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Coefficient Estimation of IIR Digital Filters Using a Real-Coded Genetic Algorithm

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Abstract : This paper proposes a methodology to estimate the system coefficients for the infinite impulse response (IIR) digital filters using real code GA. In the traditional real coded GA, it adapts the general genetic operations, whereas in this paper the proposed real coded GA applies improved genetic operations in order to search the optimal solution in given problems. Each of unknown IIR digital coefficients collected as forms of a chromosome. Two illustrative examples including the band pass and band stop IIR digital filters are demonstrated to verify the proposed method.

Key words : IIR digital filters, Real coded genetic algorithm

1. Introduction

In the field of the digital signal processing, the digital filter design is not only a basic but also an important research topic. The most important meaning of the filter is that it can remove out desired frequency and reserve other parts when an input signal is to pass the filter.

Common categories of filters include the low pass, high pass, band pass, band stop, and so on. For example, the main function for the band pass filter is that allows a band of frequencies to pass, whereas a band stop filter is to allow all frequencies outside a band to pass. For the digital signal filtering, two types of digital filters including the finite impulse response(FIR) and infinite impulse response (IIR) digital filters are often considered. The new output of the FIR filter is produced by only using the present and past input data. However, the new output of the IIR filter is not only influenced by the present and past inputs, but also by the past outputs. For a given filtering characteristic, an FIR filter may require many terms to achieve the desired characteristic, whereas an IIR filter generally needs fewer terms to achieve the same goal[1].

The GA that is based on the natural evolution is a powerful searching tool for optimally solving an optimization problem over the search domain[2], [3]. There are two

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different kinds of coding method, including the binary and real number coding, for genetic operations.

In the binary-coded GA, all system parameters must be encoded as a representation of binary bits, and these binary bits are evolved to generate new offspring. Kristinsson and Dumont[4] applied the binary-coded GA to estimating the locations of poles and zeros of a transfer function, and then used this estimated model to design a discrete time pole placement adaptive controller. Ma and Cowan[5] presented a binary-coded GA to the adaptation of IIR filters, including cascade, parallel, and lattice structures. Moreover, Chen and Hung[6] were proposed a global estimation for multi-channel time-delay and signal parameters by a binary-coded genetic algorithm in the frequency domain.

In contrast to the binary-coded GA, real-coded GA has been also applied to a wide variety of fields[7-11]. All genes in a chromosome are the form of real numbers. It is more suitable for solving the real optimization problems, and may overcome the drawback of loss of precision resulted from a real number changing to the binary bits occurred in the binary-coded GA. Besides, a real-coded GA is also easy to be implemented by the computer programs over the binary-coded GA.

In the digital filter design, Tzeng and Lu[9-10] made use of the real-coded GA for designing arbitrary complex FIR digital filter considered in the frequency domain and for designing two-dimensional FIR filters for sampling structure conversion. Xu and Daley[11] proposed a parallel genetic algorithm to design a direct form of a finite word length FIR low pass digital filter.

In this paper, we propose an improved real-coded GA to accurately identify IIR digital filter coefficients. Unlike the traditional real-coded GA which is used to general genetic operations, the proposed real-coded GA is applied to improved genetic operations.

Simulation results of two kinds of IIR digital filters are shown to good estimation performance better than the traditional real-coded GA.

2. IIR digital filters

The coefficients estimation of IIR digital filters is considered in this paper. These require past and present inputs as well as past outputs to generate new output. A convenient expression for the IIR digital filter is given by

$$\begin{aligned} y[n] &= \sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k] \\ &= a_1 y[n-1] + a_2 y[n-2] + \cdots + a_N y[n-N] \\ &\quad + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \cdots + b_M x[n-M] \end{aligned} \tag{1}$$

where x is the input signal that will be filtered, y is the output of the IIR digital filter or a filtered signal by this filter, N is the number of past outputs required, normally referred to as the order of the filter, M is the number of past inputs required, a_k and b_k are weights that determine the contribution of each output and input sample. These weights are called filter coefficients.

In this study, the numbers of N and M of the IIR digital filter are assumed to be previously known for simplification. Assume that an undetermined IIR digital filter that will match the system of Eq. (1) is given by

$$\begin{aligned}\hat{y}[n] &= \sum_{k=1}^{N}\hat{a}_k y[n-k] + \sum_{k=0}^{M}\hat{b}_k x[n-k] \\ &= \hat{a}_1\hat{y}[n-1] + \hat{a}_2\hat{y}[n-2] + \cdots + \hat{a}_N\hat{y}[n-N] \\ &\quad + \hat{b}_0\hat{x}[n] + \hat{b}_1\hat{x}[n-1] + \hat{b}_2\hat{x}[n-2] + \cdots + \hat{b}_M\hat{x}[n-M]\end{aligned} \qquad (2)$$

where $\hat{y}$ is the output of the undetermined filter, $\hat{a}_k$ and $\hat{b}_k$ are the estimated coefficients searched by the use of the proposed real-coded GA to approximate the actual values of a_k and b_k.

For simplification, let new estimated coefficients vectors like Eq. (3).

$$\Phi = [\phi_1, \phi_2, \cdots, \phi_m] = [\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_N,\ \hat{b}_0, \hat{b}_1, \hat{b}_2, \cdots, \hat{b}_M] \qquad (3)$$

where $m = N + M + 1$ is the total number of unknown filter coefficients. From the genetic algorithm point of view, Φ is called a chromosome and all ϕ_i, for $i \in m$ and $m = 1,2,\cdots,m$, in ϕ stand for genes.

3. Coefficient estimation of IIR digital filters

3.1 Real-coded genetic algorithm (RCGA)

This subsection deals with a RCGA as a tool for solving optimization problems arising in the next subsections. A RCGA starts with an initial population which comprises a set of real number chromosomes for the parameters to be searched as

$$P(k) = \{s_i(k) \mid i \in [1,\ N_p]\} \qquad (4)$$

where N_p denotes the population size and $s_i(k)$ the ith chromosome as an example of

$$s_i(k) = (12.451\ \ 120.530 \cdots 0.493) \qquad (5)$$

All individuals in $P(k)$ are evaluated in terms of their fitness. Three genetic operations of reproduction, crossover, and mutation are used to form the population $P(k+1)$ in the next generation. Normally this procedure comprising evaluation and three genetic operations continues until the solution converges with the specific accuracy or the iteration reaches a stopping condition.

A. Reproduction

Reproduction is an artificial version of 'natural selection'. It performs to inherit good-working individuals from generation to generation. If an individual has a relatively higher fitness value, it has more chance to obtain its offspring in the next generation by a selection operator. There are several well-known selection operators for reproduction. For example, there are roulette wheel reproduction, tournament reproduction, gradient-like reproduction and so on. In this paper, we adopt a gradient-like operator[12], [13].

B. Crossover

Crossover emulating sexual mating in the natural biological systems involves the process of blending chromosome information. It exchanges genes between two parent chromosomes to produce their offsprings. There are many kinds of crossover operators. For instance, there are flat crossover, simple crossover, arithmetical crossover and modified simple crossover. In this paper, we use the modified simple crossover[13] described as follows:

$$\begin{aligned} s^1 &= (r_1^1 \cdots r_i^1 | r_{i+1}^1 \cdots r_k^1) \\ s^2 &= (r_1^2 \cdots r_i^2 | r_{i+1}^2 \cdots r_k^2) \\ \Rightarrow \quad s^{1'} &= (r_1^1 \cdots \tilde{r}_i^1 | r_{i+1}^2 \cdots r_k^2) \\ s^{2'} &= (r_1^2 \cdots \tilde{r}_i^2 | r_{i+1}^1 \cdots r_k^1) \end{aligned} \tag{6}$$

where

$$\tilde{r}_i^1 = \lambda r_i^1 + (1-\lambda) r_i^2$$
$$\tilde{r}_i^2 = \lambda r_i^2 + (1-\lambda) r_i^1$$

$r_i \in \Re$ represents the ith gene, and λ is a uniformly distributed random number between 0 and 1.

C. Mutation

Mutation is a random process in which one genotype is replaced by another to generate a new chromosome. Each genotype has the probability of mutation P_m. There are many kinds of mutation operators. For example, uniform mutation, boundary mutation and dynamic mutation are generally used in genetic algorithm. In this work, dynamic mutation[12] is adopted, which is described by Eq. (7).

$$\begin{aligned} s' &= (r_1 \ r_2 \cdots r_i \ r_{i+1} \cdots r_k) \\ \Rightarrow \bar{s} &= (r_1 \ r_2 \cdots \bar{r}_i \ r_{i+1} \cdots r_k) \end{aligned} \tag{7}$$

and $\bar{r}_i = \begin{cases} r_i + \Delta(k, r_i^{(U)} - r_i), & \text{if } \tau = 0 \\ r_i - \Delta(k, r_i - r_i^{(L)}), & \text{if } \tau = 1 \end{cases}$

where $\bar{r}_i$ is the ith gene and $r_i^{(U)}$, $r_i^{(L)}$ are the upper and lower limits of $\bar{r}_i$, respectively. τ is a random number which has 0 or 1.

This operator can be set larger P_m than other operators because it uses flexible P_m which becomes gradually small according to increasing generation.

D. Elitism

The above three operators can destroy the fittest chromosome in the current generation due to their stochastic features. Elitism is used to ensure that the fittest chromosome survives into the next generation since it allows the solution to get better over generation.

3.2 Coefficient estimation of IIR digital filters using a RCGA

To begin with the proposed genetic operations, an objective function should be defined and in accordance with this function to guide its search. The sum of square error (SSE) is taken as an objective function, which is given by

$$J(\Phi) = \sum_{n=0}^{T} (y[n] - \hat{y}[n])^2 = \sum_{n=0}^{T} e^2[n] \tag{8}$$

where T is the sampling number. The purpose of this paper is to find the optimal model coefficients in Eq. (2) such that the SSE in Eq. (8) is minimized as much as possible. Fig. 1 shows the simple estimation architecture for the IIR digital filter by using the proposed genetic algorithm.

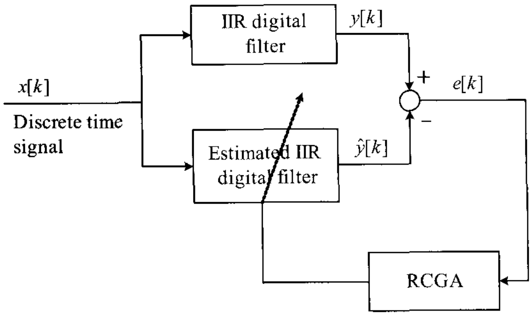

Fig. 1 Architecture of RCGA Based IIR digital filter estimation

3.3 Search Space for Filter Coefficients

The coefficient estimation for the IIR digital filter starts with a given population which contains a number of random chromosomes generated from the defined search space. Each chromosome in the population represents a set of possible solution for IIR digital filter estimation problem. A search space for ϕ is defined by

$$\Omega_\phi = \{\phi \in \Re^M \mid \phi_{1\min} \le \phi_1 \le \phi_{1\max},\ \phi_{2\min} \le \phi_2 \le \phi_{2\max},\ \cdots,\ \phi_{m\min} \le \phi_m \le \phi_{m\max}\} \quad (9)$$

All genes ϕ_i, for $i \in m$, in a chromosome will be evolved in the search space Ω_ϕ while the generation is increasing.

4. Examples and Simulations

Two kinds of illustrative examples including the band pass and band stop IIR filters are given to show the validity of the proposed method. In the following simulations, the parameters employed in the traditional RCGA operations are listed in Table 1, and the proposed RCGA operations are listed in Table 2.

The traditional RCGA is adapted simple genetic operations such as 'roulette wheel reproduction', 'one-point crossover' and 'simple mutation'. The proposed RCGA is different from the traditional RCGA adapted simple genetic operations.

Table 1 Control parameters of the traditional RCGA.

Control parameters	T	N_p	P_c	P_m	$[\phi_{i\min}, \phi_{i\max}]$
Values	150	30	0.9	0.02	[-1.0, 1.0]

Table 2 Control parameters of the proposed RCGA.

Control parameters	T	N_p	P_c	P_m	η	b	$[\phi_{i\min}, \phi_{i\max}]$
Values	150	30	0.9	0.2	1.8	5	[-1.0, 1.0]

In table 1 and table 2, P_c means the probability of reproduction. In table 2, η is the parameter of the gradient-like reproduction and recommended to have real number between 0 and 2[13]. Also b is the parameter which indicates un-uniform rate of the dynamic mutation.

Example 1. Consider an actual band pass IIR digital filter that will be estimated given by

$$y[n] = -0.5926y[n-1] - 0.1193y[n-2] + 0.4404x[n] - 0.4404x[n-2] \quad (9)$$

From Eq. (9), we know that the actual filter coefficients are a_1 =-0.5926, a_2 =-0.1193, b_0 = 0.4404, and b_2 =-0.4404. The test input signal for IIR filter coefficient estimation is given by

$$u[n] = 0.59\sin[n] + 0.1\sin(n+5) + \text{UniRand}(-0.5, 0.5) \quad (10)$$

where 'UniRand(-0.5,0.5)' means uniformly distributed random number which has between -0.5 and 0.5.

Fig. 2 shows input signal defined by Eq. (10) and output signal(filtered signal), and the band pass IIR digital filter expressed by Eq. (9) shows very well the filtering performance of input signal.

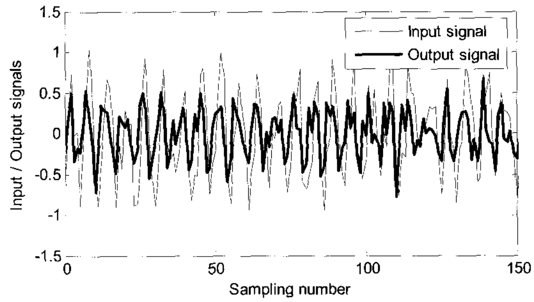

Fig. 2 Input and output signals for example 1.

To demonstrate the effectiveness, the proposed and traditional RCGA are compared as listed in Table 3. Fig. 3 shows the convergence trajectories of filter coefficients by the proposed RCGA.

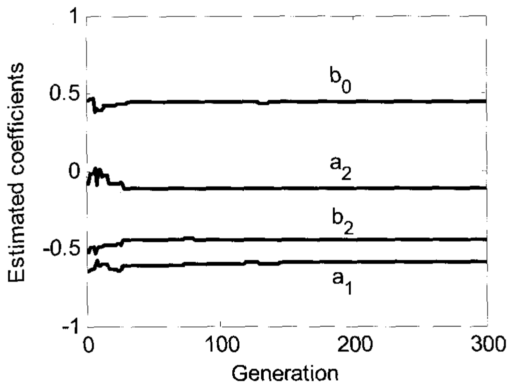

Fig. 3 Estimation results for example 1

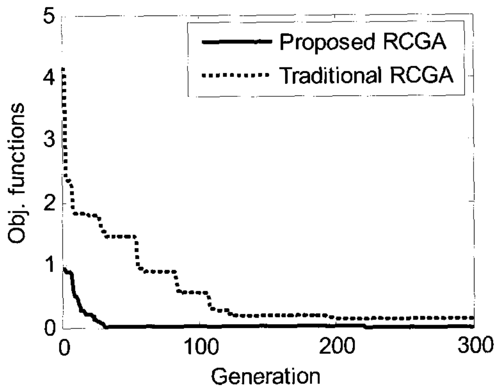

Fig. 4 Obj. function convergences for example 1

Table 3 Comparison between the proposed RCGA and traditional RCGA in Example 1.

Actual value	a_1	a_2	b_0	b_2
	-0.5926	-0.1193	0.4404	-0.4404
The proposed method	-0.5926	-0.1193	0.4404	-0.4404
The traditional method	-0.5652	-0.1727	0.4612	-0.4152

With respect to the same number of generations, Fig. 4 shows the convergence trajectories of the objective function by using these two methods. It is obvious that all of filter coefficients are approximated accurately for both RCGAs. Through Table 3 and fig. 4, however, we can say that the proposed method gives a better satisfactory estimation than the traditional RCGA.

Example 2. Another kind of band stop IIR digital filter estimation is also illustrated. An actual difference equation is given by

$$y[n]=0.5926y[n-1]-0.1193y[n-2]+0.5597x[n] + 0.5926x[n-1]+0.5597x[n-2] \qquad (10)$$

Also, we have a_1 = -0.5926, a_2 = -0.1193, b_0 = 0.5597, b_1 = 0.5926, and b_2 = 0.5597.

The test input signal has the same signal such as Eq. (10), and Fig. 5 shows input signal and output signal.

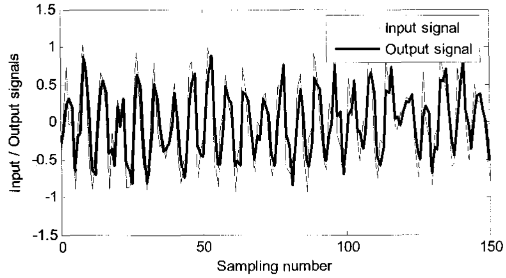

Fig. 5 Input and output signals for example 2.

After carrying out the proposed and traditional method, simulation results are listed in Table 4. Fig. 6 shows the convergence trajectories of filter coefficients by the proposed RCGA and Fig. 7 shown the convergence trajectories of the objective function by using these two methods.

Table 4 Comparison between the proposed RCGA and traditional RCGA in Example 2.

	a_1	a_2	b_0	b_1	b_2
Actual value	-0.5926	-0.1193	0.5597	0.5926	0.5597
The proposed method	-0.5927	-0.1193	0.5596	0.5927	0.5598
The traditional method	-0.3226	-0.2773	0.5211	0.4183	0.5616

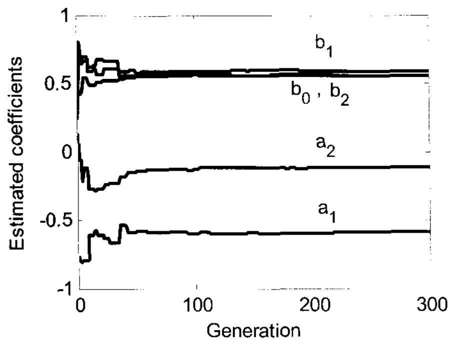

Fig. 6 Estimation results for example 2

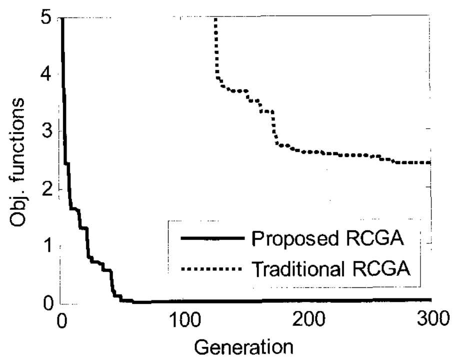

Fig. 7 Obj. function convergences for example 2

With respect to the same number of generations in Fig. 7, we can conclude that the proposed method gives a better satisfactory estimation than the traditional RCGA.

5. Conclusions

In this paper, the methodology for IIR digital filter coefficient estimation has been proposed. In the IIR digital filter estimation, all of filter coefficients are directly regarded as genes and a collection of genes constitute a chromosome. With the use of the proposed method, three genetic operations of a gradient-like reproduction, modified simple crossover, and dynamic mutation are utilized to evolve these chromosomes in the population such that the defined objective function is minimized.

Different from the traditional RCGA which is adapted simple genetic operators, the proposed RCGA can provide a better estimation results from estimation accuracy and objective function convergence point of view.

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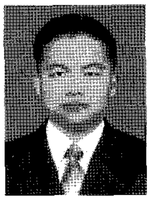

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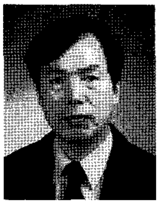

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