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EFFECT OF COMPLIANCE ON NEWMARK-TYPE RIGID BLOCK DEFORMATION ANALYSIS

Newmark-방식 강체블럭 변위해석에 대한 유연도의 영향

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Abstract

This study investigates the effect of spatial averaging and compliance taken account of in the analysis of earthquake-induced permanent deformation of slopes. At present, the rigid block analysis originally proposed by Newmark is widely used in the deformation analysis, mainly because of its computational efficiency. This type of approach, however, adopts the so-called decoupled approach, in which seismic response and deformation analyses are carried out separately. Original Newmark block analysis assumes the potential sliding mass to be noncompliant, and has been criticized to be potentially unconservative. This paper reviews the impact of the noncompliance assumption of the potential sliding mass in the Newmark-type analysis. The gross effects of earthquake shaking on the potential sliding mass are estimated by spatial averaging method and analyzed in frequency domain. The results indicate that there is a simple criterion that can be used to determine the level of compliance of the potential sliding mass.

요 지

지진으로 인해 사면에 발생한 영구적 변형 계산시 고려된 공간평균과 유연도의 영향을 조사하였다. 현재 변위계산에는 Newmark이 제안한 강체블럭해석기법이 이 기법의 효율적인 계산능력으로 인해 광범하게 사용되고 있다. 그러나 이 해석기법은 지진응답해석과 변위해석을 별도로 수행하는 소위 분리해석법을 채택하고 있다. 당초의 Newmark 해석기법은 활동토사를 강성체로 가정했으며 이로 인하여 비보수적 결과가 도출될 수 있다는 비판을 받아왔다. 본 논문은 Newmark-형식의 해석에서 강성체 가정의 영향을 검토하였다. 활동토사에 작용하는 지진하중의 전체 효과를 공간평균기법을 사용하여 평가하였으며 그 결과를 주파수 영역에서 분석하였다. 해석결과로부터 활동토사의 유연도 수준을 결정하는 경우 사용할 수 있는 단순한 지표를 제시하였다.

Keywords : Compliance, Newmark-Type Analysis, Rigid Block Analysis, Seismic Slope Deformation

1. INTRODUCTION

In general, the post-earthquake serviceability of a slope depends more on deformations than on the index of stability (e.g., the factor of safety) given by pseudo-static method of analysis. Newmark (1965) and Seed and Goodman (1964) used the analogy of a block resting on an inclined plane, to propose a simple way for estimating a permanent displacement of the sliding mass due to earthquake shaking. Newmark's sliding block method was originally developed to give a quick estimate of the magnitude of the movements to be expected in a sliding wedge of rock or earth in a slope subjected to earthquake shaking.

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The analysis of the seismically induced slope deformations requires the evaluation of the dynamic site response and of the dynamically induced slope deformation. Typically, simplified decoupled procedures have been being used in practice for seismic slope deformation, mainly because of their computational efficiency. Ideally, the ground response and slope deformation analyses should be performed in so-called coupled mode such as dynamic FEM analyses. The coupled analyses are, however, generally time-consuming, and cannot be used routinely, even in current fast computing environment. In the decoupled approach, on the other hand, seismic response and deformation analyses are carried out separately. Hence, a number of investigations (e.g., Lin and Whitman 1983, Rathje 1997, Matasovic et al. 1998) have explored the effectiveness of the decoupled approach and other simplifying assumptions.

Original Newmark analysis assumed the potential sliding mass to be noncompliant and hence the dynamic site response analysis was not performed. The paper reviews the impact of the noncompliance assumption of the potential sliding mass in the Newmark-type rigid block analysis. The effect of spatial averaging in the decoupled analysis of earthquake-induced deformation of slope is investigated. Finally, the paper proposes a relatively simple criterion that can be used to determine the level of compliance.

2. SIMPLIFIED DECOUPLED PROCEDURES

Newmark (1965) used the analogy of a block resting on an inclined plane, to propose a convenient way for estimating a permanent displacement of the sliding mass caused by earthquake shaking. It is assumed that the whole sliding mass moves as a single rigid body with resistance mobilized along the sliding surface as shown in Figure 1 (noncompliance assumption). Seed and Goodman (1964) proposed a similar approach.

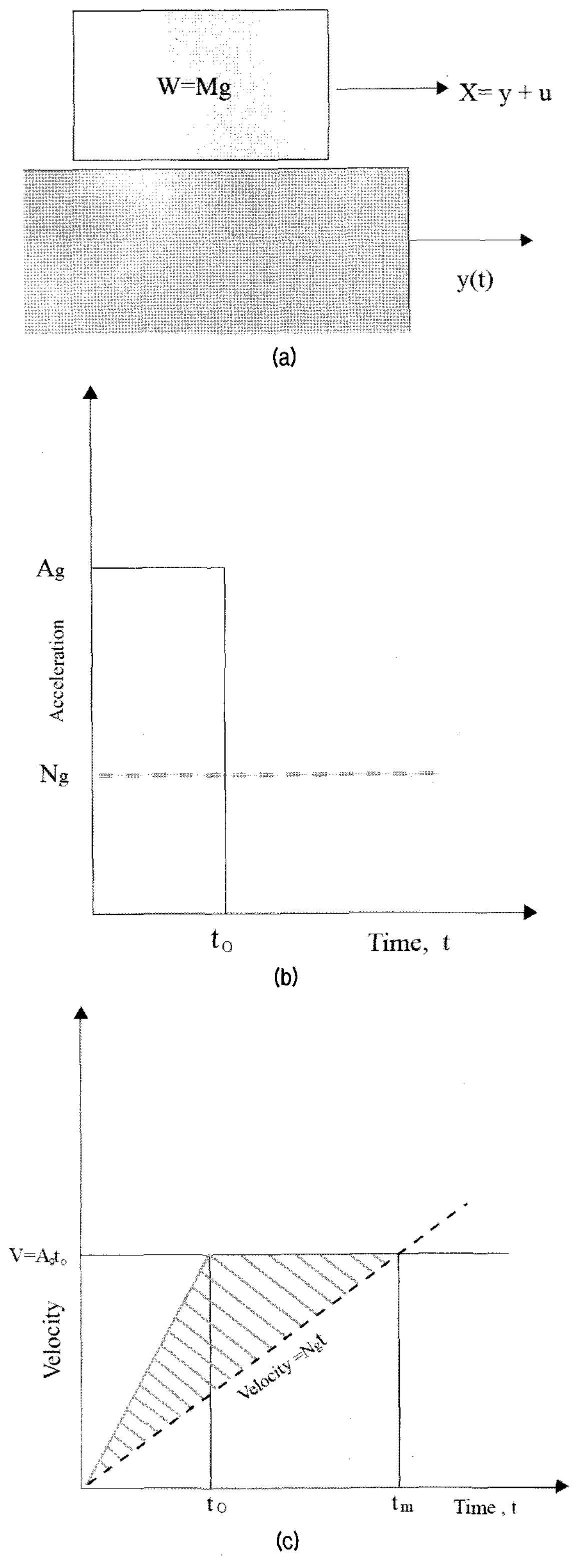

Figure 1. Estimation of Displacement During Single Slip Event: (a) Rigid Block on a Moving Support; (b) Rectangular Block Acceleration Pulse; (c) Velocity Response to Rectangular Block Acceleration (Redrawn from Newmark 1965)

2.1 Newmark-type Rigid Block Approaches

The computation of the permanent displacement of unsymmetrical resistance from an earthquake record is illustrated from the plot in Figure 2. The curve v_g represents the velocity of the ground beneath the sliding

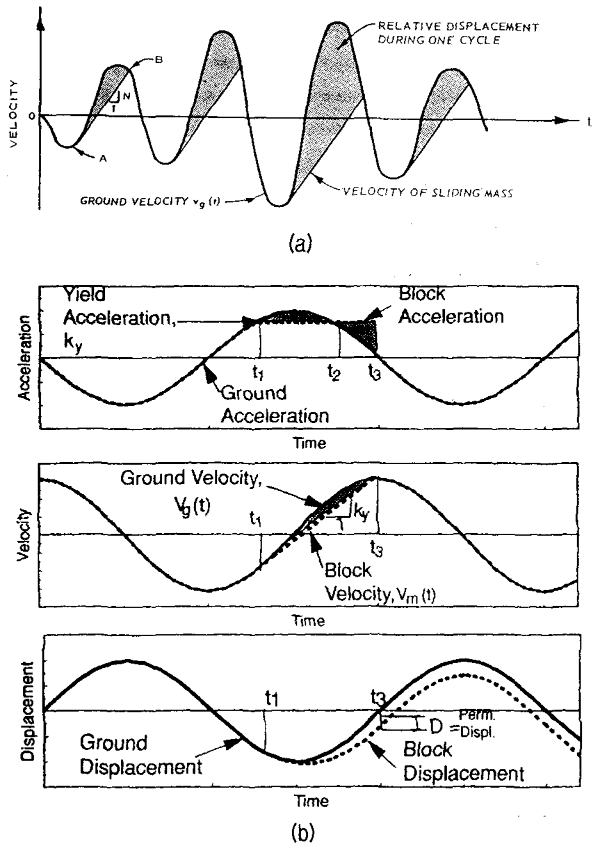

Figure 2. Computation of Permanent Displacement-Unsymmetrical Resistance: (a) from Franklin and Chang (1977); (b) from Bray et al. (1995)

mass, while the resistance along the sliding surface is represented by a slope, $dv/dt=N_g$. Whenever the ground acceleration (slope of the ground velocity) exceeds the yield acceleration Ng, the velocity curve of the sliding mass departs from that of the ground and follows a linear path, $v_b=Ng \cdot t$, until the two velocities again become equal, at which time relative movement ceases. The total permanent displacement is then given by the sum of the areas between the two velocity curves. The ground velocities, ground displacements, and permanent displacements can be computed numerically, using any *quadrature rules* appropriate.

2.2 Limitations of the Decoupled Approach

A number of investigations (e.g., Lin and Whitman 1983, Gazetas and Uddin 1994, Kramer and Smith 1997, Rathje 1997, Matasovic et al. 1998) have explored the effectiveness of the decoupled approach and other simplifying assumptions. It is known that the decoupled approach in general provides little conservative results compared to those of the coupled approach. Other factors such as the vertical component of acceleration were also explored and found to be insignificant in practical purposes (Yan 1996, Ling and Leshchinsky 1997, Matasovic et al. 1998). It was however reported (e.g., Bray et al. 1998) that the direct use of input acceleration time history without considering system compliance (i.e., without considering the averaging effect of ground motion acting on the potential sliding mass) can be significantly unconservative (i.e., produces significant small displacement).

2.3 Average Seismic Coefficients

Earthquake-induced inertia forces alternate in direction many times in soil masses within a slope. It is these pulsating forces, superimposed on the initial self-weights of soil masses, which disturb the stability of slopes. It has been a serious interest among earthquake engineers to estimate the gross (overall) effects of earthquake shaking on the potential sliding masses of slopes (e.g., Seed and Martin 1966, Chopra 1967). Overall effects of earthquake shaking on the potential sliding masses can be assessed either in terms of the peak value (pseudo static analysis) or time history (deformation-based analysis) of the inertia forces acting on a slope during the earthquake. Deformation -based analyses are, in general, more involved than the pseudo static analyses and require the estimate of time history of the inertia forces, which is used as input signal in time-domain deformation analysis.

The total lateral force acting on the potential sliding mass bounded by the slip surface at any particular instant (Figure 3) is given by (e.g., Seed and Martin, 1966):

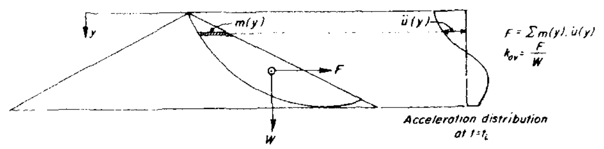

Figure 3. Concept of Average Seismic Coefficient (Seed and Martin 1966)

$$F = \sum m(y) \cdot \ddot{u}(y) \quad (1)$$

where $m(y)$ is the mass of a slice and $\ddot{u}(y)$ is the corresponding horizontal acceleration of the slice at a depth of y. The lateral force may alternatively be expressed by a product of an average seismic coefficient k_{ave} and the total weight W of the sliding mass as:

$$F = k_{ave} \cdot W \quad (2)$$

So, the average seismic coefficient at any instant is given by:

$$k_{ave} = \frac{F}{W} = \frac{1}{W} \sum m(y) \cdot \ddot{u}(y) \quad (3)$$

The time history of average seismic coefficient k_{ave} can thus be obtained by evaluating k_{ave} at other instants of time during the earthquake. One special case is when the potential sliding mass can be represented with reasonable accuracy by a parallelogram as shown in Figure 4. If the shear stresses are assumed to be constant at the base of the sliding mass, as verified to be the reasonable assumption in case of the deep slip surface by Elton et al. (1991) and Bray et al. (1996), the average seismic coefficient for the sliding parallelogram is given by:

$$k_{ave} = \frac{\tau_h \cdot b}{\gamma \cdot bh} = \frac{\tau_h}{\gamma \cdot h} \quad (4)$$

The time history of average seismic coefficient k_{ave} can thus be obtained by replacing the shear stress τ_h at any particular instant with a time history $\tau_h(t)$ of the shear stress, defined as:

$$k_{ave}(t) = \frac{\tau_h(t)}{\gamma \cdot h} \quad (5)$$

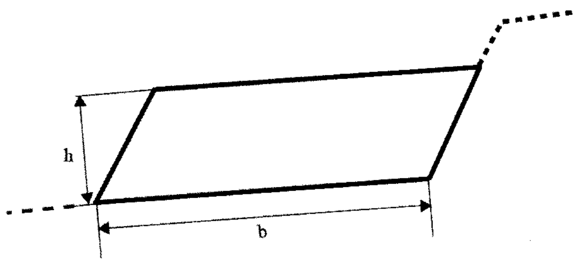

Figure 4. Parallelogram Idealization of Potential Sliding Mass

The shear stress time history at the base of potential sliding mass may be evaluated with 1-D wave analyses. It is noted that the magnitude of the average seismic coefficients for a potential sliding mass at any given depth is independent of the base width b for these special cases. Some engineers prefer to use a horizontal equivalent acceleration *(HEA)* time history, which is given as a product of the seismic coefficient and gravitational acceleration as:

$$HEA(t) = k_{ave}(t) \cdot g = \frac{\tau_h(t)}{\rho \cdot h} \quad (6)$$

in which g is the gravitational acceleration and ρ is the density of the sliding mass.

In cases of more complicated geometry, the average seismic coefficient k_{ave} for a potential sliding mass can be estimated by performing 2-D or 3-D dynamic response analyses of the slope cross section (e.g., Chopra 1966, Makdisi and Seed 1978, Idriss et al. 1973). The average seismic coefficient k_{ave} is, in general, computed as:

$$k_a = \frac{\sum_{i=1}^{n} w_i a_i}{\sum_{i=1}^{n} w_i} \quad (7)$$

3. SPATIAL AVERAGE of MOTION ACTING on HORIZONTAL GROUND

Figure 5 shows the case of a uniform elastic soil layer overlying a halfspace of rock. The vertical propagation of horizontal shear waves through visco-elastic medium (Kelvin-Voigt solid) is given as:

$$U(z,t) = (Ee^{ikz} + Fe^{-ikz})e^{i\omega t} = Ee^{i(kz+\omega t)} + Fe^{-i(kz-\omega t)} \quad (8)$$

where k and ω are the complex wave number and angular frequency respectively.

The acceleration can be obtained as the derivative of the equation as:

$$\ddot{U}(z,t) = -\omega^2 (Ee^{ikz} + Fe^{-ikz})e^{i\omega t} \quad (9)$$

where the complex wave number k is given as:

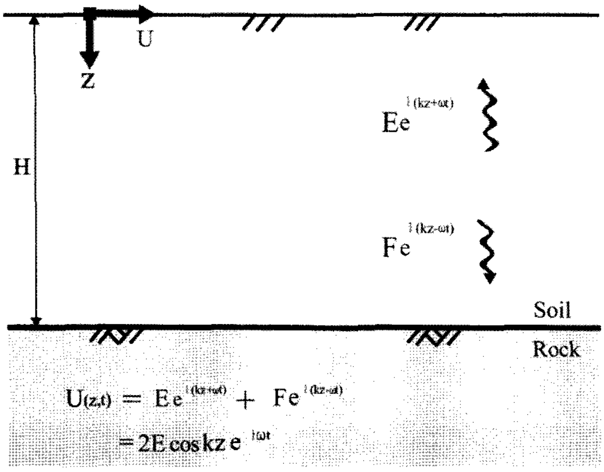

Figure 5. Wave Propagation in Uniform Elastic Soil Layer Overlying a Halfspace of Rock

$$k^2 = \frac{\rho\omega^2}{G + i\omega\eta} = \frac{\rho\omega^2}{G^*} = \frac{\omega^2}{v_s^{*2}} \tag{10}$$

$$G^* = G + i\omega\eta = G(1 + 2i\zeta) \tag{11}$$

The boundary condition where shear stresses are zero at free surface gives: $E=F$, and Equation 9 can be written as (Schnabel et al. 1972):

$$\ddot{U}(z,t) = -E\omega^2(e^{ikz} + e^{-ikz})e^{i\omega t} = -2E\omega^2 \cos kz\, e^{i\omega t} \tag{12}$$

The gross (overall) effects of earthquake shaking on the potential sliding masses are of our ultimate interest. Overall effects of earthquake shaking on the potential sliding masses may be assessed by integrating all the acceleration time history along the height H of the soil layer, given as:

$$\begin{aligned} HEA(t) &= \frac{1}{H}\int_0^H \ddot{U}(z,t)\, dz \\ &= \frac{1}{H}\int_0^H -2E\omega^2 \cos kz\, e^{i\omega t}\, dz \\ &= \frac{-2E\omega^2 e^{i\omega t}}{H}\int_0^H \cos kz\, dz \\ &= \frac{-2E\omega^2 e^{i\omega t} \sin kH}{kH} \end{aligned} \tag{13}$$

Alternatively, Equation 13 can be derived from Equation 5. The frequency response (or transfer) function for the *HEA* (i.e., average acceleration) and the motion at the bottom of the soil layer (i.e., input acceleration time history) can thus be given as:

$$\begin{aligned} H(\omega) &= \frac{HEA(t)}{\ddot{U}(z = H, t)} \\ &= \frac{\sin kH}{kH \cos kH} \\ &= \frac{\tan kH}{kH} \end{aligned} \tag{14}$$

Equations 13 and 14 are valid regardless of the bedrock stiffness (i.e., rigid or elastic).

4. EFFECTS of SPATIAL AVERAGING

Figure 6 shows the ratio between the amplitudes of the *HEA* and bottom acceleration (i.e., amplification function $|H(\omega)|$) along with that between the amplitudes of the free surface and bottom acceleration, when the soil is undamped ($\zeta=0$). The figure indicates that the spatial average of motion is always less than the motion at the free surface. The average motion is amplified ($|H(\omega)|$) at small kH (i.e., small $\omega H/v_s$ such as stiff soil, low height, and low frequency motion) up to around the first fundamental frequency of the soil deposit and the motion is de-amplified ($|H(\omega)| \leq 1$) as kH increases, except around natural frequencies (i.e., $\omega_n = \frac{v_s}{H}\left(\frac{\pi}{2} + n\pi\right)$ n=0,1,2,...., ∞). Therefore, the spatial averaging acts like a linear filter that amplifies low frequency content and de-amplifies high frequency content of motion. These findings generally support the previous landfill investigations (e.g., Bray et al. 1998) and their finding that the direct use of input acceleration time history in Newmark block analysis without considering system compliance can be significantly unconservative (i.e., produces significant small displacement). That is mainly because the computed slope displacement is influenced mainly by low frequency average motion that amplifies and is not sensitive to high frequency motion (e.g., beyond 10-20 rad/s) that de-amplifies due to the averaging (Figure 7). However, it should be noted that the direct use of an input acceleration time history, which does not consider the averaging effect of ground motion acting on the potential sliding mass, can be conservative in case of a soil deposit of very long period, although this unusual case should be assessed by methods other than Newmark-type approach. On the other

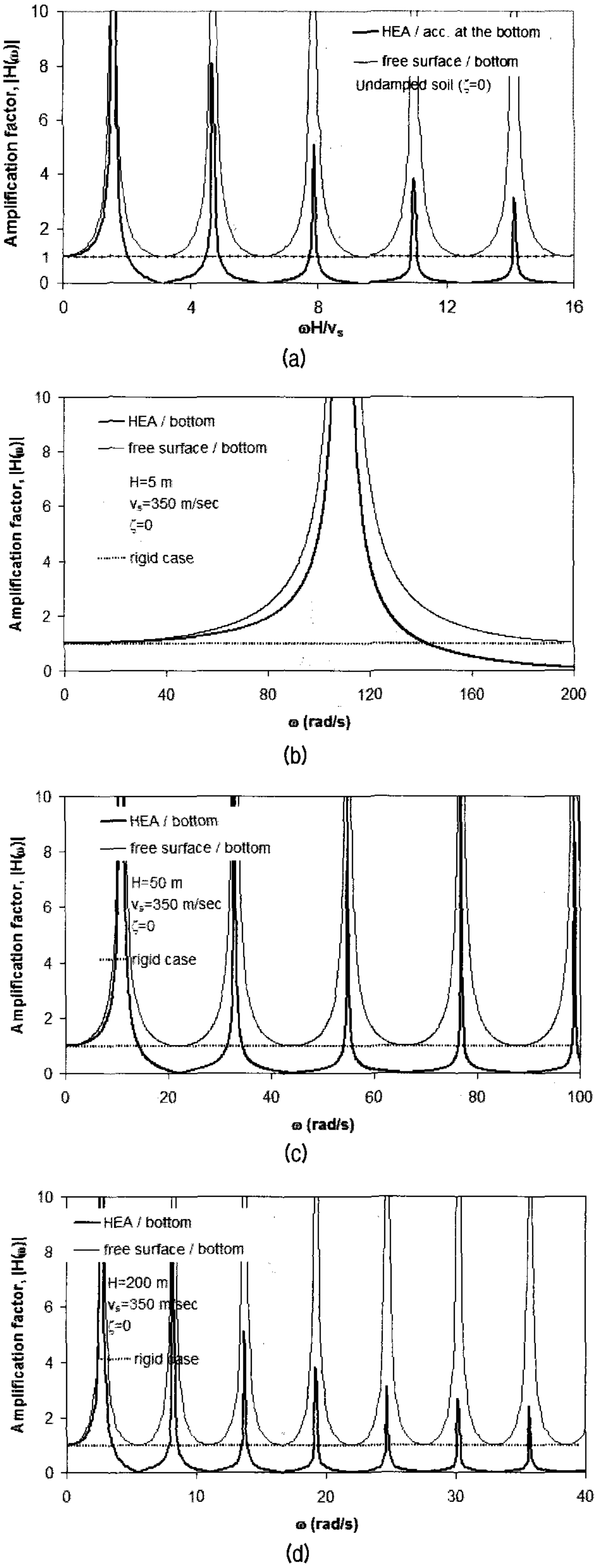

Figure 6. Ratio Between the Amplitudes of the *HEA* (Horizontal Equivalent Acceleration) and Bottom Acceleration (i.e., Amplification Function $|H(\omega)|$) Along with That Between the Amplitudes of the Free Surface and Bottom Accelerations for Undamped Soil (ζ=0): (a) with Respect to kH; (b) with Respect to ω for Height=5 m; (c) for Height=50 m; (d) for Height=200m

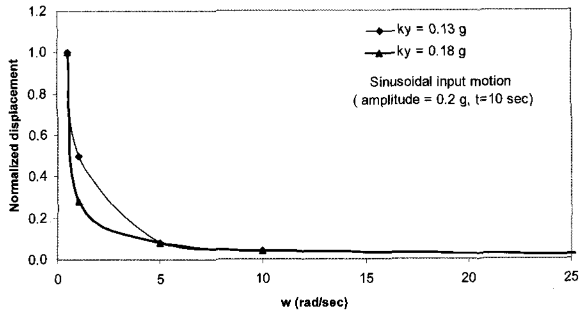

Figure 7. Effects of Motion Frequency on the Computed Displacement

hand, the direct use of rock outcrop motion as input motion in the Newmark deformation analyses may be justified, if the fundamental period of the slope is sufficiently short (i.e., shallow soil deposit with high shear velocity). The upper bound of this fundamental period is of our special interest. We need to find the maximum value of the period that satisfies Equation 14 :

$$H(\omega) = \frac{\tan kH}{kH} \approx 1.0 \tag{15}$$

It can be easily shown that for $kh \le 0.51$, $|H(\omega)| \le 1.10$ (i.e., less than 10 % difference). Therefore, the maximum value of the period of the slope can be estimated as:

$$kH = \frac{\omega}{v_s^*} H = \frac{\omega}{4} \frac{4H}{v_s^*} = \frac{\omega T}{4} \le 0.51 \tag{16}$$

Since the computed displacement is not sensitive to high frequency motion (beyond 10-20 rad/s), the maximum value of the characteristic site period can be found as:

$$T \le \frac{2.0}{\omega} = \frac{2.0}{10} = 0.2 \text{ second} \tag{17}$$

The rigid bedrock motion is the same as the bedrock outcropping motion and for high frequency range (i.e., $\omega >> \frac{2\pi}{T}$) the difference between the elastic bedrock motion and bedrock outcropping motion is negligible, while the ratio between the amplitudes of the bedrock motion and the outcropping motion is always less than 1 (e.g., Schnabel et al. 1972). Therefore, it can be concluded that the direct use of rock outcrop motion as input motion in the Newmark deformation analyses is justified, if the fundamental period of the slope is less than 0.2 second.

5. SUMMARY and CONCLUSIONS

This study investigates the effect of spatial averaging and compliance taken account for in the analysis of earthquake-induced deformation of slope.

The results suggest that the spatial averaging acts like a linear filter that amplifies low frequency content and de-amplifies high frequency content of motion.

The results generally support the previous finding that the direct use of input acceleration time history in Newmark-type block analysis without considering system compliance can be significantly unconservative. On the other hand, the direct use of rock outcrop motion as input motion in the Newmark deformation analyses may be justified, if the fundamental period of the slope is sufficiently short (i.e., shallow soil deposit with high shear velocity)

The results lead to a relatively simple criterion that can be used to determine the level of compliance. It is concluded that the direct use of rock outcrop motion as input motion in the Newmark-type deformation analyses is justified, if the fundamental period of the slope is less than 0.2 second. The study, however, does not take account for the effect of damping, which is desirable to be included in the study in the future.

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