Effects of Impulsive Noise on the Performance of Uniform Distributed Multi-hop Wireless Sensor Networks

Jae-Sung Roh, *Member, KIMICS*

***Abstract*—**Wireless sensor networks represent a new and exciting communication paradigm which could have multiple applications in future wireless communication. Therefore, performance analysis of such a wireless sensor network paradigm is needed in complex wireless channel. Wireless networks could be an important means of providing ubiquitous communication in the future. In this paper, the BER performance of uniform distributed wireless sensor networks is evaluated in non-Gaussian noise channel. Using an analytical approach, the impact of Av. BER performance relating the coherent BPSK system at the end of a multi-hop route versus the spatial density of sensor nodes and impulsive noise parameters A and Γ is evaluated.

***Index Terms*—**Impulsive Noise, Uniform Node Distribution, Multi-hop radio, Wireless Sensor Network.

I. INTRODUCTION

Recently, a new universal radio interface has been developed enabling electronic devices to communicate wirelessly via short-range low power Ad hoc connections. Wireless sensor networks have limited resources and tight energy budgets. These constraints make in network processing a prerequisite for scalable and long lived applications. The technology enables the design of low-power, small-sized, low-cost wireless products that can be embedded in existing portable devices. Eventually, these embedded wireless products will lead toward ubiquitous connectivity and truly connect everything to everything. The radio technology will allow this connectivity to occur without any explicit user interacttion [1]-[5].

Wireless sensor networks represent a new and exciting communication paradigm which could have multiple applications in future wireless communication systems. Fundamental performance limits of such a communication paradigm need to be studied. The concept of average bit error rate has been introduced to quantify the achievable transmission performance of information in the network. Multi-hop wireless sensor networks are attracting the attention of many researchers, for their potential to provide ubiquitous connectivity. In particular, in future wireless sensor networks, sensor nodes are likely to be mobile. Maintaining multi-hop routes is a challenging task, especially in the case of mobile sensor nodes. Recently, a novel communication-theoretic framework for wireless networks has been proposed [6]-[9]. In particular, the impact of the physical layer characteristics on the network performance, jointly with the used medium access control (MAC) protocol and the specific routing strategy, has been evaluated. While in [7] and [8] a network communication scenario with static nodes in AWGN channel placed at the vertices of a uniform square grid is considered. And in this paper, theoretic framework is extend in order to incorporate the effects of impulsive noise on the sensor node performance in wireless channels. Rather than relying heavily on computer simulations, semi-analytical approach is propose where the impact of wireless channel conditions is evaluated from a communication-theoretic perspective. Also, an ideal AWGN communication channel scenario and a realistic AWGN plus impulsive noise communication channel scenario is consider.

In this paper, intermediate retransmission mechanism is not used and there are N mobile sensor nodes in the wireless network and that they are confined to a fixed area A. Each sensor node transmits information in terms of messages. In particular, the messages have fixed length M [b/msg]) and the transmission data-rate at each sensor node, denoted as R_b [b/s], is fixed. Neglecting the propagation time, the duration of a message transmission between two communicating sensor nodes is M/R_b. In order to derive an analytical model which captures the impact of sensor node wireless channel from a communication-theoretic perspective, first recall some basic results from the framework proposed in [7] and [8]. It is assumed that N nodes are uniformly placed at the vertices of a square grid in a circular area: each node has therefore four nearest neighbors. Any multi-hop route in the network is given by a sequence of links between nearest neighbors.

Apart from the fading and interference detriments, most practical communication systems can be corrupted by impulsive noise, which is generated by natural or man-made electromagnetic sources, such as automotive ignition, industrial equipment, and consumer products [10]-[13]. In evaluating the performance of various communication systems, the usual Gaussian channel noise assumption seems to be inadequate. This is mostly due to the presence of an additive combination of a zero-mean Gaussian random noise component and an impulsive noise random component which is considered

---

Manuscript received October 22, 2007.

Jae-Sung Roh is with the Department of Information & Communication, Seoil College, Seoul, 131-702, Korea (Tel: +82-2-490-7206, Fax: +82-2-490-7407, Email: jsroh@seoil.ac.kr)

to be a train of Poisson arriving spikes of low probability and very high amplitude. This impulsive noise can be detected at high frequency band. Therefore, to provide pragmatic information for the wireless ad hoc network designer, it is of great interest to study the system performance in impulsive noise channels. The present study is motivated by the performance evaluation of the wireless sensor network in presence of impulsive noise and AWGN channels.

II. PERFORMANCE OF WIRELESS SENSOR NETWORK

A. Model of Uniform Distributed Sensor Node

In this paper, an average uniform model for the spatial distribution of the sensor nodes is considered. In particular, a circular network area A_S is assumed, where N sensor nodes are distributed at the vertices of a square grid. Indicating by ρ_S the sensor node spatial density can be written as

$$\rho_S = \frac{N}{A_S} \tag{1}$$

Upon the assumption that the number of hops is uniformly distributed between 1 and the maximum number over a network diameter, the average number of hops is $\overline{n_h}\left(=\sqrt{N/\pi}\right)$. Hence, an expression for average BER is as follow

$$\overline{P_e} = 1 - \left(1 - \overline{P_b}\right)^{\overline{n_h}} \tag{2}$$

Indicating by $\overline{P_b}$ the BER at the end of a single link, assuming that there is regeneration at each intermediate sensor node, and the uncorrected errors made in successive links accumulate. Expression (2) shows the dependence of the BER, over an average multi-hop route in wireless sensor network, on the number of sensor nodes N and the link BER $\overline{P_b}$.

Assume that N sensor nodes are placed at the vertices of a square grid inside a circular area A. Considering a global circular network area A, in a realistic wireless communication network scenario, sensor nodes could organize themselves in randomly shaped clusters.

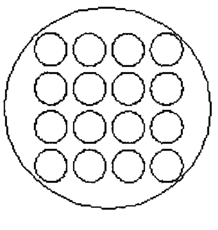


Fig. 1 Uniform distribution wireless sensor network.

In order to derive an analytical model, a geometric regularity is considered in the cluster formation. In particular, assume that all the clusters are circular and have the same dimension and the centers of the clusters are at the vertices of a square grid. This scenario is depicted in Figure 1, and the corresponding sensor node distribution will he referred to as uniformly clustered sensor node distribution. Further assume that inside each cluster the sensor nodes are distributed over a regular grid-in other words, each cluster is a small-scale version of an uniform wireless sensor network.

In particular, the performance of link BER depends on the signal-to-noise power ratio at the ending sensor node of the link. In this paper, assume that the transmitted signal is simply affected by free-space loss. Received signal power at the end of a minimum length hop can be expressed as follows.

$$P_R = \frac{G_T G_R \lambda_C^2 \rho_S P_T}{(4\pi)^2 f_L} \tag{3}$$

where P_T is the transmitted power, G_T and G_R are the transmitter and receiver antenna gains, λ_C is the wavelength corresponding to the carrier frequency, and F_L is a loss factor.

Therefore, SNR at the end of a minimum length link in AWGN plus impulsive noise channel is as follows.

$$SNR_T = \frac{P_R}{NP_{class-A}} \tag{4}$$

where $NP_{class-A}$ is class-A impulsive noise which is include the thermal noise $NP_{thermal}$. Assuming that the transmission bandwidth is B and recalling the concept of noise figure F of a receiver, the thermal noise can be expressed as

$$NP_{thermal} = FkT_oB \tag{5}$$

where k is the Boltzmann's constant, T_o is the room temperature, and for BPSK the 3-dB bandwidth is $B \approx R_b$. In the case of binary phase shift keying (BPSK) transmission, the average BER can be written as

$$\overline{P_e} = 1 - \left[1 - Q\left(\sqrt{2SNR_T}\right)\right]^{\overline{n_h}} \tag{6}$$

Equation (6) allows to explore the relationship between the BER at the end of an average communication route and very important parameters in wireless sensor network, such as the sensor node spatial density, the transmitted power, the data-rate, and the number of nodes in the uniform sensor node distribution network model. In the following, This paper assume that $G_T = G_R = f_L = 1$. The gains G_T and G_R are related to the effective areas of the antennas used at the transmitter and at the receiver, respectively. Since assume $f_L = 1$, i.e., there are no system losses not related to propagation, also there is no antenna gain in order to account for possible neglected system losses. Also consider $f_C = 2.4$ GHz, which corresponds to the carrier frequency considered in the IEEE 802.11 standard, $R_b = 2$ Mbps, $F = 9$ dB, N=100 and the transmitted power of wireless sensor network is $P_T = 15m$ Watt.

B. Non-Gaussian Class-A Impulsive Noise Model

Here, the noise process $\{n(t)\}$ to be statistically independent for all path and adopt the Gaussian-Poisson model to describe the impulsive class-A noise nature of $\{n(t)\}$, i.e.,

$$n(t) = n_G(t) + n_I(t) \tag{7}$$

where $n_G(t)$ is the Gaussian noise component and $n_I(t)$ the impulsive noise part. $n_G(t)$ and $n_I(t)$ are assumed statistically independent. Using eq. (7), $\{z_n\}$ are the noise components at the output of the post-detection filter, and have

$$z_n = \int_{-\infty}^{+\infty} h(nT_b - t)n(t)dt \tag{8}$$

where $h(t)$ is a impulse response of channel. It can be shown that $\{z_n\}$ are i.i.d and the marginal pdf of z_n with unit variance can be expressed in the form of the canonical Middleton's class-A noise model, i.e.,

$$f_z(x, y) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m! 2\pi \sigma_m^2} e^{-(x^2+y^2)/2\sigma_m^2} \tag{9}$$

where x and y denote the real and imaginary parts of z, respectively. σ_m^2 is equal to $(m/A + \Gamma/(1+\Gamma)$ so that the total variance of z_n is 1, A is the impulsive index, $\Gamma = \sigma_G^2/\Omega_{2A}$ is the mean power ratio of the Gaussian noise component to the impulsive noise component, respectively. Figure 2 shows the generation of impulsive class-A noise. As the A and Γ are increase, the characteristic of impulsive noise is more dominant than that of the small values of A and Γ.

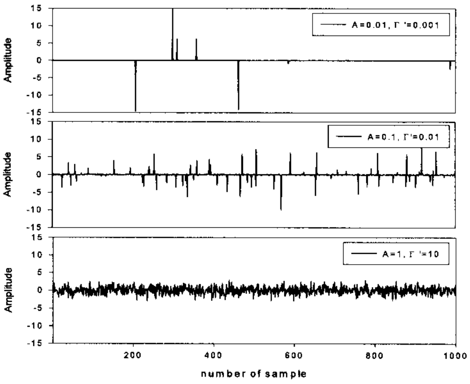


Fig. 2 Generation of non-Gaussian class-A impulsive noise.

In the case of BPSK signal transmission, the average BER according to Class-A impulsive noise parameters in the uniform distributed sensor node in wireless sensor network can be written as

$$\overline{P_e} = 1 - \left[1 - e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} Q\left(\sqrt{2SNR_T}\right)\right]^{\overline{n_h}} \tag{10}$$

From the Equation (10), the BER does not strongly depend on the number of nodes or the number of hops.

III. PERFORMANCE EVALUATION AND NUMERICAL RESULTS

The performance of a uniform distributed multi-hop wireless sensor network is evaluated in several situations. In all cases, system performance is simply considered in terms of BER versus sensor node spatial density. This paper only point out that in all the figures considered in the following, the sensor node spatial density in the horizontal axis is the sensor node spatial density for the curves relative to the uniform distribution. In all cases, the transmitted power in the case of a perfectly uniform sensor node distribution is set equal value, and the final BER as a function of the sensor node spatial density evaluate. The Av. BER is numerically evaluated using equation (10) and the obtained results are shown in Figures. Figure 3 shows the Av. BER performance of uniform distributed multi-hop wireless sensor network in class-A impulsive noise channel. For each value of spatial density of sensor node and impulsive noise parameters ($\Gamma = 0.001$ for several values of A), the Av. BER performance is shown. It can be seen from figures that decreasing A improves the performance. Therefore it can be concluded that the more structured the noise, the less impact on BER. Figure 4 shows the Av. BER performance versus spatial density of sensor node and impulsive noise parameters $\Gamma = 1$ for several values of A. $\Gamma = 1$ mean the power ratio of Gaussian to the impulsive noise is 0 dB. As the A becomes smaller, the noise impulsiveness becomes stronger, thus causing larger performance degradation. Figure 5 shows the Av. BER performance of multi-hop wireless sensor network in class-A impulsive noise channel for $A=1$ for several values of Γ. The result shows that when A is 1 and Γ becomes larger, the Av. BER performance approaches closely to the case of pure Gaussian noise. This is because when $A=1$, an impulsive noise occurs continuously, thus the impulsiveness becomes weaker as Γ becomes larger. Figure 6 shows the Av. BER versus spatial density of sensor node and Γ ($A=0.001$). Impulsive index $A=0.001$ mean that impulsive occurs for 0.1 percent of the time, and Γ varies from -10 dB to 20 dB. It can be seen from figure 6 that increasing the mean power ratio of the Gaussian noise component to the impulsive noise component, Γ improves the performance of uniform distributed multi-hop wireless sensor network in class-A impulsive noise channel.

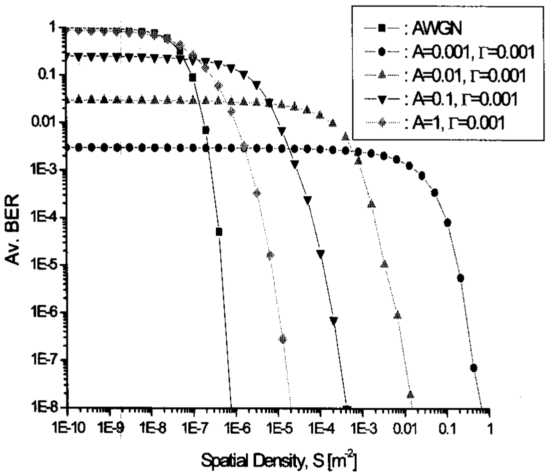


Fig. 3 Av. BER performance of uniform distributed multi-hop wireless sensor network in class-*A* impulsive noise channel ($\Gamma = 0.001$).

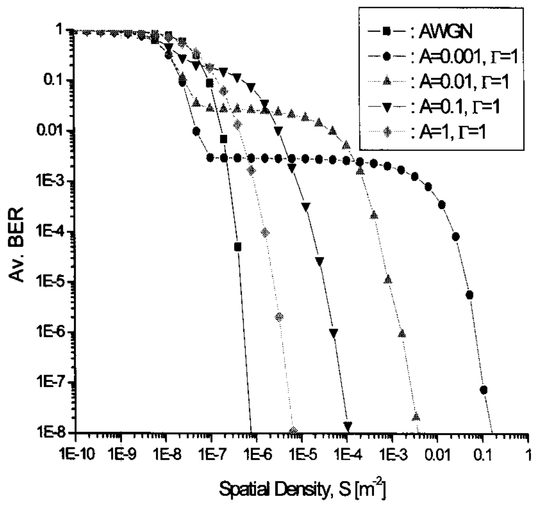


Fig. 4 Av. BERperformance of uniform distributed multi-hop wireless sensor network in class-*A* impulsive noise channel ($\Gamma = 1$).

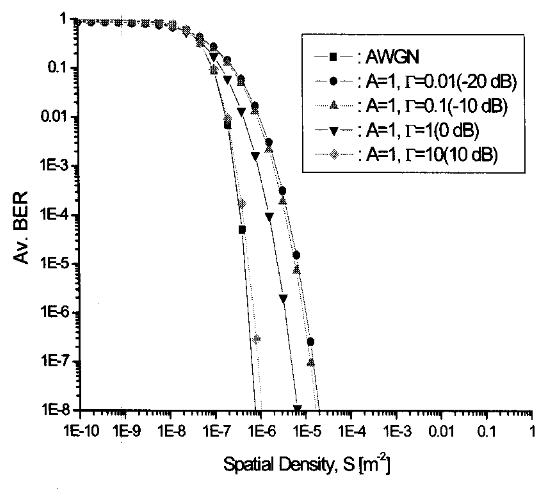


Fig. 5 Av. BER versus spatial density and Γ (*A*=*1*).

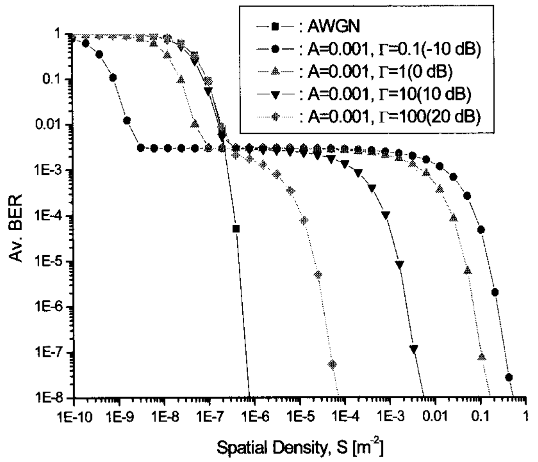


Fig. 6 Av. BER versus spatial density and Γ (*A*=0.001).

IV. CONCLUSIONS

When Wireless sensor networks represent a new communication paradigm and could be an important means of providing ubiquitous communication in the future. Based on a communication-theoretic framework, the BER performance of uniform distributed multi-hop wireless sensor networks in non-Gaussian channel is evaluated. In this paper, using the semi-analytical approach, the effect of Av. BER performance of multi-hop wireless sensor networks according to the spatial density of sensor nodes and impulsive noise parameters *A* and Γ is evaluated. Analytical expressions, relating the BER of BPSK coherent receiver at the end of a multi-hop route with non-Gaussian Class-*A* noise characteristics of the sensor nodes are derived. The analysis results can be applied to evaluating the performance of various wireless sensor communication systems.

ACKNOWLEDGMENT

This research was supported by Seoul R&BD Program.

REFERENCES

[1] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. ayirci, “A Survey on Sensor Networks,” *IEEE Communications Magazine*, 40(8):102–114, August 2002.

[2] G.J. Pottie and W.J. Kaiser, “Wireless Integrated Network Sensors,” *Communications of the ACM*, 43(5):51–58, May 2000.

[3] J. Proakis, *Digital Communications*, New York, NY: McGraw-Hill, 4‘h edition, 2001.

[4] T. S. Rappaport, *Wireless Communications: Principles and Practice*, Prentice-Hall PTR, 1999.

[5] O.K. Tonguz and G. Ferrari, *Ad Hoc Wireless Networks: A Communication-Theoretic,* Wiley, 2006.

[6] G. Fermi and O. K. Tonguz, "Performance of Circuit-Switched Ad Hoc Wireless Networks with Aloha and PR-CSMA MAC Protocols," *Proc. IEEE Global Telecomm. Conf.* (GLOBECOM), Dec. 2003.

[7] O.K. Tonguz and G. Ferrari, "A Communication-Theoretic Approach to Ad Hoc Wireless Networking," *Proc. 2006 Int'l Workshop Wireless Ad-Hoc Networks* (IWWAN '06), June 2006.

[8] G. Ferrari, S. Panichpapiboon, N. Wisitpongphan, R. Chokshi, and O.K. Tonguz, "Impact of Mobility on the BER Performance of Multi-Hop Ad Hoc Wireless Networks," *Proc. IEEE Global Telecomm. Conf.* (GLOBECOM), pp. 882-886, Nov. 2004.

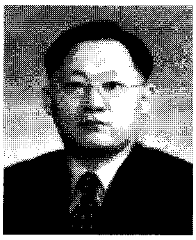
[9] G. Ferrari and O.K. Tonguz, "Performance of Ad Hoc Wireless Networks with Aloha and PR-CSMA MAC Protocols," *Proc. IEEE Global Telecomm. Conf.* (GLOBECOM), pp. 2824-2829, Dec. 2003.

[10] D. Middleton, "Man Made Noise in Urban Environment and Transportation Systems: Models and Measurement," *IEEE Trans. Commun.*, vol. COM-21, pp. 1232–1241, Oct. 1973.

[11] K. L. Blackard, T. S. Rappaport, and C.W. Bostian, "Measurements and Models of Radio Frequency Impulsive Noise for Indoor Communication," *IEEE J. Select. Areas Commun.*, vol. 11, Sept. 1993.

[12] A. D. Spaulding and D. Middleton, "Optimum Reception in an Impulsive Interference Environment—Part I: Coherent Detection," *IEEE Trans. Commun.*, vol. COM-25, pp. 910–923, Sept. 1977.

[13] A. D. Spaulding and D. Middleton, "Optimum reception in an impulsive interference environment—Part II: Incoherent detection," *IEEE Trans. Commun.*, vol. COM-25, pp. 924–934, Sept. 1977.



Jae-Sung Roh

Regular member, received the BS, MS, and PhD degrees in dept. of Information and Communication Engineering from Korea Aerospace University, Korea in 1990, 1992, and 2000, respectiv-ely. Since 2000, he has been a professor in the department of Information and Communication Engineering at Seoil College, Korea. jsroh@seoil.ac.kr.