Journal of the Korean Society of Marine Engineering, Vol.31, No.7, pp. 833~841, 2007(ISSN 1226-9549)

The Coupling of Conduction with Free Convection Flow Along a Vertical Flat Plate in Presence of Heat Generation

M.A.Taher* · Yeon-Won Lee†

(Manuscript : Received September 28, 2007 ; Revised November 6, 2007)

Abstract : The aim of this paper is to analyze the conjugate problems of heat conduction in solid walls coupled with laminar free convection flow adjacent to a vertical flat plate under boundary layer approximation. Using the similarity transformations the governing boundary layer equations for momentum and energy are reduced to a system of partial differential equations and then solved numerically using Finite Difference Method (FDM) known as the Keller-box scheme. Computed solutions to the governing equations are obtained for a wide range of non-dimensional parameters that are present in this problem, namely the coupling parameter P, the Prandtl number Pr and the heat generation parameter Q. The variations of the local heat transfer rate as well as the interface temperature and the friction along the plate and typical velocity and temperature profiles in the boundary layer are shown graphically. Numerical solutions have been consider for the Prandtl number Pr = 0.70

Key words : Conduction-Convection, Heat generation, Vertical flat plate, Prandtl number.

1. Introduction

In the traditional area of convective heat transfer between a solid wall and a fluid flow the wall conduction resistance is usually neglected, i.e. the wall is assumed to very thin. The wall conditions are either prescribed as a wall temperature or wall heat flux. There is another class of wall conditions where the convective fluid and conduction through the wall interacts. This interaction is particularly very important for natural convection problems where the wall temperature and bulk fluid temperature difference is the driving force for the flow. This type of problem is usually called conjugate heat transfer problems. In this case the heat transfer mechanism involves the combine effects of conduction and fluid motion. Nowadays, a considerable amount of research work has been done in order to understand the heat transfer characteristics over a wide range of flow configurations and fluid properties. However, in many real engineering systems the wall conduction resistance

† Corresponding Author(School of Mechanical Engineering, Pukyong National University), E-mail: ywlee@pknu.ac.kr, Tel: 051)620-1417

* Graduate School of Mechanical Engineering, Pukyong National University

cannot be neglected since conduction in the wall is able to significantly affect the fluid flow and the heat transfer characteristics of the fluid in the vicinity of the wall. Thus, the conduction in the solid wall and the convection in the fluid should be determined simultaneously. This type of convective heat transfer is referred to as a conjugate heat transfer process and it arises due to the finite thickness of the wall, there have been several works on this conduction- convection coupling. A good many research efforts, both experimental and theoretical, has been devoted to the conjugate problems of free convection heat transfer. Pozzi and Lupo [1], [2] discussed the coupling of conduction with both forced and free convection heat transfer along and over a heated flat plate, but they did not consider the heat generation and conduction effect simultaneously. Recently, the effects of coupling of wall conduction with laminar free convection heat transfer of micro polar fluids along a vertical flat plate has been studied by Char and Chang [3]. Furthermore, a numerical study has been conducted for natural convection heat transfer for air from two vertically separated horizontal heated cylinders confined to a rectangular encloser having vertical walls of finite thickness and horizontal wall at the heat sink temperature by Lacroix and Joyeux [4]. Probably a most relevant work to the present one has been first advanced by Miyamota et al. [5] who provided both experimental and numerical results and an analytical solution valid for a boundary layer regime along a heated vertical plate either by constant temperature or by constant flux. However, the most recent contributions on this subject may be found in the paper by Kimura et al. [6]. They have studied the effect of conjugate natural convection heat transfer from a vertical plate analytically and experimentally assuming the existence of vertically averaged interfacial temperature between plate and fluid. The effect of internal heat generation analysis is of most important for the proper sizing of fuel elements in the nuclear reactors cores to prevent burnout. The performance of aircraft also depends upon the case with which the structure and engines can be cooled. Molla et al. [7] described the effect of internal heat generation /absorption along a uniformly heated vertical wavy surface with uniform surface temperature. Also the heat transfer characteristics in the laminar boundary layer of viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation were investigated by Vajravelu and Hadjinolaou [8]. Actually the mechanism of heat transfer in these situations is frequently encountered in our environment and in engineering services and has been extensively reviewed by Pop and Ingham [9].

The purpose of the present paper is to study the conduction-convection coupling with laminar free convection along a vertical flat plate in presence of heat generation. As far we know the problem has not been considered before. The non-dimensional boundary layer equations are solved by using implicit finite difference method[10]. The objective of the present work is to be obtained the evolution of the surface shear stress in terms of local skin friction and the non-dimensional surface temperature, velocity distribution as well as temperature for varying different values of non-dimensional parameters that present in our problem namely the coupling parameter P and heat generation parameter Q. Numerical solutions have been considered for the Prandtl number Pr=0.70.

Nomenclature

b	Plate thickness [m]
f	Dimensionless stream function
g	Acceleration due to gravity [ms^{-2}]
Gr	Grashof number
k	Thermal conductivity [$Js^{-1}m^{-1}K^{-1}$]
Pr	Prandtl number
P	Coupling conduction parameter
Q	Heat generation parameter
q_w	Heat flux at the surface[$Js^{-1}m^{-2}$]
T	Temperature of the fluid in the boundary layer [K]
u, v	Dimensionless velocity components along x and y directions [ms^{-1}]
x, y	Axis in the direction along and normal of the surface respectively [m]
ψ	Stream function [m^2s^{-1}]
η	Pseudo-similarity variable
ρ	Density of the fluid [kgm^{-3}]
μ	Viscosity of the fluid [$kgm^{-1}s^{-1}$]
υ	Kinematic viscosity [m^2s^{-1}]
τ_w	Shearing stress at the wall [$kgm^{-1}s^{-2}$]
θ	Dimensionless temperature function
β	Coefficient of thermal expansion

2. Formulation of the Problem

2.1 Governing Equations and Boundary Conditions

Consider a vertical flat plate whose length and thickness are L and b respectively. The temperature of the fluid far away from the plate is T_∞ whereas that of the outside surface of the plate is maintained at a constant temperature T_b and $T_b > T_\infty$. The flow configuration and the coordinates system are shown in Fig. -1. In the formulation of the present work the following common assumptions are made: the flow is steady, laminar, incompressible and two dimensional; the effects of viscous dissipation is neglected.

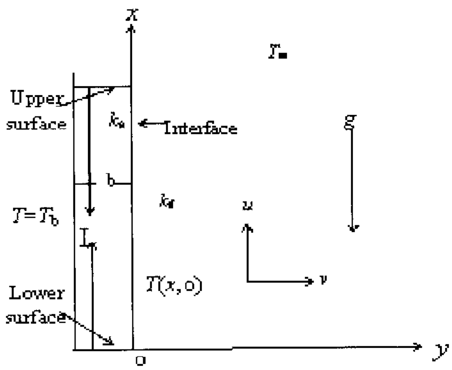


Fig. 1 Physical model and coordinate systems.

Thus the governing equations for mass continuity, momentum and energy are obtained from the boundary layer equations by introducing the Boussinecq and boundary layer approximation as the following forms:

$$\frac{\partial}{\partial \hat{x}}(\hat{u})+\frac{\partial}{\partial \hat{y}}(\hat{v})=0 \qquad (1)$$

$$\rho\left(\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{u}}{\partial \hat{y}}\right)=\mu\frac{\partial^2 \hat{u}}{\partial \hat{y}^2}+\rho g\beta(T-T_\infty) \qquad (2)$$

$$\hat{u}\frac{\partial T}{\partial \hat{x}}+\hat{v}\frac{\partial T}{\partial \hat{y}}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial \hat{y}^2}+\frac{Q_0}{\rho C_p}(T-T_\infty) \qquad (3)$$

Where $(\hat{x}, \hat{y})$ are the dimensionless coordinate along and normal to the tangent of the plate and $(\hat{u},\hat{v})$ are the velocity components along $(\hat{x},\hat{y})$ directions, g is the acceleration due to gravity, β is the coefficient of thermal expansion, $\nu = \mu/\rho$ is the kinematic viscosity, T is the local temperature, and Pr is the Prandtl number, k is the thermal conductivity and C_p the specific heat at constant pressure. The amount of heat generated or absorbed per unit volume is $Q_0\,(T-T_\infty)$, Q_0 being a constant, which may take either positive or negative. The source term represents the heat generation when $Q_0 > 0$ and the heat absorption when $Q_0 < 0$ and ρ is the density of the fluid.

The boundary conditions for the equations (1) to (3) are

$$\hat{u}=\hat{v}=0, T=T_b \text{ at } \hat{y}=0 \qquad (4.1)$$

$$\hat{u} \to 0, T \to T_\infty \quad \text{as} \quad \hat{y} \to \infty \qquad (4.2)$$

The temperature T_{so} in the plate as given by A. Pozzi and M. Lupo [1,2] is $T_{so}=T(x,0)-\{T_b-T(x,0)\}\frac{y}{b}$. It is noted that $T(x, 0)$ is the unknown temperature at the interface. Thus the heat flux continuity condition at the interface may be written as

$$k_s\frac{\partial T_{so}}{\partial y}=k_f\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (4.3)$$

where k_s and k_f are the thermal conductivity of the plate and the fluid respectively.

To facilities the analysis, it is necessary to make the above equations dimensionless and use the following new variables

$$x=\frac{\hat{x}}{L},\ y=Gr^{1/4}\left(\frac{\hat{y}}{L}\right),\ u=\frac{L}{\nu}Gr^{-1/2}\hat{u},\ v=\frac{L}{\nu}Gr^{-1/4}\hat{v}$$

$$\theta=\frac{T-T_\infty}{T_b-T_\infty} \qquad (5)$$

where $L = Gr^{1/4}\left(\frac{bk_f}{k_s}\right)$ is the characteristic length and θ is the non dimensional temperature and $Gr=g\beta(T_b-T_\infty)\frac{L^3}{\nu^2}$ is the Grashof number, which represents the ratio of the buoyancy force to the viscous force acting on the fluid.

Introducing the above dimensionless variables into equations (1) to (3), we have

$$\frac{\partial}{\partial x}(u)+\frac{\partial}{\partial y}(v)=0 \qquad (6)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\theta \qquad (7)$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+\frac{Q_0}{\mu c_p}\frac{L^2}{Gr^{1/2}}\theta \qquad (8)$$

The boundary conditions (4) associated with equations (6) to (8) become

$$u=v=0,\ (\theta-1)=p\frac{\partial\theta}{\partial y} \quad \text{at } y=0$$

$$u \to 0,\ \theta \to 0 \text{ as } y \to 0 \qquad (9)$$

where p is the coupling parameter define by $P=\left(\frac{\kappa_f}{\kappa_s}\cdot\frac{b}{L}\right)Gr^{1/4}$

2.2 Similarity Transformation

For the purpose of analytical development and ultimate numerical solution, it is better to reformulate equations(6) to (9) using similarity-like variables. To do this, we assume the following dimensionless variables f and the similarity variable as

$$\psi = x^{4/5}(1+x)^{-1/20} f(x,\eta)$$

$$\eta = yx^{-1/5}(1+x)^{-1/20}, \theta = x^{1/5}(1+x)^{-1/5}H \qquad (10)$$

Here is the non-dimensional stream function, which is related to the velocity components in the usual way as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, is the dimensionless similarity variable and $H(x)$ is the dimensionless temperature.

We may proceed to transform the conservation of momentum and energy equations (7) and (8) into the new co-ordinates. To facilitate the transformation, it is useful to have the velocity components explicitly expressed in terms of the new variables. Therefore we obtained

$$f''' + \frac{16+15x}{20(1+x)} f f'' - \frac{6+5x}{10(1+x)} f'^2 + H = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \qquad (11)$$

$$\frac{1}{\Pr}H'' + \frac{16+15x}{20(1+x)} fH' - \frac{1}{5(1+x)} fH$$

$$+ Qx^{2/5}(1+x)^{1/10} H = x\left(f'\frac{\partial H}{\partial x} - H'\frac{\partial f}{\partial x}\right) \qquad (12)$$

where $Q = \frac{L^2 Q_0}{C_p \mu d^{1/2}}$ is the heat generation parameter. The corresponding boundary conditions for the present problem then turn into

$$f = f' = 0,$$

$$H' = \frac{1}{p}\left(x^{1/5}(1+x)^{1/20} H - (1+x)^{1/4}\right) \quad at\ \eta = 0$$

$$f' \to 0, \quad H \to 0 \quad \text{as } \eta \to \infty \qquad (13)$$

It has been seen that the leading edge of the plate i.e. $x \simeq 0$, equations (11) and (12) reduce to the following ordinary differential equations:

$$f''' + \frac{4}{5} f f' - \frac{3}{5} f'^2 + H = 0 \qquad (14)$$

$$\frac{1}{\Pr}H'' + \frac{4}{5} fH' - \frac{1}{5} fH = 0 \qquad (15)$$

along with the boundary conditions.

$$f = f' = 0, \quad H'p + 1 = 0 \quad at \quad \eta = 0$$

$$f' \to 0, \quad H \to 0 \quad \text{as} \quad \eta \to \infty \qquad (16)$$

The physical quantities of primary interest in this paper include the surface temperature θ_w $(x,0)$, define by the equation of (10b) and the surface shear stress in terms of local skin friction coefficient C_{fx}, which is define by $C_f = \frac{G_r^{-3/4} L^2}{\mu \nu}\tau_w$, where $\tau_w = \mu\left(\frac{\partial \hat{u}}{\partial \hat{y}}\right)_{\hat{y}=0}$ and thus we have

$$C_{fx} = x^{2/5}(1+x)^{-3/20} f''(x,0) \qquad (17)$$

If we know the values of the functions $f(x, \eta)$, H (x, η) and their derivatives for different values of coupling parameter P and the heat generation parameter Q, we may calculate the numerical values of the surface temperature $\theta_w(x, 0)$, surface shear stress $\tau_w(x)$ as well as velocity and temperature distribution at the surface that are very important from the physical point of view.

3. Results and Discussion

In this paper, we wish to give contribution to the study of heat generation effect on the coupling conduction with free convection boundary layer flow along a vertical flat plate. Here we discuss the numerical results obtained from equations (11) and (12) with the boundary conditions (13), the dimensionless skin friction $\tau_w(x)$ from equation (17) and also the non-dimensional wall temperature $\theta_w(x)$ using the very efficient implicit finite difference method (FDM) known as the Keller box scheme [10]. It can be seen that solutions are affected by two parameters, namely coupling parameter P and heat generation parameter Q. Numerical solutions have been considered for Prandtl number Pr = 0.70.

Fig. 2-3 deal with the effect of different values of heat generation parameter Q on the surface shearing stress in terms of local skin friction and the non-dimensional wall temperature, similarity velocity distribution as well as temperature distributions for the fluid having Prandtl number Pr = 0.70, with coupling parameter P = 0.5.

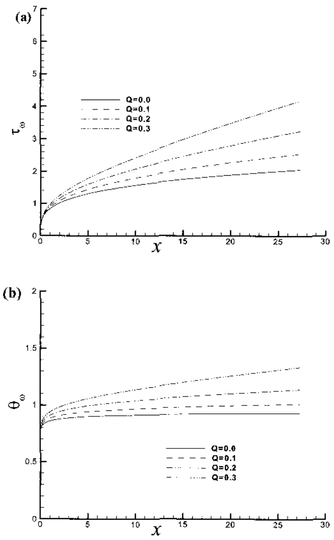


Fig. 2 (a) Skin friction $\tau w(x)$ and (b) Wall temperature $\Theta w(x)$ different values of Q.

Fig. 2 presents the variations of the dimensionless interface frictions and surface temperature against x for various values of heat generation parameter Q while P = 0.5 and Prandtl number Pr = 0.70. It can easily be seen that with the effect of heat generation parameter Q leads increase both the shearing stress and the surface temperature. This means that for increasing the values of heat generation parameter Q, the surface shearing stress and the surface temperature increases more significantly and is due to the decrease of the thermal boundary layer thickness. This means that when the body is at low temperature compared with fluid, say air; the surface shearing stress is very high.

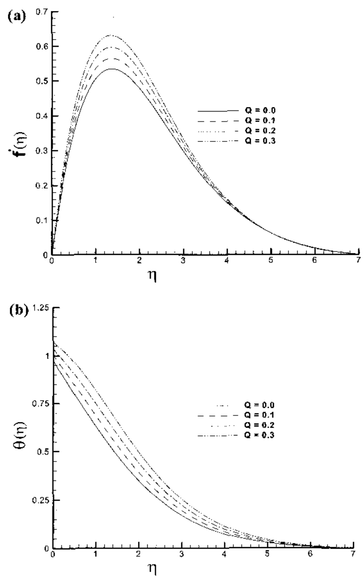


Fig. 3: (a) Velocity $f'(\eta)$ and (b) Temperature $\theta(\eta)$ profiles for different values of Q.

The distributions of the fluid velocity and the temperature fields are illustrated in Fig. 3 against similarity variable η for different small values of heat generation parameter Q (= 0.0, 0.1, 0.2, and 0.3) while the coupling parameter P = 0.5 with Pr=0.70. It is observed that for increasing the values of heat generation parameter Q both the velocity and temperature distributions increases by the mass and conduction-convection phenomena and increase with the heat generation parameter.

The influenced of different values of coupling parameter P (= 0.3, 0.5, 0.7, 0.9) on the surface shearing stress in terms of local skin friction, non dimensional surface temperature as well as fluid velocity and temperature profiles are shown in Figs. 4-5.

coupling parameter P while heat generation parameter Q=0.5 and the Prandtl number Pr=0.70. It is clearly seen that for increasing the values of coupling parameter P, the surface shearing stress and non dimensional temperature increases. Fig.4 (b) shows that at $x=0$ and also for small values of P, according to boundary conditions (13), the surface temperature approximately start at 1.0 and then increasing very slowly (solid line)along the streamwise direction. However for larger values of P(>0.3),the surface temperature start at approx. 0.80 and then increases more significantly along with the values of x.

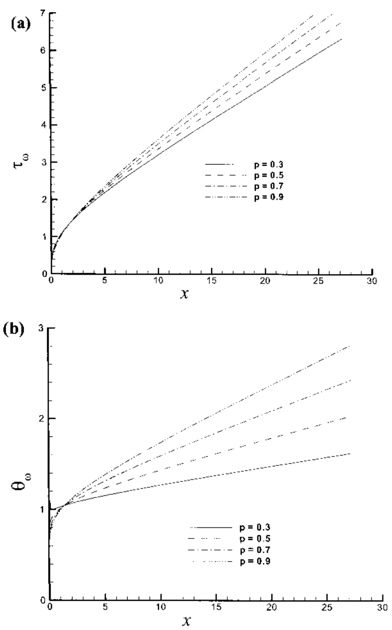


Fig. 4: (a) Skin friction $\tau_w(x)$ and (b) Wall temperature $\theta_w(x)$ different values of P.

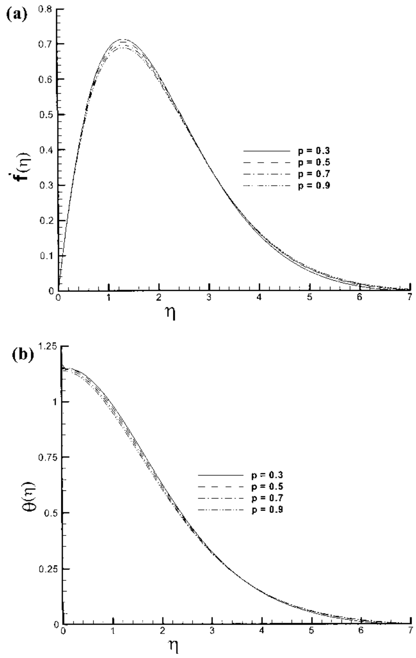


Fig. 5: (a) Velocity $f'(\eta)$ and (b) Temperature $\theta(\eta)$ profiles for different values of P.

Fig. 4 shows that variation of the shearing stress and the non dimensional surface temperature with x for different values of

Finally, Fig. 5 show the development of the fluid velocity and temperature for different values of coupling parameter P

plotted against the similarity variable η with the heat generation parameter Q = 0.5 and the Prandtl Pr = 0.70. In this case the change of velocity and temperature distributions is negligible for increasing the values of coupling parameter P from 0.3 to 0.9. Here it is found from Figure 5(a) that the velocity distribution decreases slightly as the coupling parameter P increase in the boundary layer region. But near the surface of the plate velocity increase and become maximum and then decreases and at a certain point velocity profiles coincide and finally approaches to zero according to the outer boundary condition and there exist a local maximum of the velocity within the boundary layer. Though the temperature distributions decrease but at the plate temperature profiles maximum and decreases away from the plate and finally takes asymptotic values.

4. Conclusions

In this paper some aspects of the coupling of conduction inside and free convection along a vertical flat plate in presence of heat generation have been analyzed. From the presented investigation, we may conclude that for increasing the heat generation parameter Q, the heat transfer characteristics, namely the shearing stress and surface temperature as well as fluid velocity and temperature distributions increase significantly. However, for increasing the values of coupling conduction parameter P in presence of heat generation parameter, the shearing stress and non-dimensional temperature increase considerably whereas the change of the fluid velocity and temperature distributions are negligible.

Reference

[1] A. Pozzi, and M. Lupo, "The coupling of conduction with forced convection over a flat plate", Int. J. Heat and Mass Transfer., 32, pp.1207-1214, 1989.

[2] A. Pozzi, and R.Tognaccini, "Coupling of conduction and convection past an impulsively started semi- infinite flat plate",Int. J. Heat and Mass Transfer, 43, pp.1121-1131, 2000.

[3] M.Char, and C. L. Chang, "Effect of wall conduction on natural convection flow of micropolar fluids along a flat plate", Int. J. Heat Mass Transfer, 40(15) pp 3641-3652, 1997.

[4] M. Lacroix, and A. Joyeux, "Coupling of conduction with natural convection from heated cylinder in a rectangular enclosure", Int. Comm. Heat Mass Transfer, 23(1), pp.143-151, 1996.

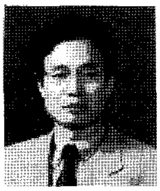
[5] M.Miyamoto, J. Sumikawa, T. Akiyoshi, and T. Nakamura, "Effect of axial heat conduction in a vertical flat plate on free convection heat transfer", Int. J. Heat Mass Transfer, 23, pp. 1545-1533, 1980.

[6] S. Kimura, A. Okajima and T. Kiwata, "Conjugate natural convection flow from a vertical heated slab", Int. J. Heat Mass Transfer, 41, pp. 3203-3211, 1998

[7] M. M. Molla, M. A. Hossain, and L. S. Yao, "Natural convection flow along a vertical wavy surface with heat generation/absorption", Int. J Thermal Science, 43, pp. 57-163, 2004.

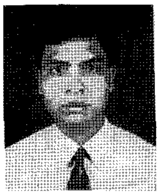
[8] K. Vajravelu, and A. Hadjinicolaou, "Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation" Int. Comm. Heat Mass Transfer, 20, pp. 417- 430, 1993.
[9] I.Pop, and D. B. Ingham, "Convective heat transfer mathematical and computational modeling of viscous fluids and porous media", Pergamon, An imprint of Elsevier Science, 2001
[10] B. Keller, "Numerical methods in boundary layer theory," Annual Review Fluid Mechanics, 10, pp.417- 433, 1978

Author Profile



Yeon-Won Lee

He graduated from Kyungpook National University (B.A. 1981, M.S. 1983) in Korea. He received his Ph.D. degree from the University of Tokyo (Doctor of Engineering 1993) in Japan. He worked at POSCO E&C for 3 years as a Team Leader of Division of Mechanical Design. He visited the Institute of Industrial Science of the University of Tokyo as a visiting professor in 1997. He has been serving as a professor of the School of Mechanical Engineering in Pukyong National University since 1993. His research interests are CFD, various natural energy problems including wave energy conversion, and MEMS technology.



M. A. Taher

He Graduated (B. Sc. 1997 and M. Sc. 1998) in Applied Mathematics from the University of Rajshahi, Bangladesh. He was recruited as a lecturer of Mathematics Department in Dhaka University of Engineering & Technology (DUET), Gazipur-1700, Dhaka, Bangladesh since 2004. At present, he has been studying as a Ph. D. student, School of Mechanical Engineering (CFD and MEMS laboratory) in Pukyong National University, Busan, Korea.