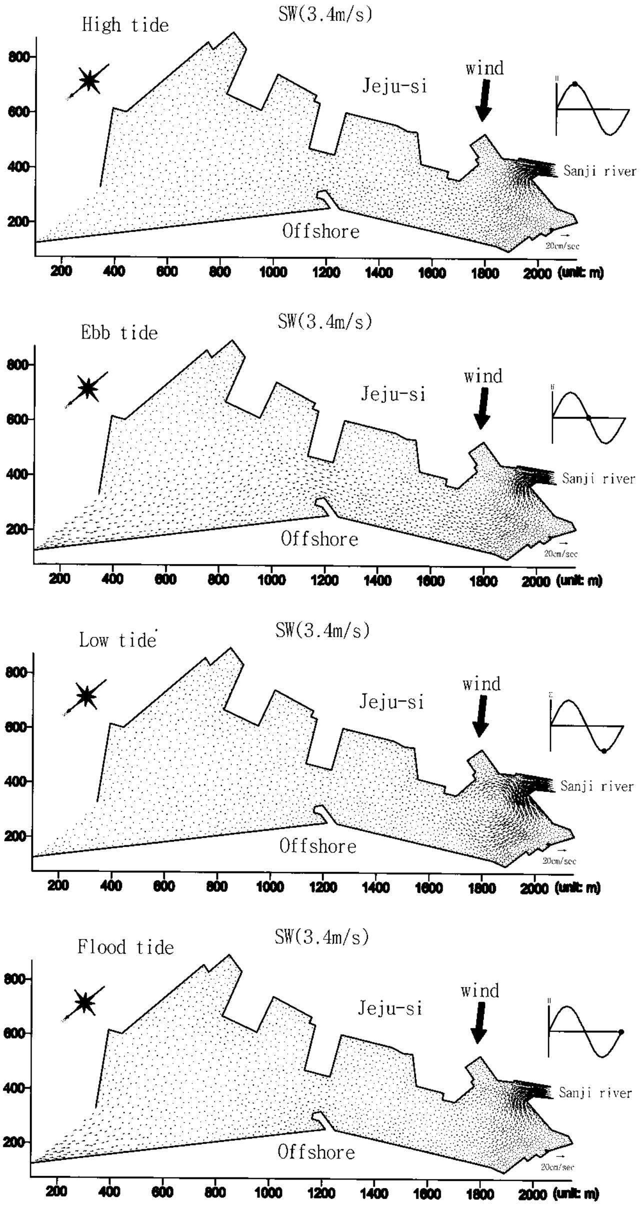


Fig. 7 Velocity vectors in Jeju harbor with seasonal wind effect in summer(SE, 3.4m/s)

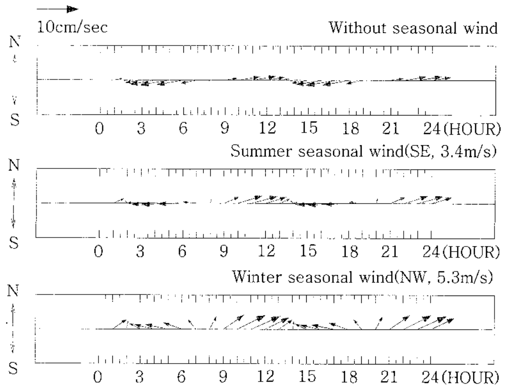
Fig. 8 shows calculated tidal velocity during 24 hours at nodal point C and E in Fig. 2. Flow considering winter seasonal wind at bottom in Fig. 8 at (a) and (b) is the biggest than that of Case 1 without seasonal wind and Case 2 with summer seasonal wind.

Fig. 9 shows tidal current ellipse at nodal point C and E in Fig. 2 in each case. It shows that tidal current ellipse is bigger formated on winter season(Case 3) than that of summer season(Case 2).

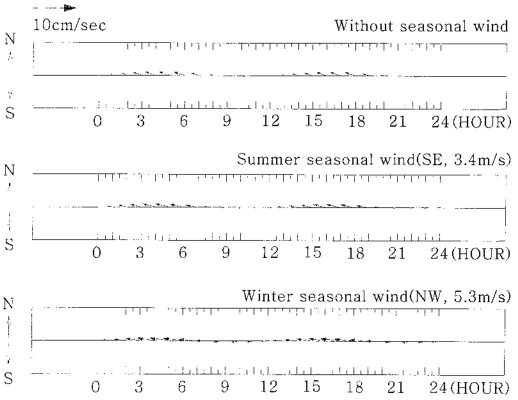
As shown in Fig. 8 and Fig. 9, tidal current has bigger flows on entrance of Jeju harbor at nodal point C than its of inside of Jeju harbor at nodal point E.

Table 2 The comparison of velocity vectors considering wind stress and unconsidered seasonal wind stress in nodal points C, D, E and F(MOMAF, 1998)

Case	Wind Direction	Wind Speed (m/sec)	velocity vector(cm/sec)			
			C	D	E	F
1	None	None	6.58	1.22	5.29	3.63
2	SE (summer)	3.4	6.80	1.49	5.7	6.45
3	NW (winter)	5.3	6.88	5.44	6.13	7.06



(a) nodal point C



(b) nodal point E

Fig. 8 Comparison of calculated current at nodal point C and E

5. Conclusion and Remark

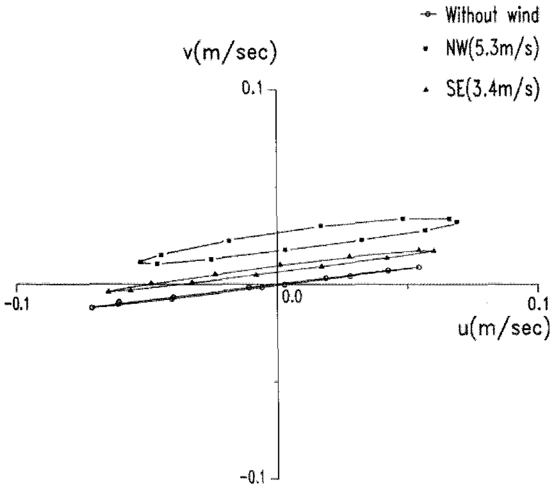
In this study, the effect of seasonal wind acting tidal current is investigated in shallow water zone using finite element method.

The conclusions obtained from this study considering the effect of seasonal wind and unconsidered the effect of seasonal wind are these:

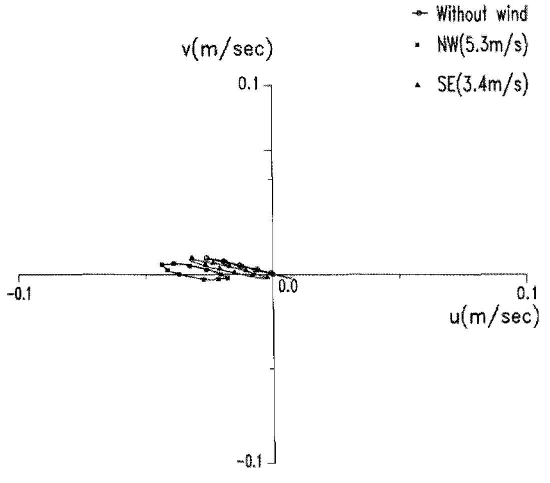
- Tidal current velocity vector is bigger than that of without seasonal wind.
- The vortex of tidal current is strongly rotating counterclockwise with the effect of seasonal wind.
- The vortex of tidal current is more strongly rotating counterclockwise in winter season than that of in summer season.

Therefore, tidal current sensitively flows due to wind stress on Jeju harbor inside.

When analysis of tidal current is performed in shallow water zone, it is necessary that the effect of wind stress should be considered to exactly reappear a real tidal current phenomenon.



(a) nodal point C



(b) nodal point E

Fig. 9 Comparison calculated current ellipse at nodal point C and E

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