한국도로학회 논문집
第9券4號 2007年 12月
pp. 21 ~ 32

A Delay and Sensitivity of Delay Analysis for Varying Start of Green Time at Signalized Intersections: Focused on through traffic

신호교차로의 출발녹색시간 변화에 따른 직진교통류의 지체 및 지체민감도 분석

안 우 영*
Ahn, Woo Young

요 지

신호교차로의 교통류 해석을 위해 널리 사용되는 선형모형(Vertical queueing model)은 자유속도에 따라 모든 차량이 정지선에 도착하고, 대기행렬은 정지선에서 수직으로 형성된다고 가정한다. 이러한 모형자체의 단순성 때문에 선두차량은 자유속도와 가속도에 의해 계산된 출발유효녹색시간(start of effective green time)에 정지선을 통과하게 되며, 모든 추종차량은 동일한 궤적을 갖게 된다고 가정한다. 본 연구의 목적은 Vertical queueing 모형의 단순성과 비현실성을 보완하기 위해 물리학의 Kinematic수식을 응용하여 신호교차로의 출발녹색시간(start of green time)을 함수로 한 자동차추종모형(Kinematic Car-following model at Signalized intersections: KCS traffic model)을 개발하고, 이에 따른 지체 및 지체민감도를 비교함에 있다. 출발녹색시간 변화에 따른 지체분석결과 Vertical queueing 모형에서 산출된 지체값이 KCS 교통류모형에 비해 과다하게 추정됨을 알 수 있었다. 반면, 지체민감도 분석결과 KCS 교통류모형이 Vertical queueing 모형에 비해 민감하게 변함을 알 수 있었다.

핵심용어 : 선형모형, 출발유효녹색시간, KCS교통류모형, 출발녹색시간, 지체, 지체민감도

Abstract

The linear traffic model (Vertical queueing model) that is adopted widely in traffic flow estimation assumes that all vehicles have the identical motion before joining a queue at the stop-line. Thus, a queue is supposed to form vertically not horizontally. Due to the simplicity of this model, the departure time of the leading vehicle is assumed to coincide with the start of effective green time. Thus, the delay estimates given by the Vertical queueing model is not always realistic. This paper explores a microscopic traffic model (a Kinematic Car-following model at Signalised intersections: a KCS traffic model) based on the one dimensional Kinematic equations in physics. A comparative evaluation in delay and sensitivity of delay difference between the KCS traffic model and the previously known Vertical queueing model is presented. The results show that the delay estimate in the Vertical queueing model is always greater than or equal to the KCS traffic model; however, the sensitivity of delay in the KCS traffic model is greater than the Vertical queueing model.

Keywords : linear traffic mode, start of effective green time, KCS traffic model, start of green time, delay, sensitivity of delay

* 정회원 · 국립공주대학교 건설환경공학부 교수

1. INTRODUCTION

The traditional linear traffic model(Vertical queueing model) that is adopted widely in traffic signal control represents all vehicles have the same trajectory before joining a queue at the stop-line. Thus, a queue is supposed to form vertically at the stop-line without occupying any space on the link. Due to the simplicity of this model, all vehicle motions are identical. In this model, the departure time of the leading vehicle is assumed to coincide with the start of the effective green time(Clayton, 1940; Webster 1966; Allsop, 1970) and then departure times for the successive following vehicles are estimated in accordance with the saturation departure time at the stop-line. The delay estimates given by the Vertical queueing model are not always realistic, because this model does not consider any braking motion until the stop-line. Namely, it is more focused on the queue delay rather than the approach delay. The TRANSYT(Roberson, 1969) uses Vertical queueing concepts in fixed-time signal optimization. Also, in traffic-responsive signal control, Miller (1963), Gartner (1983), and Heydecker (1990) use constant mean travel time from the detector to the stop-line, and some others like Bang (1976) uses occupancy rate of the loop detector and the average speed to estimate arrival time at the stop-line.

In traffic-responsive signal control, the motion of each vehicle from the upstream detector to the downstream stop-line is needed for full interpretation of the detector outputs for performance evaluation. Hence, the concepts of Kinematics in physics are applied to derive a Kinematic Car-following model at Signalised intersections (a KCS traffic model). The model developed in this paper requires one upstream detector, the position of detector is not an issue in this research. According to this model, departure times of all detected vehicles at the stop-line, and also their delays are estimated on the basis of the on-line detector data. The model is developed to represent the individual vehicle motion in relation to the general car-following concept. Thus, it can be applied in the dynamic signal optimisation at a microscopic level (Ahn, 2004).

In the following section, the formulae are derived for two different vehicle groups: a trajectory equation for the leading vehicle and a trajectory equation for the following vehicles, in which the motion of vehicles responding to the current signal indication is formulated analytically as a function of the start of green time and the detection time. The delay and sensitivity of delay difference between the simpler Vertical queueing model and the more detailed KCS traffic model are compared.

2. KINEMATIC EQUATIONS IN PHYSICS

Kinematics is the study of motion irrespective of the forces; it deals with the mathematical description of motion in terms of position, speed, acceleration (or braking) and time. If any three of those variables are known, then the fourth variable can be calculated by using Kinematic equations. According to these concepts, we can describe the motion of vehicles in the vicinity of signalised intersections. If a vehicle is moving, the speed v is defined as the displacement of the vehicle divided by the time over which the displacement occurs. Furthermore, acceleration rate a refers to the rate of change of speed over time, which is defined as the change of speed divided by the change of time. The acceleration is equal to the second derivative of x with respect to time t :

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \tag{1}$$

By using the Equation (1), the speed equation as a function of time, the position equation as a function of speed, and the position equation as a function of time are obtained as follows:

$$v_f = v_i + a(t_f - t_i) \quad (2)$$

$$x_f = x_i + \frac{v_f^2 - v_i^2}{2a} \quad (3)$$

$$x_f = x_i + v_i(t_f - t_i) + \frac{a}{2}(t_f - t_i)^2 \quad (4)$$

where v_i is the initial speed, v_f is the final speed, x_i is the initial position, and x_f is the final position.

3. NOTATION

The following notations will be used for the trajectory of the leading vehicle $n=1$ and the following vehicles n ($2 \leq n \leq N$), where N is the serial number of the most recent detected vehicle. Let

a be the acceleration rate ($a>0$),

b be the braking rate ($b>0$),

v_o be the free-flow speed,

t_g be the start of green time(including the reaction time),

v_g be the speed of the leading vehicle $n=1$ at time t_g,

X_g be the position of the leading vehicle $n=1$ at time t_g,

X_d be the position of the detector ($X_d \leq 0$),

X_s be the position of the stop-line ($X_s = 0$),

$\overline{X}_v$ be the maximum boundary of downstream at which any delayed vehicles can regain the free-flow speed ($\overline{X}_v = X_s + v_0^2/2a$),

$\overline{X}_b$ be the maximum boundary of upstream ($\overline{X}_b = X_d - v_0^2/2b$),

t_d^n be the time at which the vehicle n is detected at position X_d,

v_d^n be the speed of the vehicle n at time which it detected at position X_d,

X_b^n be the braking position of the vehicle n ($X_b^n = X_b^{n-1} - L$, where L is the safety margin),

t_b^n be the time at which the vehicle n starts to brake,

v_b^n be the speed of the vehicle n at position X_b^n ($v_b^n = v_0$)

t_a^n be the time at which the vehicle n starts to accelerate ($2 \leq n \leq N$),

v_a^n be the speed of the vehicle n at time which the acceleration starts($2 \leq n \leq N$),

X_a^n be the position of vehicle n starts to accelerate ($2 \leq n \leq N$),

X_q^n be the position at which the vehicle n stops completely after braking ,

t_q^n be the time at which the vehicle n stops at position X_q^n ,

X_v^n be the position at which the vehicle n regains the free-flow speed, if it has been delayed ($X_v^n \leq \overline{X}_v$),

t_v^n be the time at which the vehicle n regains the free-flow speed at position X_v^n,

t_s^n be the time at which the vehicle n crosses the stop-line X_s ,

v_s^n be the speed of the vehicle n at time which it crosses the stop-line X_s,

4. KINEMATIC TRAFFIC MODEL DEVELOPMENT

In this section, trajectory equations as a function of start of green time t_g are proposed in two vehicle groups: a trajectory equation for the leading vehicle and a trajectory equation for the following vehicles. This model assumes a constant acceleration rate and braking rate, no overtaking is allowed and no vehicle exceeds the free-flow speed under any circumstances. Here, the start of green time t_g is defined as the beginning of the green time plus a reaction time τ, Gipps (1981) used τ = 2/3 seconds.

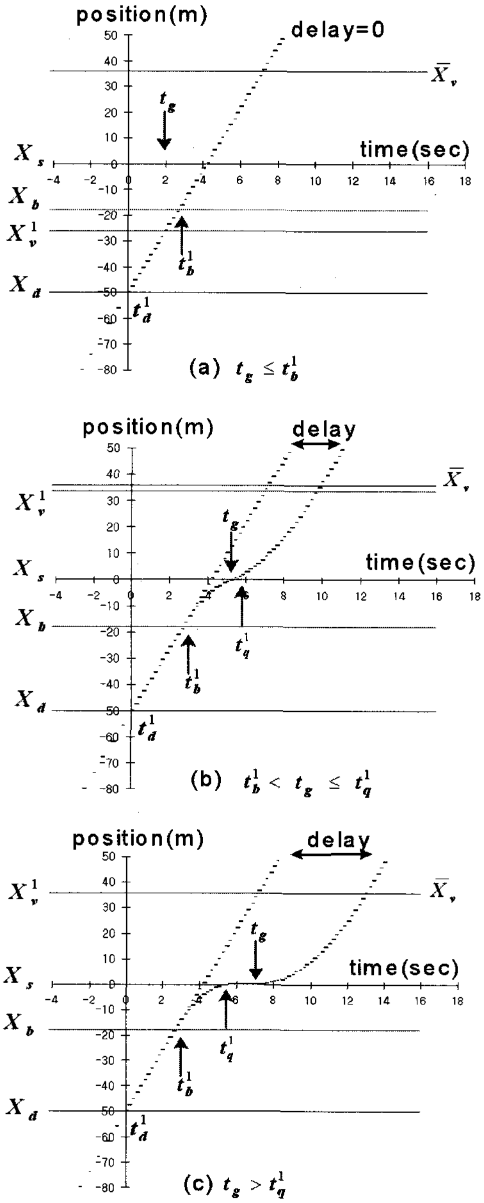


Fig 1. Trajectory of the leading vehicle in relation to varying start of green time t_g

4.1 Trajectory Equations for the Leading Vehicle

As can see in Fig 1, the motion of the leading vehicle at signalised intersection is affected by the current signal indication. In respect of the start of green time t_g, the motion can be classified broadly into four regions: free-flow, braking (or braking and stopping), acceleration and free-flow. If current signal is green, the leading vehicle will cross the detector with free-flow speed and then reaches stop-line without experiencing any delays. However, if the signal is currently red, the vehicle from the detector can travel up to the braking position with free-flow speed, and on reaching that point the vehicle has to start to brake in order to stop safely at the stop-line. Meanwhile, if the signal changes to green, the vehicle will start to accelerate until it regains the free-flow speed; otherwise, the vehicle has to go through stopping until the next green starts. In this way, we can identify that the vehicle will pass the stop-line either at the free-flow speed or under. For the leading vehicle n =1, three different trajectories are considered as follows:

If $t_g \le t_b^n$: maintain free-flow speed (see Fig. 1a),

If $t_b^n \le t_g \le t_q^n$: free-flow→braking→acceleration→ free-flow (see Fig. 1b),

If $t_g > t_q^n$: free-flow→braking→stopping→ acceleration→free-flow(see Fig. 1c).

The motion of the leading vehicle from the detector position up to the braking position is unaffected by the current signal indication, thus it maintains free-flow speed. For the leading vehicle n=1, the braking position X_b^n, the braking time t_b^n and the stopping time t_q^n are calculated as follows:

$$X_n^b = -v_0^2 / 2b \tag{5}$$

$$t_b^n = t_d^n - \frac{v_0}{2b} - \frac{X_d}{v_0} \tag{6}$$

$$t_q^n = t_b^n + \Delta t_b^n = t_d^n + \frac{v_0}{2b} - \frac{X_d}{v_0} \text{(where } \Delta t_b^n = \frac{v_0}{b}) \tag{7}$$

When the vehicle has reached its braking position, its further motion is determined by the current signal indication at the intersection. As can see in Fig. 1, there are three different trajectories to be considered with respect to the start of green time t_g for the vehicle n=1:

If $t_g \le t_b^n$ then $v_g = v_0$, $X_g = X_d + v_0(t_g - t_d^n)$,

$$t_v^n = t_g \text{ and } X_v^n = X_g \tag{8}$$

If $t_b^n < t_g < t_q^n$ then

$$v_g = v_0 - b(t_g - t_b^n),$$

$$X_g = X_b^n + v_0(t_g - t_b^n) - \frac{b}{2}(t_g - t_b^n)^2,$$

$$t_v^n = t_g + \frac{b}{a}(t_g - t_b^n) \text{ and}$$

$$X_v^n = X_b^n + v_0(\frac{a+b}{a})(t_g - t_b^n) - (\frac{a+b^2}{a})(t_g - t_b^n)^2 \tag{9}$$

If t_g starts after the vehicle has stopped $(t_g > t_q^n)$, we can simply assume that

$$v_g=0,\ X_g=0,\ t_v^n = t_g + \frac{v_0}{a} \text{ and } X_v^n = \frac{V_0^2}{2a} \tag{10}$$

Finally, for the leading vehicle n=1, the crossing time t_s^n and its speed v_s^n at the stop-line can be obtained by comparing the fixed position X_s and the varying position X_v^n :

If $X_v^n \le X_s$, which means that the vehicle is crossing the stop-line with free-flow speed, then

$$t_s^n = t_v^n - \frac{X_v^n}{v_0} \text{ and } v_s^n = v_0 \tag{11}$$

If $X_v^n > X_s$, which means that the vehicle is crossing the stop-line during the acceleration, then

$$v_s^n = \sqrt{v_g^2 - 2aX_g} \text{ and } t_s^n = t_g + \frac{\sqrt{v_g^2 - 2aX_g - v_g}}{a} \tag{12}$$

4.2 Trajectory Equations for the Following Vehicles

The variables estimated for the leading vehicle are used as parameters for the following vehicles trajectory in the following section. The basic concept we use in the following calculations are the following vehicles cannot depart the downstream free-flow position $\overline{X}_v$ with less than the minimum headway. Once we have characterised the full trajectory of the leading vehicle n=1 as a function of the start of green time t_g, the trajectory of all successive following vehicles n=2,3,$\cdots$ N can be calculated directly based on the motion in front.

As can see in Fig 2, when the following vehicle n=2 crosses the detector at time t_d^n,, the first order of task is finding the possible departure time $\bar{t}_v^n = Max[\hat{t}_v^n, \bar{t}_{d,v}^n]$ at position $\overline{X}_v$, here $\hat{t}_v^n = \bar{t}_v^{n-1} + 1/s$ (where s is a saturation flow) is the earliest departure time at position $\overline{X}_v$ and $\bar{t}_{d,v}^n = t_d^n + (\overline{X}_v - X_d)/v_0$ is the free-flow travel time from the detector to the position $\overline{X}_v$. By comparing variables $\hat{t}_v^n$ and $\bar{t}_{d,v}^n$, we can decide whether or not the following vehicle will be delayed. If $\bar{t}_{d,v}^n \ge \bar{t}_v^n$, the detected vehicle is identified as undelayed so it can reach the position $\overline{X}_v$ from the detector with free-flow speed. Thus, it is not necessary to find acceleration variables. However, if $\bar{t}_{d,v}^n < \bar{t}_v^n$, the vehicle is identified as delayed, then we need standard motion test to find acceleration variables, in which we can test whether or not any stopped lost time due to a queue is involved. Here, the standard arrival time is the longest approach time from the braking position X_b^n to the position $\overline{X}_v$ supposing that the vehicle has not stopped. The braking position is calculated by of adding minimum safe spacing *L*. The final information we are seeking for each following vehicle $n(2 \le n \le N)$ is its crossing time t_s^n, speed v_s^n at stop-line X_s and departure time $\bar{t}_v^n$ at position $\overline{X}_v$: the time t_s^n will be used to estimate the number of vehicles that can pass the stop-line if the green is extended by a certain control decision time, and $\bar{t}_v^n$ will be used to estimate its delays. In this analysis, the final information we are seeking for each following vehicle $n(2 \le n \le N)$ is its crossing time t_s^n, speed v_s^n at stop-

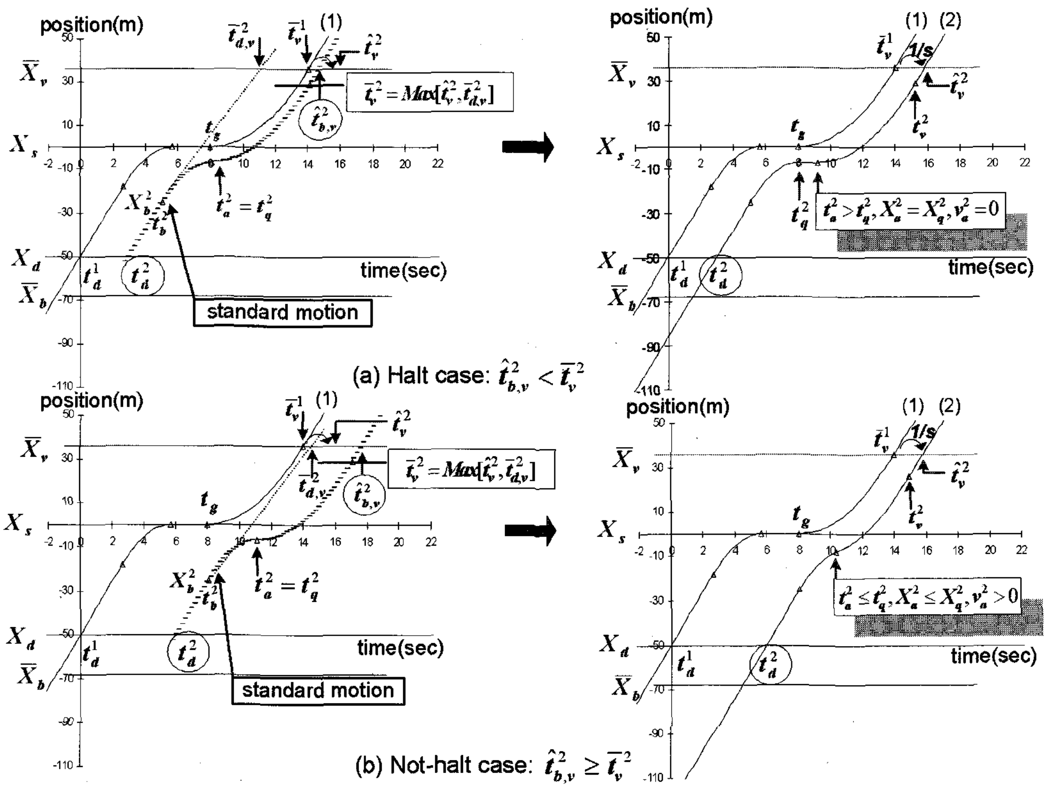


Fig 2. A Standard motion test for the following vehicles

line X_s and departure time $\bar{t}_v^n$ at position $\bar{X}_v$: the time t_s^n will be used to estimate the number of vehicles that can pass the stop-line if the green is extended by a certain control decision time, and $\bar{t}_v^n$ will be used to estimate its delays. The calculation algorithm for the following vehicles trajectory is as follows:

Step 0 : (Leading vehicle trajectory)

Identify the leading vehicle trajectory $n=1$ with respect to the current signal indication, and all identified variables are used as parameters in the following steps

⇒ if $N=1$, stop processing (no following vehicles)

Step 1 : (Departure time at $\bar{X}_v$)

Find the earliest departure time $\hat{t}_v^n$ and free-flow arrival time $\bar{t}_{d,v}^n$ at position $\bar{X}_v$, then the departure time will be $\bar{t}_v^n = Max[\hat{t}_v^n, \bar{t}_{d,v}^n]$, where $\hat{t}_v^n = \bar{t}_v^{n-1} + 1/s$ and $\bar{t}_{d,v}^n = t_d^n + (\bar{X}_v - X_d)/v_0$

Step 2 : (Motion definition for undelayed case or delayed case)

Find the expected motion of the each following vehicle:

2-1) If $\bar{t}_{d,v}^n \geq \bar{t}_v^n$ which corresponds to undelayed motion, then standard motion test is not required

2-2) If $\bar{t}_{d,v}^n < \bar{t}_v^n$ which corresponds to delayed motion, then standard motion test is required

Step 3 : (Braking position and time)

$X_b^n = X_b^{n-1} - L(t_b^n$ is only possible to find when $X_b^n \geq \overline{X}_b$; otherwise, out of range)

From Step 2, if the motion is identified as an undelayed case and:

If $(X_b^n \geq \overline{X}_b)$, then $t_b^n = t_d^n + (X_b^n - X_d)/v_0$,

Else, t_b^n is unknown

⇒ **go to Step 4** to find the variable of crossing time to the stop-line.

From Step 2, if the motion is identified as a delayed case :

If $X_d \leq X_b^n < X_s$, then $t_d^n = t_d^n + (X_b^n - X_d)/v_o$

If $\overline{X}_b \leq X_b^n < X_d$, then

$$t_b^n = t_d^n - (v_0 - \sqrt{v_0^2 + 2b(X_b^n - X_d)})/b$$

⇒ **go to Step 5** to find additional variables.

Step 4 : (Crossing time to the stop-line for undelayed vehicle)

The crossing time t_s^n at stop-line X_s can be calculated by using a free-flow motion equation, that is $t_s^n = t_d^n + (X_s - X_d)/v_0$

⇒ if $n < N$, **go to Step 1**; otherwise, stop processing

Step 5 : (Standard motion test for delayed vehicle: halt case or not-hale case)

The standard motion test is only necessary if the vehicle is identified as delayed and $X_b^n \geq \overline{X}_b$

From the braking position standard motion test is needed to find whether or not it will come to a halt

5-1) The standard motion arrival time, that is

$$\hat{t}_{d,v}^n = t_b^n + v_0(a+b)/2ab + (\overline{X}_v - X_b^n)/v_0$$

5-2) Motion definition: halt-case or not-halt case

If $\hat{t}_{d,v}^n < \bar{t}_v^n$, the motion is identified as a halt-case, then the stopping is involved

If $\hat{t}_{d,v}^n \geq \bar{t}_v^n$, the motion is identified as a not-halt case, then the stopping is not involved

Step 6 : (Acceleration position, time and speed variables for delayed vehicle)

6-1) For a halt-case, acceleration position is equal to the stopped position, then

$$X_a^n = X_q^n (\text{where } X_q^n = X_b^n + \frac{v_0^2}{2b}),\quad v_a^n = 0 \quad \text{and}$$

$$t_a^n = \bar{t}_v^n - \frac{v_0}{2a} - \frac{(\overline{X}_v - X_a^n)}{v_0}$$

6-2) For a not-halt case, acceleration starts meanwhile of braking, then

$$t_a^n = t_b^n + \sqrt{\frac{(X_b^n - \overline{X}_v) + v_0(\bar{t}_v^n - t_b^n)}{(ab+b^2)/2a}}$$

$$X_a^n = X_b^n + v_0(t_a^n - t_b^n) - \frac{b}{2}(t_a^n - t_b^n)^2 \quad \text{and}$$

$$v_a^n = v_0 - b(t_a^n - t_b^n)$$

Step 7 : (Position and time of regaining the freeflow speed for delayed vehicle)

If $X_b^n \geq \overline{X}_b$, the time t_v^n and the position X_v^n can be calculated by using acceleration variables; otherwise, they can be calculated corresponding to the departure time $\bar{t}_v^n$ at $\overline{X}_v$

7-1) If $(X_b^n \geq \overline{X}_b)$, $t_v^n = t_a^n + (v_0 - v_a^n)/a$ and

$$X_v^n = X_a^n + v_a^n(t_v^n - t_a^n) + \frac{a}{2}(t_v^n - t_a^n)^2$$

7-2) If $(X_b^n < \overline{X}_b)$ which is defined as a out of boundary case, then

$$t_v^n = t_d^n + \sqrt{\frac{2[(X_d - \overline{X}_v) + v_0(\bar{t}_v^n - t_d^n)]}{a}} \quad \text{and}$$

$$X_v^n = X_d + v_d^n(t_v^n - t_d^n) + \frac{a}{2}(t_v^n - t_d^n)^2$$

where the estimated speed of $v_d^n = v_0 - a(t_v^n - t_d^n)$

Step 8 : (Crossing time and speed to the stop-line for delayed vehicle)

The departure time t_s^n at the stop-line X_s can be calculated by comparing position variables X_s and X_v^n

8-1) If ($X_b^n \ge \overline{X}_b$) and

If $X_v^n \le \overline{X}_s$, than $t_s^n = t_v^n - (X_v^n / v_0)$

If $X_v^n > \overline{X}_s$, than $t_s^n = t_a^n + (v_s^n - v_a^n)/a$,

where $v_s^n = \sqrt{(v_a^n)^2 - 2aX_a^n}$

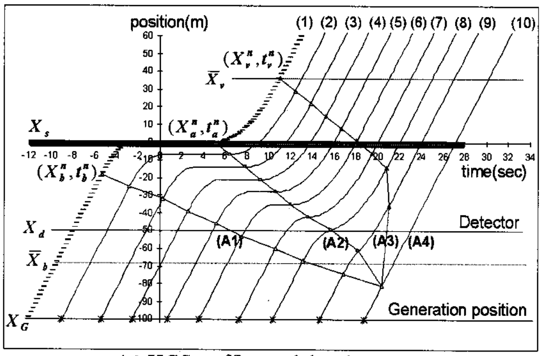
8-2) If $(X_b^n < \overline{X}_b)$ and

If $X_v^n \le X_s$, than $t_s^n = t_v^n - (X_v^n / v_0)$

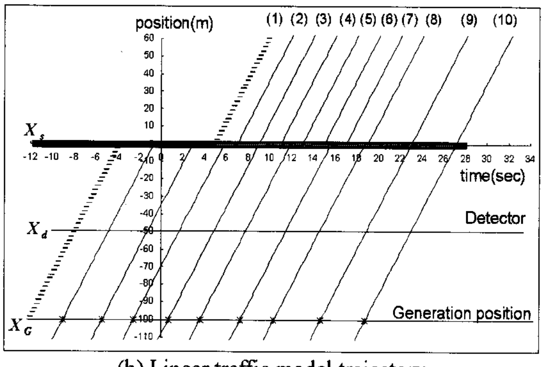
If $X_v^n > X_s$, than $t_s^n = t_v^n + (v_0 - v_s^n)/a$,

where $v_s^n = \sqrt{(v_d^n)^2 - 2aX_d}$

$\Rightarrow$if $n < N$ **go to Step 1**; otherwise, stop processing.



(a) KCS traffic model trajectory



(b) Linear traffic model trajectory

Fig. 3 Trajectory of following vehicles

As can see in Fig. 3a, vehicles are generated from the position X_G that is located further upstream than the detector. There are four different sets of motions have been identified at the position of the detector during one green period. Position and variables are expanding backwards from the first vehicle's trajectory through the upstream of the detector. In respect of the start of green time, any detected vehicles until the time A1 have crossed the detector with free-flow speed, the times between A1 and A2 vehicles have crossed it while braking, the times between A2 and A3 vehicles have crossed it while accelerating, and any following vehicles after the time A4 will cross the detector with free-flow speed. Presumably, the time A4 is the queue (or delay) dissipation time of this stage.

5. DELAY AND SENSITIVITY OF DELAY ANALYSIS

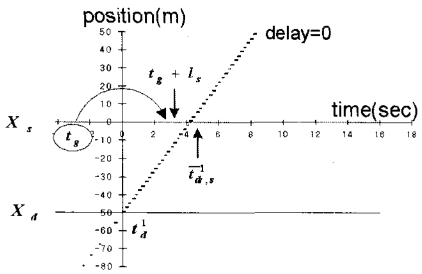
In this section, the sensitivity of delay difference between the Vertical queueing model and the KCS traffic model is discussed in relations to varying start of green time t_g . The delay in the Vertical queueing model is identified as the time difference between the queue departure time and its free-flow travel time to the stop-line. In this model, the departure time of the leading vehicle is assumed to coincide with the start of the effective green time and queue forms vertically at the stop-line without occupying any space on the link. The delay in the KCS traffic model is calculated on the basis of the time t_g in accordance with vehicular characteristic variables, such as, acceleration rate a, braking rate b, free-flow speed v_o and physical queue length L.

5.1 Delay and Sensitivity of Delay for the Leading Vehicle

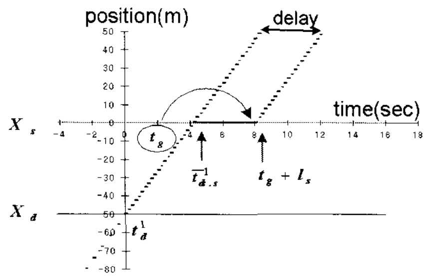
As can see in Fig. 4, the delay in the Vertical Queueing model is identified as the time difference between the start of effective green time $t_g + l_s$(where $l_s = v_0 / 2a$) and its free-flow travel time to the stop-line $\bar{t}^n_{d,s} = t^n_d - X_d / v_0$ The Vertical queueing model delay for the leading vehicle n=1 with respect to the time t_g is expressed as $D^n_V(t_g)$, which are calculated as follows :

$$\text{If } (t_g + l_s \le \bar{t}^n_{d,s}), \text{ than } D^n_V(t_g) = 0 \tag{13}$$

$$\text{If } (t_g + l_s > \bar{t}^n_{d,s}), \text{ than } D^n_V(t_g) = (t_g + l_s) - \bar{t}^n_{d,s} \tag{14}$$



(a) $t_g + l_s \le \bar{t}^1_{d,s}$

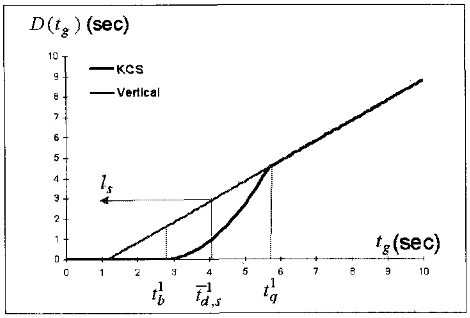


(b) $t_g + l_s > \bar{t}^1_{d,s}$

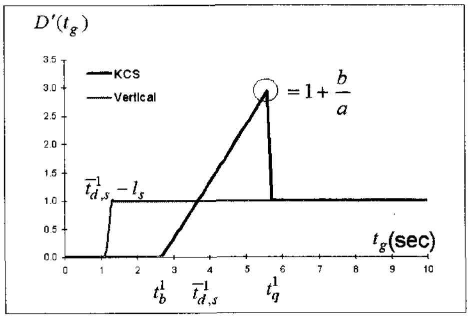
Fig 4. Delay for the leading vehicle (Vertical queueing model)

As can see in Fig. 5, if t_g starts before the free-flow travel time minus the start lag, no delay is incurred in the Vertical queueing model, and the resulting sensitivity is 0. However, from that time if t_g is increased by an amount ε, delay is increased by an identical amount ε, so that the resulting sensitivity is 1. By differentiating the delay Equation 13 and 14 with respect to the time t_g, we can get the sensitivity of delay as follows:

$$D^{n'}_V(t_g) = \begin{Bmatrix} 0 \; t_g \le \bar{t}^n_{d,s} - l_s \\ 1 \; t_g > \bar{t}^n_{d,s} - l_s \end{Bmatrix} \tag{15}$$



(a) Delay



(b) Sensitivity of delay

Fig 5. Delay and Sensitivity of delay for the leading vehicle in varying time t_g

As seen in Fig. 1, the delay for the KCS traffic model is calculated on the basis of the time t_g in accordance with vehicular characteristic variables, such as, acceleration rate, braking rate and free-flow speed. The delay for the leading vehicle n=1 with respect to the time t_g is expressed as $D^n_K(t_g)$, which are calculated as follows:

$$\text{If } t_g \le t^n_b, \text{ than } D^n_K(t_g) = 0 \tag{16}$$

If $t^n_b \le t_g \le t^n_q$, than

$$D^n_K(t_g) = (\bar{t}^n_b - \bar{t}^n_d) - \frac{(X^n_b - X_d)}{v_0} + \frac{(ab + b^2)}{2av_0}(t_g - \bar{t}^n_b)^2 \tag{17}$$

If $t_g > t^n_q$, than

$$D_K^n(t_g) = (t_g - \bar{t}_d^n) + \frac{v_0}{2a} + \frac{X_d}{v_0} \quad (18)$$

As can see in Fig. 5, if t_g starts before braking, no delay is incurred and the resulting sensitivity is 0. If t_g starts between braking and stopping, the delay increases quadratically, and the resulting sensitivity increases linearly. Once the vehicle has stopped, if t_g is increased by an amount ε, delay is increased by an identical amount ε, so that the resulting sensitivity is 1. By differentiating the delay Equations 16, 17 and 18 with respect to the varying time t_g, we can get the sensitivity of delay as follows:

If $t_g \le t_b^n$, than $D_K^{n'}(t_g) = 0$ (19)

If $t_b^n \le t_g \le t_q^n$, than

$$D_K^{n'}(t_g) = \frac{(ab+b^2)}{av_0}(t_g - t_b^n) \quad (20)$$

If $t_g > t_q^n$, than $D_K^{n'}(t_g) = 1$ (21)

The maximum sensitivity of delay for the leading vehicle n=1 in the Vertical Queueing model is always 1. In contrast, the maximum sensitivity of delay in the KCS traffic model is $1+(b/a)$ that is obtained by differentiating the t_v^1 in Equation 9 that obtained when $t_g = t_q^1$.

5.2 Delay and Sensitivity of Delay for the Following Vehicles

In this section, the following results are based on the simulation. Two different cases of traffic are considered: a low density case and a high density case. In the present examples, the maximum number of vehicles that can be held in the queue with this given condition is 8-vehicle.

The detector is located X_d = -50m from the stop-line, and the minimum spacing of each following vehicle is

Table 1. Detection time generation : using shift exponetial distribution of headways

		Vehicle n =							
		1	2	3	4	5	6	7	8
Low density	U	-	0.613	0.321	0.824	0.569	0.851	0.312	0.67
a=0.5	H	-	3.97	5.27	3.38	4.12	3.32	5.33	3.79
h_0=3.0	t_d^n	0.0	4.0	9.3	12.7	16.8	20.1	25.4	29.2
High density	U	-	0.613	0.321	0.824	0.569	0.851	0.311	0.671
a=0.9	H	-	2.54	3.26	2.21	2.62	2.18	3.29	2.44
h_0=2.0	t_d^n	0.0	2.5	5.8	8.0	10.6	12.8	16.1	18.5

L= 7m, saturation departure time is 1.8 sec/vehicle (2,000 vehicles/hour) and two cases of vehicles are generated based on *shifted exponential distribution of headways H*, which is given by

$$H = h_0 - \frac{1}{a}\ln(u) \quad (22)$$

where

u : is the random value generation, in which variables are generated with equal probability between [0, 1],

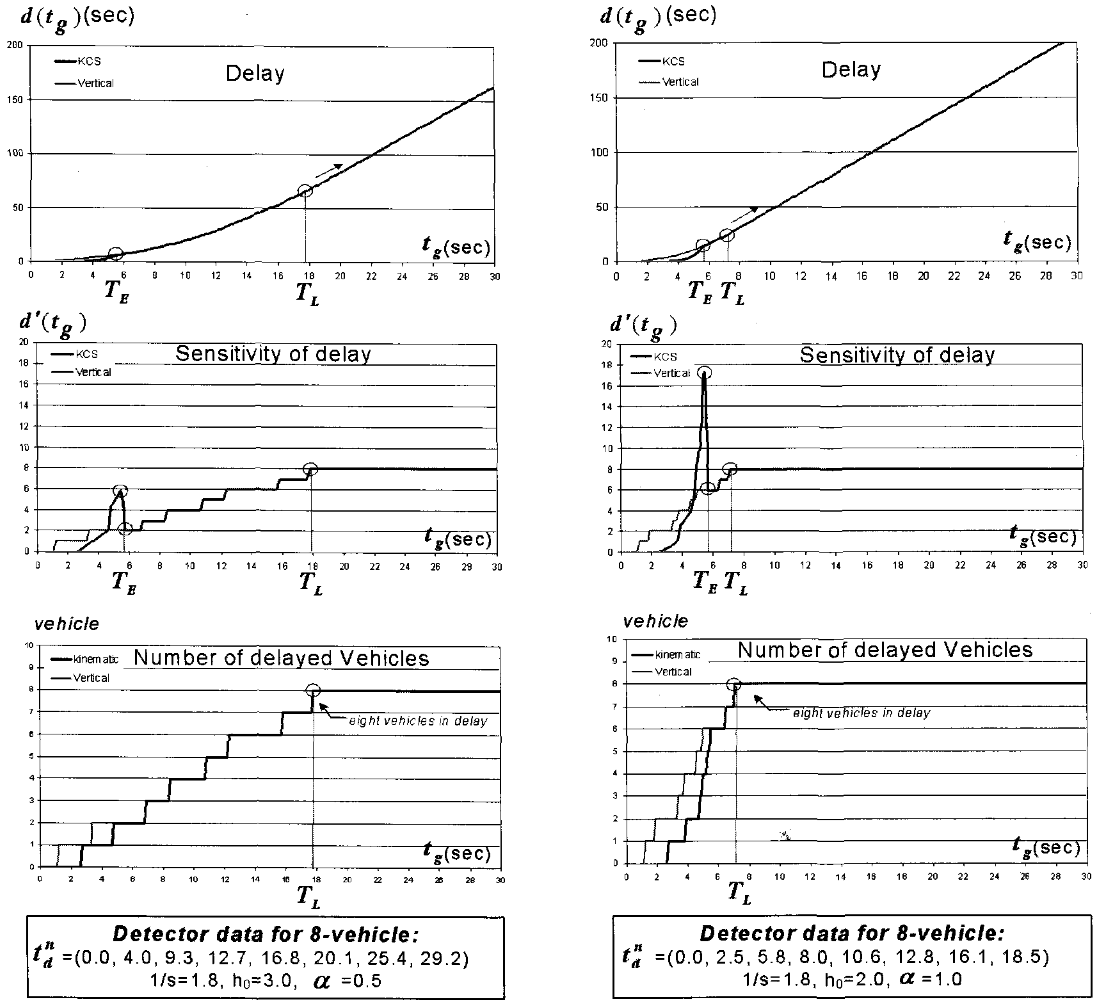
h_0 : is the minimum gap of following ($h_0 \ge \tau + L/v_0$, where τ is 0.67sec),

a : is the density parameter (the bigger a generates greater flow).

Using Equation 22, we can generate the detection time of vehicles without causing any headway violation. The time t_d^n can be given by $t_d^n = t_d^{n-1} + H(n \ge 2)$.

Based on Table 1 data, the sensitivity of delay for 8-vehicle is tested with respect to the variations in the start of green time: in the range of $t_g = 0 \sim 30$ sec and t_g is incremented by 0.1 sec.

As can see in Fig. 6, let T_E be the time at which the total delay of eight vehicles for the KCS traffic model and the Vertical queueing model become same, namely $T_E = t_q^1$, where T_L is the time at which total delay becomes increasing linearly. If t_g starts before T_L, which means that all eight vehicles have not delayed yet;



(a) Low density case (b) High density case

Fig 6. Sensitivity of delay for the following vehicles in varying time t_g

in this range, we can suppose that some vehicles may go through the stop-line without experiencing any delay, thus the sensitivity fluctuates. However, if t_g starts after T_L, which means that all eight vehicles have been delayed, the sensitivity of delay in this range with respect to t_g is equal to the total number of delayed vehicles. The sensitivity of delay for the Vertical queueing model is equal to the number of delayed vehicles in the time range. But it differs for the KCS traffic model, if t_g starts before T_E, in some time range it shows a sensitivity higher than the total number of delayed vehicles. For the high density case simulation (see Fig. 6b), the Vertical queueing model delay is greater than or equal to the KCS traffic model delay. T_E=5.7 sec and T_L=7.1 sec. At time t_g = 5.5 sec, the KCS traffic model shows a sensitivity of delay D_K= 17.28, and the Vertical queueing model shows D_V = 6, which is equivalent to numbers of delayed vehicles. At this time, the sensitivity of delay for the

KCS traffic model is about three times greater than for the Vertical queueing model. At time $t_g = 7.1$ sec, the sensitivity of delay for both models is equal to 8. From that time, we can see that all vehicles will experience some delay.

6. CONCLUSIONS

As explored in this paper, the delay estimated in the Vertical queueing model and the KCS traffic model differs around the time of the green start, but so long as there is a queue the delay estimates from these models are identical. With respect to variations in the start of green time and provided that the braking rate exceeds the acceleration rate, the delay estimated in the Vertical queueing model is always greater than or equal to the KCS traffic model; however, the maximum sensitivity of delay in the KCS traffic model is greater than that in the vertical queueing model. The KCS traffic model proposed has some attractions compared to the simpler Vertical queueing model. The delay estimate in the KCS traffic model is on the basis of the vehicular characteristics, such as vehicle length, acceleration rate, braking rate and free-flow speed; however, the Vertical queueing model takes adjusted time parameter of the start of effective green time. Thus, the delay estimated from the Vertical queueing model is not realistic and always greater than or equal to the KCS traffic model. From these results, it is clear that the leading vehicle trajectory is the most important determinant for estimating the motion of vehicles at signalized intersections. Namely, the delay can be minimized in signal operation, if the start of green time begins before the vehicle starts to brake. The work described so far in this study is only at on early stage, in the sense that more work should be done for estimating the motion of vehicles in various intersection configurations.

References

Ahn, W-Y. (2004) "Dynamic signal optimisation for isolated road junctions", *Ph.D. thesis, University College London.*

Allsop, R.E. (1970) "Optimisation techniques for reducing delay to traffic in signalised road junction". *Ph.D. thesis, University College London.*

Bang, K.L. (1976) "Optimal control of isolated traffic signals", *Traffic Engineering and Control, 17 (7),* pp. 288-292.

Clayton, A.J.H. (1940) "Road traffic calculation", J. Inst. *Civil Engineering. 16 (7),* pp.247-284.

Gartner, N.H. (1983) "OPAC: A demand-responsive strategy for traffic signal control", *TRB, Transportation Research Record 906,* pp.75-84.

Gipps, P.G. (1981) "A behavioural ca-following model for computer simulation", *Transportation Research, 15(2),* pp105-111.

Heydecker, B.G. (1990) "A continuous-time formulation for traffic-responsive signal control", *Proceedings of the 11th International Symposium on Transportation and Traffic Theory, Yokohama,* pp. 599-618.

Miller, A. J. (1963) "A computer system for traffic networks", Proceedings of the Second International Symposium on the Theory of Road traffic Flow (ed. Almond, J.), *OECD, Paris,* pp.200-220.

Robertson, D.I. (1969) "TRANSYT: a traffic network study tool", *RRL Report, LR253.*

Webster, F.V. and Cobbe, B.M. (1966) "Traffic signals", *HMSO, London.*

접 수 일 : 2007. 5. 28
심 사 일 : 2007. 5. 31
심사완료일 : 2007. 6. 25