Performance of the Long Code MMSE Detector With Pilot Channel in the Presence of Rayleigh Fading

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***Abstract*—**In this paper we propose a new structure of the long code MMSE receiver with pilot channel, which maintians excellent symbol detection capability even in the presence of Rayleigh fading. We explain analytically how the stability of the receiver weight vector, which is critical to the system performance, can be achieved by compensating the error signal as well as received signal vector distorted by fading channel. Computer simulation shows while maintaining better performance than the conventional matched filter receiver, the proposed long code MMSE receiver can extend its period up to $16 \times T_b$ in a fading environment.

***Index Terms*—**MMSE, Rayleigh fading, DS/CDMA, AWGN, MAI, Doppler shift frequency, LMS algorithm.

I. INTRODUCTION

Conventional matched filter demodulator adapting feedback power control scheme to solve near-far problem is widely accepted as a DS/CDMA receiver of commercial operation. However feedback channel from the receiver to the transmitter to control transmitted power level is nuisance overhead in a system structural as well as operational point of view. Several interference canceling schemes applicable to DS/CDMA receiver without power control has been proposed[1][2][3]. Among them MMSE criterion is considered possible candidate which can replace conventional matched filter receiver in future DS/CDMA system. A major advantage of MMSE scheme, compared to other previously proposed interference canceling schemes, is that explicit knowledge of interference parameters is not required, since filter parameter is to be simply adapted to achieve MMSE criterion. Throughout several papers already published[4][5], the performance of the MMSE receiver is proved better than that of conventional matched filter receiver when the received signal is distorted by AWGN as well as moderate multipath fading.

However MMSE demodulators, unlike conventional matched filter receiver, have to calculate its weights code per each symbol in MMSE manner. If the characteristic of the channel transfer function and MAI(Multiple Access Interference) is varying so fast due to multipath fading, the adaptive receiver weight vector is likely behind the object movement. For example, the signal received by vehicle running about 100km per hour in multipath environment, which can be modeled by typical Rayleigh distribution, shows the level of received signal fluctuates 20dB peak-to-peak at the Doppler shift frequency $f_d \approx 100$.[6] In the presence such kind of Rayleigh fading, the performance of the MMSE receiver may be getting worse than that of the conventional matched filter receiver because of its adaptive property of receiver weight vector. In worst case it may need another training sequence to initialize receiver weight vector and to make it back to normal operation.

IS-95, the CDMA standard for digital cellular telephony, specify long code as its spreading code. In the case of long code MMSE receiver, the convergence problem of the receiver weight vector due to multipath fading is getting even worse than that of code-on-pulse case, where spreading code repeats every symbol. In this paper we propose a new structure of the long code MMSE receiver equipped with pilot channel to compensate signal distortion due to fading. We also show with analytical and experimental measure how these compensation affects convergence and stability of the receiver weight vector and eventually enhance BER performance of the MMSE receiver compared to conventional matched filter receiver.

II. SYSTEM MODEL

The received signal without fading can be represented by the sum of K simultaneous CDMA transmission plus additive white Gaussian noise. The received signal due to the *j*th specific user can be expressed as

$$r_j(t) = \sum_{i=-\infty}^{\infty} \sqrt{2P_j}\, b_{i,j} s_j\left(t - iT - v_j\right)\cos\left(\omega_c t + \theta_j\right) \quad 1 \le j \le K \qquad (1)$$

where T is the symbol interval, $b_{i,j} \in \{1,-1\}$ is the *i*th symbol of the *j*th user, $P_j, v_j, \theta_j, \omega_c$ and $s_j(t)$ are power, delay, phase, carrier frequency and spreading signal respectively. Spreading waveform is given by

$$s_j(t) = \sum_{k=0}^{N-1} a_j[k]\Pi\left(t - kT_c\right) \qquad (2)$$

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where $a_j[k] \in \{1,-1\}$ is jth user's kth spreading code, $\Pi(t)$ is rectangular chip waveform which has unit energy and duration T_c, N is processing gain and $T_c = T/N$ is chip duration.
The received signal is

$$r(t) = \sum_{j=1}^{K} r_j(t) + n(t) \tag{3}$$

where $n(t)$ is white Gaussian noise with power spectral density $N_0/2$. For simplicity we assume $\nu_j = \theta_j = 0$. Assuming the receiver is synchronized to the transmission, the kth sample at the output of the chip matched filter is

$$r[k] = \int_{kT_c}^{(k+1)T_c} r(t)\Pi(t)\cos\omega_c t\, dt \tag{4}$$

If the order of transversal filter of MMSE detector is the same as the process gain N, then the vector of received sample is $\mathrm{r}^T = (r[0], \ldots, r[N-1])$.

In the presence of Rayleigh fading, the received sample of chip matched filter output, expressed by equation (4), should be modified. A commonly used multipath model is two ray model with independent Rayleigh distribution and the impulse response of the model is expressed as

$$h(t) = \alpha_1 \exp(j\phi_1)\delta(t) + \alpha_2 \exp(j\phi_2)\delta(t-\tau) \tag{5}$$

where α_1 and α_2 are independent and Rayleigh distributed, ϕ_1 and ϕ_2 are independent and uniformly distributed over $[0, 2\pi]$, and τ is the time delay between the two ray. By setting α_2 equal to zero, the special case of a flat Rayleigh fading channel is obtained as

$$h'(t) = \alpha_1 \exp(j\phi_1)\delta(t) \tag{6}$$

In the Rayleigh fading environment kth sample at the output of the chip matched filter will be

$$r_f[k] = \int_{kT_c}^{(k+1)T_c} \left\{ \sum_{j=1}^{K} \left(r_j(t) * h_j(t)\right) + n(t) \right\} \Pi(t)\cos\omega_c t\, dt \tag{7}$$

where $h_f(t)$ is Rayleigh fading transfer function for jth signal.

III. MMSE RECEIVER SYSTEM PERFORMANCE WITH-WITHOUT FADING

Since the functional purpose of MMSE receiver is detecting transmitted symbol $b_j \in \{1,-1\}$ from the received vector r, it is necessary to express vector r with b_j to check the system performance analytically.

$$\mathrm{r} = \sum_{j=1}^{K} b_j \mathrm{v}_j + \mathrm{n} \tag{8}$$

If the order of transversal filter is the same as the process gain N then the sampling period of the filter is the same as the chip period. Without fading, vector v_j is the sequence achieved by sampling jth spreading signal $s_j(t)$ at T_c time interval. We assume b_1, v_1 are desired symbol and desired signal vector respectively, b_j, v_j, $2 \le j \le K$ are interference symbol and vectors. It is also assumed that the transmitted symbols are independent and noise vector n is Gaussian with zero mean.

Applying sign function to the inner product of weight vector c and received signal vector r, received symbol estimated will be

$$\hat{b}_1 = \mathrm{sgn}(\mathrm{c}^T\mathrm{r}) \tag{9}$$

where T means transpose of vector.
Mean squared error of the estimated symbol to the transmitted symbol is

$$MSE = E(\mathrm{c}^T\mathrm{r} - b_1)^2 = (\mathrm{c}^T\mathrm{v}_1 - 1)^2 + \sum_{j=2}^{K}(\mathrm{c}^T\mathrm{v}_j)^2 + \mathrm{c}^T\Gamma\mathrm{c} \tag{10}$$

where Γ is covariance matrix of noise vector n. In the reference [7], Output SNIR and BER as a function of interference symbol $\mathrm{b}_I = (b2, \ldots, b_K)^T$ are give as follows

$$SNIR = \frac{(\mathrm{c}^T\mathrm{v}_1)^2}{\mathrm{c}^T\Gamma\mathrm{c} + \sum_{j=2}^{K}(\mathrm{c}^T\mathrm{v}_j)^2} \tag{11}$$

$$P_e(\mathrm{b}_I) = Q\left(\frac{\mathrm{c}^T\mathrm{v}_1 + \sum_{j=2}^{K} b_j(\mathrm{c}^T\mathrm{v}_j)}{(\mathrm{c}^T\Gamma\mathrm{c})^{1/2}} \right) \tag{12}$$

In the presence of fading, transient MSE_f, $SNIR_f$, $P_e(\mathrm{b}_I)_f$ have the same form as the non-fading case but the vector v_j is replaced by vector p_j

$$MSE_f = E(\mathbf{c}^T\mathbf{r} - b_1)^2 = (\mathbf{c}^T\mathbf{p}_1 - 1)^2 + \sum_{j=2}^{K}(\mathbf{c}^T\mathbf{p}_j)^2 + \mathbf{c}^T\Gamma\mathbf{c} \tag{13}$$

$$SNIR_f = \frac{(\mathrm{c}^T\mathrm{p}_1)^2}{\mathrm{c}^T\Gamma\mathrm{c} + \sum_{j=2}^{K}(\mathrm{c}^T\mathrm{p}_j)^2} \tag{14}$$

$$P_e(\mathbf{b}_l)_f = Q\left(\frac{\mathbf{c}^T\mathbf{p}_1 + \sum_{j=2}^{K} b_j(\mathbf{c}^T\mathbf{p}_j)}{(\mathbf{c}^T\Gamma\mathbf{c})^{1/2}}\right) \tag{15}$$

Where

$$\mathbf{p}_j = (p_j(1), \ldots, p_j(K)) \tag{16}$$

$$p_j(k) = \int_{kT_c}^{(k+1)T_c} \left\{\frac{(r_j(t) * h_j(t))}{b_j}\right\} \Pi(t)\cos\omega_c t\, dt \tag{17}$$

IV. LONG CODE MMSE RECEIVER WITH PILOT CHANNEL

IS-95 specify a long code rather than code-on-pulse for its spreading sequence. For the system of long code of periodicity $m \times T_b$ (T_b is period of a symbol), spreading code does not repeat every symbol, instead it repeats every *m*th symbol. If the characteristic of either channel transfer function or MAI vary fast during a period of long code, $m \times T_b$, the receiver weight vector calculated for previous symbol may be too far apart, in vector space, to be updated for the weight vector of the symbol just arrived. That is one of the main reason most of the MMSE research works are based on code-on-pulse case. Among the very few papers mentioning about long code MMSE receiver[8],[9], [9] handles the problem seriously by suggesting the long code MMSE receiver structure with coded word which can operate in flat fading channel.

We suggest here a new MMSE receiver structure with pilot channel which can overcome the effect of fast time variance of channel and MAI. To investigate the rationale of improving system performance of long code MMSE receiver utilizing pilot channel, it is necessary to take a closer look of weight update procedure called LMS algorithm. LMS adaptive algorithm applied in MMSE receiver system for *j*th user is

$$\mathbf{c}_{i+1} = \mathbf{c}_i - \mu\left\{\mathbf{c}_i^T\left(\sum_{j=1}^{K}\mathbf{p}_j\right) - b\right\}\left(\sum_{j=1}^{K}\mathbf{p}_j\right), j = 1,\ldots,K \tag{18}$$

In equation (18), terms inside brace, called error signal, can be rewritten as

$$e = \mathbf{c}_i^T\left(\sum_{j=1}^{K}\mathbf{p}_j\right) - b = (\mathbf{c}_i^T\mathbf{p}_1 - b) + \mathbf{c}_i^T\left(\sum_{j=2}^{K}\mathbf{p}_j\right) = (c_i^T p_1 - b) + I_m \tag{19}$$

where I_m is multiple access interference. Using equation (19), instantaneous gradient ∇_{ins} can be gotten as

$$\nabla_{ins} = \{(\mathbf{c}_i^T\mathbf{p}_i - b) + I_m\}\left(\mathbf{p}_1 + \sum_{j=2}^{K}\mathbf{p}_j\right) \tag{20}$$

It is relatively simple to estimated channel transfer function using pilot channel in CDMA.[10] The discrete version of estimated channel transfer function $h'(t)$ is $h'(n) = \alpha_1 \exp(j\phi)\delta(n)$. From this estimated channel fading function, it is easy to get channel compensation function W for the desired signal.

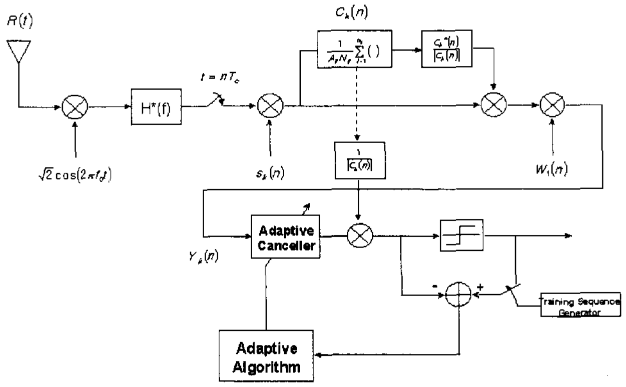

Fig. 1 Proposed receiver block diagram

$$W = \frac{h'(n)^*}{|h'(n)|^2} = \frac{1}{\alpha_1}\exp(-j\phi)\delta(n) \tag{21}$$

To compensate distortion of each estimated symbol due to multipath fading, W is multiplied to symbol output, then we can get fading compensated desired signal. Resultantly the error signal is as follows

$$\begin{aligned} e' &= W\,\mathbf{c}_i^T\left(\sum_{j=1}^{K}\mathbf{p}_j\right) - b = (W\,\mathbf{c}_i^T\mathbf{p}_1 - b) + W\,\mathbf{c}_i^T\left(\sum_{j=2}^{K}\mathbf{p}_j\right) \\ &= (\mathbf{c}_i^T\mathbf{v}_1 - b) + I'_m \end{aligned} \tag{22}$$

Due to the orthogonal property of $\mathbf{p}_j$ and high correlation property between $\mathbf{c}_i$ and $\mathbf{p}_1$, desired signal enhancing effect in equation (22) is much greater than unwanted increase of interference, expressed $I'_m - I_m$. Thus by compensating fading effect on the received symbol signal, we can avoid undesired fluctuation of error signal that is critical in updating MMSE receiver weight. Since the error signal fluctuation is directly reflected to the consequent weight value, it is very important to stabilize error signal by compensation.

Not only error signal but also received signal vector itself is critical to determine instantaneous gradient as can be seen in equation (20). So we need to compensate received signal vector itself. However it is impossible to make gain compensation for the desired signal vector only, because receive signal vector is sum of all K user signals which consist of the desired signal and MAI. When $1/\alpha$ is multiplied to received signal vector to compensate gain distortion of the desired signal, compensation is done not only for the desired signal but also for co-channel interference signal, so that there is no merit in terms of signal to interference ratio. But phase compensation is different. By multiplying phase compensation function of $W_p = h(n)^* / |h(n)| = \exp(-j\phi)$ to the received signal, we can get at least phase compensated desired signal among the mixture of MAI and this phase compensated desired signal vector partly contribute to keep the weight vector stable in the presence of Rayleigh fading.

V. LONG CODE MMSE RECEIVER STRUCTURE AND ITS PERFORMANCE

Block diagram of the Long code MMSE receiver structure with pilot channel is given in figure 1.
Walsh code is used to discriminate the pilot channel from the traffic channel and using this known Walsh code we can estimate the channel transfer function in the presence of Rayleigh fading. Even though the spreading code is extended over *m*th symbol, by storing and retrieving the weight for the specific sub-period to and from memory, same transversal filter structure of code-on-pulse can be used as shown in figure 2.

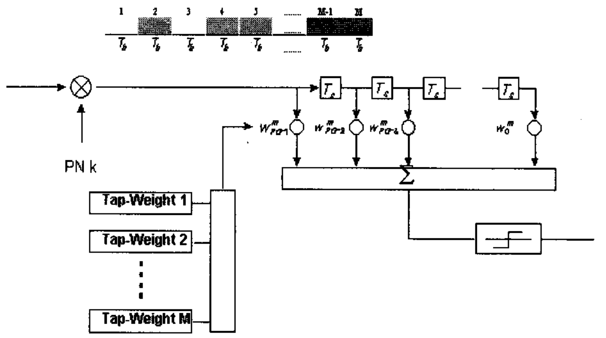

Fig. 2 MMSE Detector with multiple tap weight

To check the performance of long code MMSE receiver, SPW coded by ALTA is used. Computer simulation parameters are as follows; the number of co-users are 10, the range of the Doppler shift frequency are from 70 Hz to 130 Hz, modulation type is BPSK and E_b/N_o is assumed 20dB. Particularly to check the maximum period of long code allowable in the presence of Rayleigh fading, we perform computer simulation separately using the long code of periods 4, 8 and $16 \times T_b$.

The transfer function of Rayleigh fading channel used in computer simulation is estimated like figure 3. The computer simulation results of signal quality and scattering of symbol estimated for the conventional matched filter and MMSE receiver using $16 \times T_b$ long code with pilot channel are shown in figure 4. In this figure we can see the estimated signal quality of MMSE receiver is far better than that of conventional matched filter receiver.

The figure 5, 6 shows how the compensation in terms of received signal vector and error signal affects the stability of receiver weight vector and eventually the BER of the receiver. Closely looking the BER performance of figure 6, we can see that as long as the period of long code is not longer than $16 \times T_b$, the MMSE long code receiver has better performance than conventional matched filter receiver.

VI. CONCLUSION

Even though IS-95, the CDMA standard for digital cellular telephony, specify long code as its spreading code, most of research work on the MMSE receiver are based on the code-on-pulse type. One of the main reason, the MMSE research works are lingering around code-on-pulse is that using long code it is hard to guarantee the performance of MMSE receiver compared to conventional matched filter receiver.

MMSE detector, calculating receiver weight vector minimizing unnecessary power contributed by MAI, can solved near-far problem of CDMA without power control. However it needs training sequence to guide weight vector to the vicinity of optimum value before it operates by decision feedback LMS algorithm. For engineers who consider seriously MMSE receiver may be one of possible candidates replacing the conventional matched filter receiver, training sequence at its initial stage as well as the weight convergence problem in the presence of Rayleigh fading is really bothersome. To solve the problems research on blind adaptive multi-user detection scheme using MOE algorithm[11] or analogy

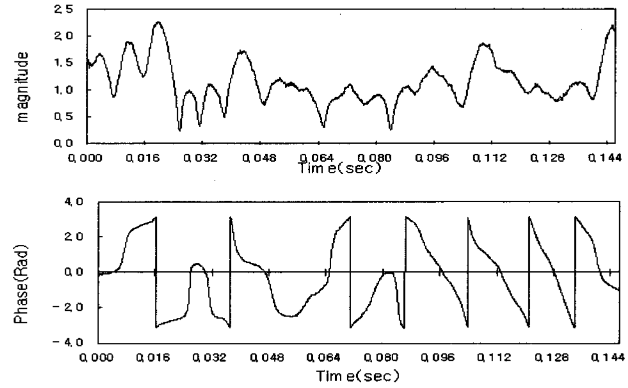

Fig. 3 The estimated transfer function of Rayleigh fading channel environment (Magnitude and Phase)

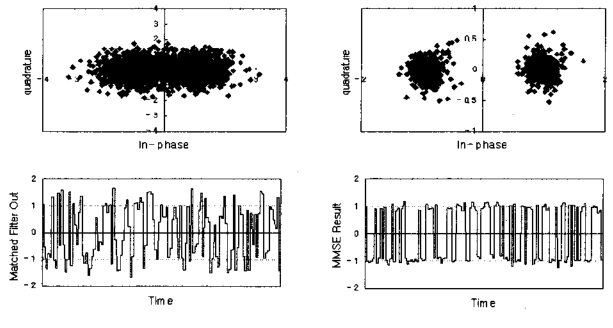

Fig. 4 Matched filter output and Adaptive MMSE Detector output (scattering plot and signal plot)

of adaptive array is exploited [12].

Nevertheless we address the weight convergence problem in the long code MMSE receiver with pilot channel in an effort to extend the period of spreading sequence. Seeing the results of our research work, adding of pilot channel is critical to extend period of spreading code.

By referencing pilot channel, we can estimate the time varying channel transfer function of desired symbol, further we can compensate distortion brought by fading channel. The receiver weight vector based on LMS algorithm is quite dependent on received signal vector as well as error signal, so that weight vector with compensation is likely more stable than that without compensation. The interesting question is how long we can extend the period of spreading code? By the computer simulation ,assuming specific parameters, it is shown that up to $16 \times T_b$ long code MMSE receiver can have better performance than that of conventional matched filter receiver.

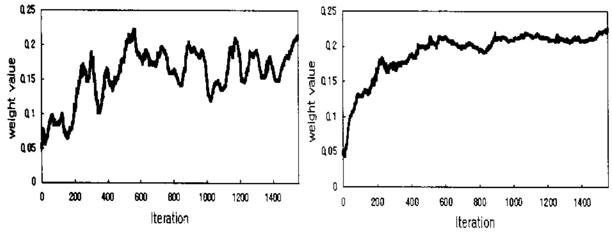

Fig. 5 Tap weight convergence (left : with only phase compensation, right : phase & gain compensation)

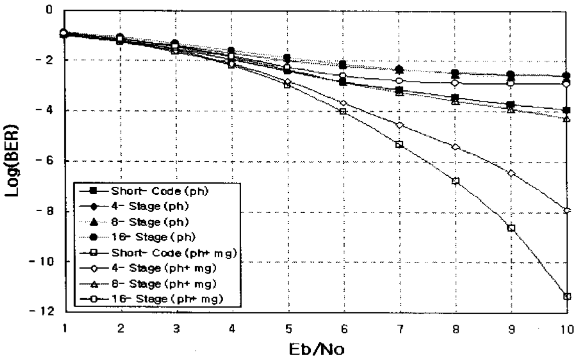

Fig. 6 Bit error rate (ph : phase compensation, ph+mg : phase & magnitude compensation)

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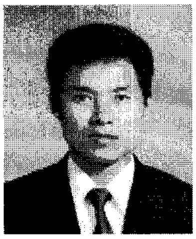

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