

# Performance of DCSK under the Coexistence of non-Chaotic Transmit Reference System

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## ABSTRACT

In wireless communications, chaotic communications have been a field of interest due to its low complexity in hardware implementation and low power consumption in chaotic signal generation. Among the modulation schemes using the chaotic signal, Differential Chaos Shift Keying (DCSK) is a robust non coherent technique. As in the conventional communication systems, chaos-based systems are required to provide reasonable bit error performance in the presence of a narrow-band signal coming from any other systems. The frequency band of this foreign narrow band signal may lie within the bandwidth of the chaos-based systems. This situation may occur when chaotic signal transmission is done in the presence of other conventional communication system. This paper has evaluated the performance of the non coherent differential chaos shift keying (DCSK) system under the presence of conventional non-chaotic transmit reference system. Both systems are assumed to have same data rates. The mathematical expressions for the bit error rate (BER) are derived with computer simulations to verify the analytical results.

**Key Words** : Chaotic communications, Differential chaos shift keying (DCSK), Conventional communications, Transmit reference system.

## I. Introduction

Chaotic communications have been a subject of major interest in the field of wireless communications due to the wideband spectrum, random like signals, non-repetition of the signals, easiness of generating the chaotic signals, low power consumption property and less complex system. In chaotic communications, the information is directly mapped to some property of a chaotic signal thus avoiding the need of additional spectrum spreading. Other significant advantages of chaotic communications are no use of carriers and the possibility of highly secured communications.

Many chaos based communication systems have been proposed as chaos shift keying (CSK), chaos frequency modulation (CFM), chaos pulse position modulation (CPPM)<sup>[2],[3],[7]</sup> direct chaotic communications (DCC)<sup>[6]</sup> and so on . For our analysis, differential chaos shift keying (DCSK) modulation scheme has been considered.

In this paper, we have considered the problem of coexistence of chaotic system with other conventional narrow band system. Here a non-chaotic transmit reference system running in parallel with the given DCSK system is considered as the source of interference to our DCSK system. Also the interfering system has bandwidth lying within the bandwidth of the DCSK system.

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## II. System Description

In DCSK for the transmission of one bit information, two chaotic signals are sent for each symbol period <sup>[3],[4]</sup>. The first signal is used as a reference signal whereas the second signal is the signal bearing information. The information bit is extracted at the receiver by differentially coherent demodulation.

Transmission of reference chip via the same channel is generally considered as a loss in transmitted energy per bit. This may be valid for the AWGN channel only. But real channels have linear or non-linear distortion and the modulated carrier should be correlated with a reference signal distorted in the same manner as the modulated carrier to obtain the best system performance. And correlation with original distortion free reference results in performance degradation. The reference signal can be considered as a test signal used to measure the channel characteristics.

In our proposed analysis, we have considered a foreign transmit reference system interfering with DCSK system with same data rate. The transmit reference system we have considered has the similar transmitter and receiver structure as that of DCSK. The only difference is the nature of the signal. In DCSK, the transmitted signal for each bit is chaotic in nature while in conventional transmit reference system non chaotic sinusoidal pulse as used in BPSK is used. We have made some mathematical analysis on BER performances of DCSK system in the presence of other transmit reference system interfering with it and computer simulations are also done to verify our result.

Fig.1 shows the structure of DCSK combined with non chaotic system.

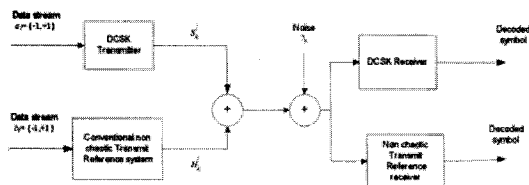


Fig. 1 Conventional DCSK system combined with conventional non-chaotic transmit reference system.

### 2.1 DCSK Modulation

The discrete chaotic sequence  $s_k^l$  for  $l$ th bit is represented as:

$$s_k^l = \begin{cases} x_k & \text{for } k = 2\beta(l-1) + 1, 2\beta(l-1) + 2, \\ & \dots, 2\beta(l-1) + \beta \\ \alpha_l x_{k-\beta} & \text{for } k = 2\beta(l-1) + \beta + 1, \\ & 2\beta(l-1) + \beta + 2, \dots, 2\beta l \end{cases}, \quad (1)$$

where,  $2\beta$  is defined as the spreading factor which is the number of chaotic samples used to transmit one binary symbol and  $\alpha_l = \{-1, +1\}$ , denoting the symbol to be sent in the  $l$ th bit period. If the information bit is 1, then  $\alpha_l = +1$  and if the information bit is 0, then  $\alpha_l = -1$  which is the basic principle of DCSK system.  $x_k$  is a chaotic signal used in DCSK which is generated by a chaotic map. Here a logistic map is used for chaos generation which is of the form:

$$x_{k+1} = g(x_k) = 1 - 2x_k^2. \quad (2)$$

For bit '1' the modulator as shown in Fig.2(a) transmits the same chaotic signal twice in succession, while for bit '0' it transmits the reference signal and the message signal, where the message signal is the chaotic signal delayed by  $\beta$  time period as compared to the reference signal and is the inverted replica of the reference signal, i.e., reference signal multiplied by '-1'.

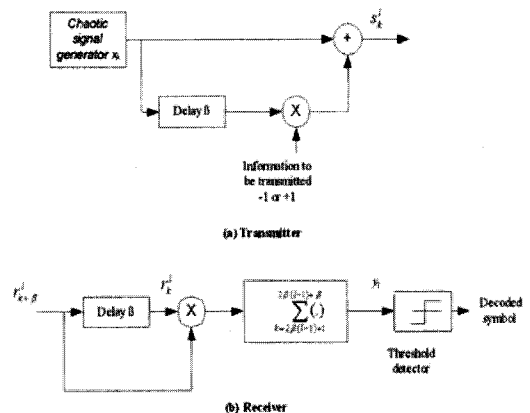


Fig. 2 Block diagram of a non-coherent DCSK system.

### 2.2 Combined DCSK and non-chaotic transmit reference system

Here in our proposed system, we have considered the coexistence of a DCSK system with the non chaotic system having its bandwidth lying within the bandwidth of DCSK system.

As for the interfering non-chaotic system, we have considered a conventional transmit reference system<sup>[5]</sup>. In this system, the transmitted signal for the  $l$ th bit is given as:

$$u_k^l = \begin{cases} \sqrt{p_b} & \text{for } k = 2\beta(l-1)+1, 2\beta(l-1)+2, \\ & \dots, 2\beta(l-1)+\beta \\ b_l \sqrt{p_b} & \text{for } k = 2\beta(l-1)+\beta+1, \\ & 2\beta(l-1)+\beta+2, \dots, 2\beta l \end{cases}, \quad (3)$$

where,  $b_l = \{-1, +1\}$  and it denotes the symbol to be sent in the  $l$ th bit period. If the information bit is 1, then  $b_l = +1$  and if the information bit is 0, then  $b_l = -1$ .  $p_b$  denotes the signal power which is assumed to remain constant during the bit interval. The principle of the transmitter and the receiver of the conventional transmit reference system is the same as that of the DCSK system. The only difference is the nature of the signal used for transmitting.

The DCSK and non-chaotic transmit reference signals are combined and corrupted by the channel noise before arriving at the receiver end. Thus the received signal,  $r_{k+\beta}^l$  for the  $l$ th bit is given as:

$$r_k^l = s_k^l + u_k^l + \eta_k, \quad (4)$$

where,  $\eta_k$  is the channel noise. In our computer simulations we have considered an additive white Gaussian noise (AWGN) channel and a two-ray multipath channel.

### 2.3 DCSK Demodulation

Signal recovery as in Fig.2(b) is done in the receiver side by multiplying the received reference and message signals, i.e., signal  $r_k^l$  and the received signal delayed by  $\beta$ ,  $r_{k+\beta}^l$ <sup>[1]</sup>. This output is then sent to the threshold detector where it is

compared to some threshold value and hence the transmitted bit is recovered. The output of the correlator for the  $l$ th bit is given as:

$$y_l = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} (r_k^l r_{k+\beta}^l), \quad (5)$$

$$= \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} [x_k + u_k^l + \eta_k] [\alpha_l x_k + u_{k+\beta}^l + \eta_{k+\beta}],$$

$$= \alpha_l \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k^2 + \sqrt{p_b} (b_l + \alpha_l) \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k + \beta b_l p_b + \sqrt{p_b} \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} (\eta_{k+\beta} + b_l \eta_k) + \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k \eta_{k+\beta} + \alpha_l \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k \eta_k + \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} \eta_k \eta_{k+\beta},$$

$$y_l = \underbrace{\alpha_l A}_{\text{required signal}} + \underbrace{\sqrt{p_b} (b_l + \alpha_l) B + \beta b_l p_b + \sqrt{p_b} C}_{\text{non-chaotic signal}} + \underbrace{D + \alpha_l E + F}_{\text{noise}} \quad (6)$$

Where,

$$A = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k^2, \quad (7)$$

$$B = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k, \quad (8)$$

$$C = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} (\eta_{k+\beta} + b_l \eta_k), \quad (9)$$

$$D = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k \eta_{k+\beta}, \quad (10)$$

$$E = \alpha_l \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k \eta_k, \quad (11)$$

$$F = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} \eta_k \eta_{k+\beta}. \quad (12)$$

Let '+1' be the transmitted bit in both the DCSK and non-chaotic transmit reference system then the output of the correlator is:

$$y_l | (\alpha_l = +1, b_l = +1) = A + 2\sqrt{p_b} B + \beta p_b + \sqrt{p_b} C + D + E + F. \quad (13)$$

Let  $E[y_l | (\alpha_i = +1, b_l = +1)]$  and  $var[y_l | (\alpha_i = +1, b_l = +1)]$  be the mean and variance of the  $l$ th symbol,  $y_l | (\alpha_i = +1, b_l = +1)$ , respectively. The error occurs when  $y_l \leq 0 | (\alpha_i = +1, b_l = +1)$ .

Assuming  $y_l | (\alpha_i = +1, b_l = +1)$  has a normal distribution and hence the error probability is given as [1]:

$$prob(y_l \leq 0 | (\alpha_i = +1, b_l = +1)) = \frac{1}{2} \operatorname{erfc} \left( \frac{E[y_l | (\alpha_i = +1, b_l = +1)]}{\sqrt{2var[y_l | (\alpha_i = +1, b_l = +1)]}} \right). \quad (14)$$

Similarly, for the case of  $\alpha_i = +1$  and  $b_l = -1$ , (6) becomes

$$y_l | (\alpha_i = +1, b_l = -1) = A - \beta p_b + \sqrt{p_b} C + D + E + F. \quad (15)$$

The corresponding error probability is given as:

$$prob(y_l \leq 0 | (\alpha_i = +1, b_l = -1)) = \frac{1}{2} \operatorname{erfc} \left( \frac{E[y_l | (\alpha_i = +1, b_l = -1)]}{\sqrt{2var[y_l | (\alpha_i = +1, b_l = -1)]}} \right). \quad (16)$$

When '+1' or bit '1' is sent by the DCSK system the total probability of error at this condition is given as [1]:

$$\begin{aligned} BER_{DCSK-I} &= prob(b_l = +1) \times prob(y_l \leq 0 | (\alpha_i = +1, b_l = +1)) \\ &+ prob(b_l = -1) \times prob(y_l \leq 0 | (\alpha_i = +1, b_l = -1)), \\ &= \frac{1}{4} \left[ \operatorname{erfc} \left( \frac{E[y_l | (\alpha_i = +1, b_l = +1)]}{\sqrt{2var[y_l | (\alpha_i = +1, b_l = +1)]}} \right) \right. \\ &\quad \left. + \operatorname{erfc} \left( \frac{E[y_l | (\alpha_i = +1, b_l = -1)]}{\sqrt{2var[y_l | (\alpha_i = +1, b_l = -1)]}} \right) \right]. \quad (17) \end{aligned}$$

Similarly, when '-1' or bit '0' is sent by the DCSK system during the  $l$ th symbol duration in the DCSK system, i.e., when  $\alpha_i = -1$  it can be shown that

$$y_l | (\alpha_i = -1, b_l = +1) = -A + \beta p_b + \sqrt{p_b} C + D - E \quad (18)$$

$$y_l | (\alpha_i = -1, b_l = -1) = -A - 2\sqrt{p_b} B - \beta p_b + \sqrt{p_b} C + D - E + F. \quad (19)$$

And hence the error probability when '-1' is sent by DCSK system is given as:

$$\begin{aligned} BER_{DCSK-II} &= prob(b_l = +1) \times prob(y_l > 0 | (\alpha_i = -1, b_l = +1)) \\ &+ prob(b_l = -1) \times prob(y_l > 0 | (\alpha_i = -1, b_l = -1)), \\ &= \frac{1}{4} \left[ \operatorname{erfc} \left( \frac{-E[y_l | (\alpha_i = -1, b_l = +1)]}{\sqrt{2var[y_l | (\alpha_i = -1, b_l = +1)]}} \right) \right. \\ &\quad \left. + \operatorname{erfc} \left( \frac{-E[y_l | (\alpha_i = -1, b_l = -1)]}{\sqrt{2var[y_l | (\alpha_i = -1, b_l = -1)]}} \right) \right], \quad (20) \end{aligned}$$

As both (17) and (20) are independent of  $l$ , the BER of the DCSK system under a combined communication environment,  $BER_{DCSK}$ , which is the overall error probability of the  $l$ th transmitted symbol, is given as:

$$\begin{aligned} BER_{DCSK} &= prob(\alpha_i = +1) \times BER_{DCSK-I} \\ &+ prob(\alpha_i = -1) \times BER_{DCSK-II} \\ &= \frac{1}{2} [BER_{DCSK-I} + BER_{DCSK-II}]. \quad (21) \end{aligned}$$

From (A.21) in the Appendix A, we obtain:

$$\begin{aligned} BER_{DCSK} &= \frac{1}{4} \operatorname{erfc} \left[ \frac{\beta p_s + \beta p_b}{\sqrt{2\beta A + 8\beta p_b p_s + 2\beta p_b N_o + 2\beta p_s N_o}} \right] \\ &+ \frac{1}{4} \operatorname{erfc} \left[ \frac{\beta p_s - \beta p_b}{\sqrt{2\beta A + 2\beta p_b N_o + 2\beta p_s N_o + \frac{\beta N_o^2}{2}}} \right]. \quad (22) \end{aligned}$$

As already mentioned above, we have used a logistic map for the chaos generation which is of the form  $x_{k+1} = g(x_k) = 1 - 2x_k^2$ . From [1] for this map,

$$\begin{aligned} p_s &= E[x_k^2] = \frac{1}{2}, \quad A = var[x_k^2] = \frac{1}{8}, \\ \Omega &= E[x_{k+1} x_k^2] = -\frac{1}{4}. \quad (23) \end{aligned}$$

So, for this particular logistic map the BER for DCSK can be obtained as:

$$BER_{DCSK} \approx \frac{1}{4} \operatorname{erfc} \left[ \frac{0.5p_s + \beta p_b}{\sqrt{0.25\beta + 4\beta p_b + 2\beta p_b N_o + \beta N_o}} \right] + \frac{1}{4} \operatorname{erfc} \left[ \frac{0.5p_s - \beta p_b}{\sqrt{0.25\beta + 2\beta p_b N_o + \beta N_o + \frac{\beta N_o^2}{2}}} \right] \quad (24)$$

(24) gives the mathematical analysis of the BER value of DCSK under its coexistence with the non-chaotic transmit reference system.

### III. Simulation Results

In this paper, we have evaluated the performance of DCSK under its coexistence with the non-chaotic transmit reference system. We have made the assumption of both the systems having the same data rate. The results of our simulations are presented in terms of BER versus the ratio  $E_b/N_o$  expressed in dB, where  $E_b$  is the energy per bit, and  $N_o$  is the single-sided spectral noise density. BER performance is analyzed under the variations of average bit energy to noise spectral density ratio ( $E_b/N_o$ ) and non-chaotic transmit reference signal power to chaotic signal power ratio  $p_b/p_s$ . In our analysis, we have taken the chaotic signal samples of spreading factor 50. We have shown the plot of analytical result as obtained in (24) and computer simulation results. The computer simulations are done under the additive white Gaussian noise (AWGN) channel and the two-ray multipath channel.

We can see from our simulation and analytical results as in Fig.3, Fig.4 and Fig.5 that for any given  $E_b/N_o$  the BER of the DCSK system increases with the increase in  $p_b/p_s$ . This is because of the reason that as  $p_b/p_s$  increases the signal power of the non-chaotic transmit reference system also increases and thus causing more interference to the DCSK system and increasing BER. We can see from Fig.3, Fig.4 and Fig.5 that the results with  $p_b/p_s$  of -15dB and -5dB are acceptable but for higher values of 5dB and 15

dB they give much worse BER performances. From Fig.3, it is observed that for the analytical result,  $BER=10^{-3}$  at  $E_b/N_o=-15$ dB while from the simulation result of Fig.4, it is observed that under AWGN channel  $BER=10^{-3}$  at  $E_b/N_o=-15.8$ dB for  $p_b/p_s=-15$ dB. This difference in value of  $E_b/N_o$  between theoretical result and simulation result may be because of the channel condition. Comparing Fig.3 with Fig.5, we can see that the influences of the non-chaotic transmit reference system highly degrades the system performance of DCSK system under the two-ray multipath channel. We note from Fig.3, Fig.4 and Fig.5 that at the condition of no interference from foreign non-chaotic signal, i.e., at  $p_b=0$ , the BER performance is similar to that of the condition when  $p_b/p_s=-15$ dB. This is because of the reason that at  $p_b/p_s=-15$ dB the power of the interfering signal is too low.

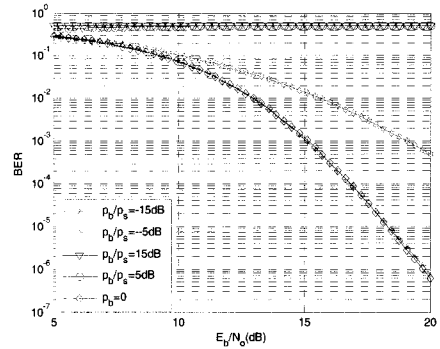


Fig. 3 Theoretical BER performance of DCSK system combined with non-chaotic transmit reference system for different values of  $p_b/p_s$ .

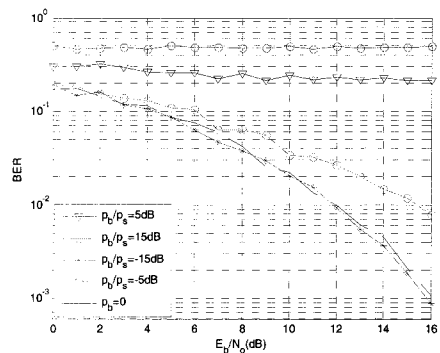


Fig. 4 BER performance of DCSK system combined with non-chaotic transmit reference system for different values of  $p_b/p_s$  under AWGN channel.

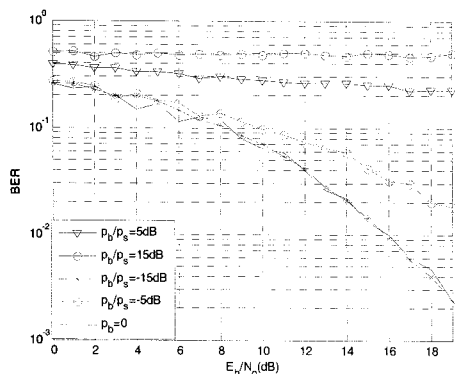


Fig. 5 BER performance of DCSK system combined with non-chaotic transmit reference system for different values of  $p_b/p_s$  under two-ray multipath channel.

#### IV. Conclusion

In this paper, we have analysed the system performance of the conventional DCSK system under the coexistence of non-chaotic transmit reference system with same data rate. The analysis is done for different values of bit energy to noise spectral density ratio ( $E_b/N_0$ ) and non-chaotic transmit reference signal power to chaotic signal power ratio  $p_b/p_s$ . Our results show how the foreign non-chaotic transmit reference system influences the system performance of DCSK system. And our analysis shows how much non-chaotic signal power the DCSK system can resist to give acceptable BER performance.

For future work, we can analyse the system performance of the foreign non-chaotic transmit reference system as well under this condition of its coexistence with DCSK system and see how much it is affected by the DCSK system.

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#### APPENDIX A

For our analysis, some assumptions are made as in [1] which leads us to the values of means, variances and covariances as:

$$\begin{aligned} E[A] &= \beta E[x_k^2] = \beta p_s & \text{var}[A] &= \beta \text{var}[x_k^2] = \beta \lambda \\ E[B] &= 0 & \text{var}[B] &= \beta p_s \end{aligned}$$

$$\begin{aligned}
 E[C] &= 0 & \text{var}[C] &= \beta N_o & \text{var}[y_i | (\alpha_i = -1, b_i = -1)] &= \text{var}[A] + 4p_b \text{var}[B] & \text{(A.11)} \\
 E[D] &= 0 & \text{var}[D] &= \frac{\beta p_s N_o}{2} & & + p_b \text{var}[C] + \text{var}[D] + \text{var}[E] + \text{var}[F] \\
 E[E] &= 0 & \text{var}[E] &= \frac{\beta p_s N_o}{2} & & + 4\sqrt{p_b} \text{cov}[A, B]. \\
 E[F] &= 0 & \text{var}[F] &= \frac{\beta N_o^2}{4} & & \\
 \text{cov}[A, B] &= (\beta - 1)E[x_{k+1} x_k^2] & \text{cov}[\chi, \gamma] &= 0 & & \\
 & \approx \beta \Omega & & & & 
 \end{aligned}$$

Where,

$$\Lambda = \text{var}[x_k^2] \quad \text{(A.2)}$$

$$\Omega = E[x_{k+1} x_k^2] \quad \text{(A.3)}$$

and  $p_s = E[x_k^2]$  denotes the average power of chaotic signal and  $\chi, \gamma \in \{A, B, C, D, E, F\}$ , and  $(\chi, \gamma) \neq (A, B)$  or  $(B, A)$ . And it can be shown that

$$\begin{aligned}
 E[y_i | (\alpha_i = +1, b_i = +1)] &= E[A] + 2\sqrt{p_b} E[B] + \beta p_b \\
 &+ \sqrt{p_b} E[C] + E[D] + E[E] + E[F]. \quad \text{(A.4)}
 \end{aligned}$$

$$\begin{aligned}
 E[y_i | (\alpha_i = +1, b_i = -1)] &= E[A] - \beta p_b + \sqrt{p_b} E[C] \\
 &+ E[D] + E[E] + E[F]. \quad \text{(A.5)}
 \end{aligned}$$

$$\begin{aligned}
 E[y_i | (\alpha_i = -1, b_i = +1)] &= -E[A] + \beta p_b + \sqrt{p_b} E[C] \\
 &+ E[D] - E[E] + E[F]. \quad \text{(A.6)}
 \end{aligned}$$

$$\begin{aligned}
 E[y_i | (\alpha_i = -1, b_i = -1)] &= -E[A] - 2\sqrt{p_b} E[B] \\
 &- \beta p_b + \sqrt{p_b} E[C] + E[D] - E[E] + E[F]. \quad \text{(A.7)}
 \end{aligned}$$

$$\begin{aligned}
 \text{var}[y_i | (\alpha_i = +1, b_i = +1)] &= \text{var}[A] + 4p_b \text{var}[B] \\
 &+ p_b \text{var}[C] + \text{var}[D] + \text{var}[E] + \text{var}[F] \\
 &+ 4\sqrt{p_b} \text{cov}[A, B]. \quad \text{(A.8)}
 \end{aligned}$$

$$\begin{aligned}
 \text{var}[y_i | (\alpha_i = +1, b_i = -1)] &= \text{var}[A] + p_b \text{var}[C] \\
 &+ \text{var}[D] + \text{var}[E] + \text{var}[F]. \quad \text{(A.9)}
 \end{aligned}$$

$$\begin{aligned}
 \text{var}[y_i | (\alpha_i = -1, b_i = +1)] &= \text{var}[A] + p_b \text{var}[C] \\
 &+ \text{var}[D] + \text{var}[E] + \text{var}[F]. \quad \text{(A.10)}
 \end{aligned}$$

Putting (A.1) into (A.4) through (A.11), we obtain

$$E[y_i | (\alpha_i = +1, b_i = +1)] = \beta p_s + \beta p_b. \quad \text{(A.12)}$$

$$E[y_i | (\alpha_i = +1, b_i = -1)] = \beta p_s - \beta p_b. \quad \text{(A.13)}$$

$$E[y_i | (\alpha_i = -1, b_i = +1)] = -\beta p_s + \beta p_b. \quad \text{(A.14)}$$

$$E[y_i | (\alpha_i = -1, b_i = -1)] = -\beta p_s - \beta p_b. \quad \text{(A.15)}$$

$$\begin{aligned}
 \text{var}[y_i | (\alpha_i = +1, b_i = +1)] &= \beta \Lambda + 4\beta p_b p_s + \beta p_b N_o + \\
 &\beta p_s N_o + \frac{\beta N_o^2}{4} + 4\beta \Omega \sqrt{p_b}. \quad \text{(A.16)}
 \end{aligned}$$

$$\begin{aligned}
 \text{var}[y_i | (\alpha_i = +1, b_i = -1)] &= \beta \Lambda + \beta p_b N_o + \beta p_s N_o + \frac{\beta N_o^2}{4}. \\
 &\text{(A.17)}
 \end{aligned}$$

$$\begin{aligned}
 \text{var}[y_i | (\alpha_i = -1, b_i = +1)] &= \beta \Lambda + \beta p_b N_o + \beta p_s N_o + \frac{\beta N_o^2}{4}. \\
 &\text{(A.18)}
 \end{aligned}$$

$$\begin{aligned}
 \text{var}[y_i | (\alpha_i = -1, b_i = -1)] &= \beta \Lambda + 4\beta p_b p_s + \beta p_b N_o + \\
 &\beta p_s N_o + \frac{\beta N_o^2}{4} + 4\beta \Omega \sqrt{p_b}. \quad \text{(A.19)}
 \end{aligned}$$

Putting (A.12) through (A.19) into (17), (20) and (21), we get the BER of DCSK as:

$$\begin{aligned}
 &BER_{DCSK} \\
 &= \frac{1}{2} \left[ \frac{1}{4} \text{erfc} \left( \frac{E[y_i | (\alpha_i = +1, b_i = +1)]}{\sqrt{2 \text{var}[y_i | (\alpha_i = +1, b_i = +1)]}} \right) \right. \\
 &\quad \left. + \frac{1}{4} \text{erfc} \left( \frac{E[y_i | (\alpha_i = +1, b_i = -1)]}{\sqrt{2 \text{var}[y_i | (\alpha_i = +1, b_i = -1)]}} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{4} \operatorname{erfc} \left( \frac{-E[y_i | (\alpha_i = -1, b_i = +1)]}{\sqrt{2\operatorname{var}[y_i | (\alpha_i = -1, b_i = +1)]}} \right) \\
 & + \frac{1}{4} \operatorname{erfc} \left( \frac{-E[y_i | (\alpha_i = -1, b_i = -1)]}{\sqrt{2\operatorname{var}[y_i | (\alpha_i = -1, b_i = -1)]}} \right)
 \end{aligned}
 \tag{A.20}$$

$BER_{DCSK}$

$$\begin{aligned}
 & = \frac{1}{8} \operatorname{erfc} \left[ \frac{\beta p_s + \beta p_b}{\sqrt{2\beta\Lambda + 8\beta p_b p_s + 2\beta p_b N_o + 2\beta p_s N_o}} \right. \\
 & \quad \left. \sqrt{+ \frac{\beta N_o^2}{2} + 8\beta\Omega\sqrt{p_b}} \right] \\
 & + \frac{1}{8} \operatorname{erfc} \left[ \frac{\beta p_s - \beta p_b}{\sqrt{2\beta\Lambda + 2\beta p_b N_o + 2\beta p_s N_o + \frac{\beta N_o^2}{2}}} \right] \\
 & + \frac{1}{8} \operatorname{erfc} \left[ \frac{\beta p_s - \beta p_b}{\sqrt{2\beta\Lambda + 2\beta p_b N_o + 2\beta p_s N_o + \frac{\beta N_o^2}{2}}} \right] \\
 & + \frac{1}{8} \operatorname{erfc} \left[ \frac{\beta p_s + \beta p_b}{\sqrt{2\beta\Lambda + 8\beta p_b p_s + 2\beta p_b N_o + 2\beta p_s N_o}} \right. \\
 & \quad \left. \sqrt{+ \frac{\beta N_o^2}{2} + 8\beta\Omega\sqrt{p_b}} \right] \\
 & = \frac{1}{4} \operatorname{erfc} \left[ \frac{\beta p_s + \beta p_b}{\sqrt{2\beta\Lambda + 8\beta p_b p_s + 2\beta p_b N_o + 2\beta p_s N_o}} \right. \\
 & \quad \left. \sqrt{+ \frac{\beta N_o^2}{2} + 8\beta\Omega\sqrt{p_b}} \right] \\
 & + \frac{1}{4} \operatorname{erfc} \left[ \frac{\beta p_s - \beta p_b}{\sqrt{2\beta\Lambda + 2\beta p_b N_o + 2\beta p_s N_o + \frac{\beta N_o^2}{2}}} \right]
 \end{aligned}
 \tag{A.21}$$

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