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# Designing a Coordinated Setup Cost Reduction Program of a Supply Chain*

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**ABSTRACT**

This paper contributes by incorporating works addressing supply chain coordination and investing in setup reduction program. Consider a two-echelon, EOQ-like inventory system consisting of a supplier and a buyer. We assume that both the supplier and the buyer can invest in setup cost reduction programs in order to benefit from small order sizes. However, the costs of investing in setup cost reduction programs are different for the two parties, leading to mismatches in individually optimal setup costs and order cycle times. We propose a supply chain coordination contract that makes use of quantity discount as an incentive transfer scheme for supply chain coordination.

Keywords: Supply Chain Coordination, Setup Cost Reduction, Economic Order Quantity

## 1. Introduction

Recent studies in Just-In-Time (JIT) Systems suggest companies in a supply chain maintain an integrated system, and continuously improve their production-delivery functions so as to achieve a high-frequency JIT delivery in a supply chain-wide synchronized manner. Many successful cases of supply chain integration through sup-

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plier-retailer partnership, mergers and acquisitions, etc. have been recently reported (Lieberman [20]; National Research Council [23]; Langabeer and Seifert [14]). However, regardless of general recognition of the importance of achieving a supply chain integration, aligning the localized operations of individual members with various self-serving orientations is a strenuous task (Parker and Anderson [25], Brouwers *et al.*, [4]). Vertical integration with acquisition may some times achieve this goal by enforcement, but this approach often leads to structural inflexibilities and organizational inefficiency. A more desirable approach may be designing a supply chain coordinating contract with incentives to induce a voluntary compliance. In this way, individual members can focus on their competitive strength while the supply chain can achieve a system-wide improvement in a more harmonious way. The notion of designing a channel coordinating contract to integrate a supply chain has been studied extensively in supply chain literatures. Examples of such studies include using Quantity Discount Contracts in a EOQ setting to eliminate mismatches in order cycle times (Jeuland and Shugan [11], Monahan [21, 22], Lee and Rosenblatt [17], Banerjee [2, 3], Dada and Srikanth [8], Joglekar [10], Drezner and Wesolowsky [9], Kohli and Park [12, 13], Weng and Wong [34], Weng [35, 36], Chen, Federgruen, and Zheng [7], and Wang [33]), and using channel policies in a Newsboy model setting to eliminate system double-marginalization phenomenon (see, for example, Buyback Contract: Pasterneck [26], Price Protection Contract: Lee, Padmanabhan, Taylor and Whang [18], Quantity Flexibility Contract: Tsay [31], Sales Rebate Contract: Taylor [30], Revenue Sharing Contract: Cachon [5], and Holding Cost Subsidy Contract: Wang and Gerchak [32]).

Our paper contributes by incorporating works addressing supply chain coordination and investing in *setup cost reduction programs (SRP)*. It has been reported that companies have used SRP to benefit from small order sizes and improved process quality (see, for example, Porteus [27, 28], Spence and Porteus [29], Paknejad, Nasri, and Affisco [24], Leschke and Weiss [19], Chaneski [6], and Albert [1]). However, previous modeling approaches mostly focus on studying an individual company's internal process quality improvement. In this article, we extend the study to include upstream (supplier) and down stream (buyer) processes so as to achieve a system-wide synchronized setup cost reductions.

This paper is structured as follows. In §2 and §3, we consider an EOQ-like inventory system consisting of a supplier and a sole/major buyer. We assume that setup cost reduction technologies are available both to the supplier and the buyer, and that the supplier's inventory and setup cost reduction investment costs differ from the buyer's. We consider a managerial scenario assuming that the buyer has independently implemented a setup cost reduction program. This event leads to a situation in which the supplier suffers a cost penalty from excessively frequent orders. We analyze the problem and provide a supply chain coordinating contract with a quantity discount as an incentive transfer scheme to entice the buyer to alter the setup cost reduction program and order policy for achieving a supply chain optimal delivery schedule. A brief discussion with numerical examples are provided in §4. §5 concludes the paper.

## 2. The Assumptions and Notations

We assume a monopolistic supplier selling a product only through its exclusive buyer following the previous modeling approaches and assumptions (Wang [33]). (i) The buyer's demand is not affected by the quantity discount. (ii) The supplier and the buyer use EOQ inventory policies governed by their deterministic demand and inventory cost structures. (iii) We assume complete information; i.e., both supplier and buyer share identical information. This assumption rules out asymmetric information as a possible explanation for optimal supply coordination contracts. (iv) Channel members (the supplier and its independent buyer) maximize their own profits.

We refer to the buyer and supplier as party 1 and party 2 in our distribution channel. Generally, then, we will use subscripts "1" and "2" to designate the buyer's and supplier's set of parameters. Let $D$ denote the yearly demand rate in units, $P$ denote the retail price, $W$ denote the delivered unit wholesale price, $h_i$ denote holding cost rate with $i = 1, 2$, $S_i$ denote setup cost per order with $i = 1, 2$, $S_i^0$ denote the setup cost prior to the investment in reducing the setup cost with $i = 1, 2$, and $Q$ denote the buyer's order quantity. We assume that both the supplier and the buyer can invest in SRP to benefit from small order sizes. The setup cost reduction investment cost function reducing the setup cost from $S_i^0$ to $S_i$ is a logarithmic form, and can be expressed as $B_i \ln\left(S_i^0 / S_i\right)$, $i = 1, 2$. In common with the previous works

(Porteus [26, 27]; Spence and Porteus [29], Leschke and Weiss [19]), we employ this function as an approximation. Here, $B_i = K \times r$ consists of two parts; a constant $K$ representing the cost of making about a 63% reduction in the setup cost, and $r$ denotes fractional per unit time opportunity cost of capital. Let $\Pi_1(Q, S_1|W)$ denote buyer's profit function for a given $W$.

$$\Pi_1(Q, S_1|W) = (P-W)D - \{DS_1/Q + Qh_1/2 + B_1 \ln(S_1^0/S_1)\}. \tag{1}$$

The individual cost terms reflect purchasing, order processing, holding, and setup reduction investment costs, respectively. The simultaneously optimal setup cost and order quantity $(S_1, Q)$ maximizing (1) are:

$$S_1^* = \min\left[S_1^0,\ S_1^R(Q^R) = 2B_1^2/Dh_1\right] \text{ and}$$

$$Q^* = \min\left[\sqrt{2DS_1^0/h_1},\ Q^R(S_1^R) = \sqrt{2DS_1^R/h_1} = 2B_1/h_1\right] \tag{2}$$

*We assume throughout the work that the buyer has independently implemented a setup reduction program, and its optimal order quantity and setup cost are respectively* $Q^* = Q^R(S_1^R)$ *and* $S_1^* = S_1^R(Q^R)$. Let $Q^R := Q^R(S_1^R)$ and $S_1^R := S_1^R(Q^R)$, and let $\Pi_2(n, S_2, W|Q^R)$ denote the supplier's profit where the order quantity is $Q^R$. Let $V$ denote the supplier's production rate in units, $m$ denote the supplier's gross profit as a percentage of sales, and $h_2$ denote the supplier's holding cost rate. We assume that the supplier follows a produce-to-stock production principle, and designs an optimal production lot size as an integer $n$-multiple of the buyer's order size, i.e., $nQ^R$. Upon subtracting the setup cost, setup cost reduction investments, and holding cost from the gross sales revenue, the supplier's profit is (see Lee and Rosenblatt [17] and Joglekar[10]):

$$\Pi_2(n, S_2, W|Q^R) = WmD - DS_2/nQ^R$$

$$-Q^R h_2[(n-1)-(n-2)D/V]/2 - B_2 \ln(S_2^0/S_2). \tag{3}$$

The optimal setup cost and integer multiplier $n$ minimizing $\Pi_2\left(n,\ S_2,\ W\middle|Q^R\right)$ are:

$$S_2^P(n) = \min\left[S_2^0,\ nS_1^R B_2/B_1\right] \text{ and}$$
$$n^P(S_2) = \sqrt{S_2 h_1/S_1^R h_2 (1 - D/V)}. \tag{4}$$

The necessary conditions in (4) and (2) reveals that s $= \sqrt{2S_2 D/h_2(1-D/V)}$ gives a classical Economic Manufacturing Quantity. We call this a *"passive" policy*. Here, the supplier only reacts to the buyer's individually optimal order policy, and does not try to actively correct the disadvantageous situation. In the next section, we will consider a scenario in which the supplier, acting as a Stakelberg leader, designs a supply chain coordinating quantity discount contract (*SQDC*) that makes use of quantity discount policy as an incentive transfer scheme to induce the buyer to modify its individually optimal order policy.

## 3. Centralized and Decentralized Supply Chains

We first examine the case of the centralized system in which both supplier's and buyer's decisions are fully coordinated under the central control of the distribution system. This is a benchmark case with which we compare a contract for supply chain coordination in a decentralized supply chain. Order and setup cost reduction programs are chosen for the benefit of the entire supply chain. Thus, any internal transaction that does not increase the joint profit is eliminated and each agent's efforts are aligned for the maximization of total system profits.

Suppose now that the supplier proposes a quantity discount contract to induce the buyer to increase its order size from the current level of $Q^R$ by a factor of $\delta$. Should the buyer accept this proposal, a reward will be given in the form of a quantity discount. With no other compensating changes, adjusting $Q^R$ by a factor of $\delta$ will cause the buyer's profit changes from that in (1) to:

$$\Pi_1(\delta Q^R,\ S_1|W) = (P - W)D - DS_1/\delta Q^R - \delta Q^R h_1/2$$

$$-B_1 \ln\left(S_1^0/S_1^R\right)+A(\delta)B_1 \ln\left(S_1/S_1^R\right) \tag{5}$$

$A(\delta)\in[0, 1]$ in (5) represents the fraction of investment retrievable from the modification of the set up reduction program. The possibility of modifying the SRP is based on Porteus (1985), who stated that the investment in reducing the setup cost can be regarded as a lease that can be broken on occasion and a new setup cost level selected. $A(\delta)$ is included in the formulation to account for the penalties of breaking the "lease," if any, resulting from reducing the investment in the set up reduction program. One example of such a penalty is poor quality control resulting from an increase in the production lot size. (see Porteus [1986a] for the relationship between lot sizing and quality control.) We assume that $A(\delta)$ is a decreasing function of $\delta$; thus, the greater the adjustment, the lower the retrievable portion of the original investment. We formulate

$$A(\delta)=\begin{cases}1-\delta/U & \text{if } 1\le\delta<U \\ 0 & \text{if } \delta\ge U\end{cases}.$$

Here, $U$ is an upper bound of the adjustment factor. Adjustment of the order quantity over $U$ times of the current order size will lead to a penalty too high to allow the initial set up reduction investment to be retrieved. For the sack of simplicity, we assume that the supplier's fraction of investment retrievable $A(\delta)\equiv 1$. Denote $\Delta := n\delta$, and let $\Pi_2(\_)$ and $\Pi(\_)$ denote the supplier profit and joint profit of the supplier and the buyer. Then,

$$\Pi_2\left(\Delta,\ \delta,\ S_2,\ W\middle|\delta Q^R\right)=WmD-DS_2/\Delta Q^R$$
$$-Q^R h_2\left[(\Delta-\delta)-(\Delta-2\delta)D/V\right]/2-B_2\ln\left(S_2^0/S_2\right), \text{ and}$$
$$\Pi\left(\Delta,\ \delta,\ S_1,\ S_2\right)=\Pi_2\left(\Delta,\ \delta,\ S_2,\ W\middle|\delta Q^R\right)+\Pi_1(\delta Q^R,\ S_1|W).$$

Let $\left(\Delta^J,\ \delta^J,\ S_1^J,\ S_2^J\right)$ denote optimal policies designed to maximize the joint supply chain profit. The following properties are observed.

**Property 1.** *Maximizing* $\Pi(\Delta, \delta, S_1, S_2)$ *with respect to* $\Delta$ *and* $S_2$ *reveals that* $S_2^J(\Delta) = \min[S_2^0, \Delta S_1^R B_2/B_1]$ *and*

$$\Delta^J(S_2) = \sqrt{S_2 h_1 / S_1^R h_2 (1 - D/V)}.$$

**Property 2.** *Maximizing* $\Pi(\Delta, \delta, S_1\ S_2)$ *with respect to* $S_1$ *and* $\delta$ *reveals that* $S_1^J(\delta) = \delta A(\delta) S_1^R \in [S_1^R, S_1^0]$ *and*

$$\delta^J(S_1) = \sqrt{S_1 h_1 / S_1^R [h_1 - h_2(1 - 2D/V) + h_1 ln(S_1/S_1^R)/U]}.$$

*Property 1* reveals that $S_2^J(\Delta = \Delta^J) = S_2^P(n = n^P)$ *and* $\Delta^J(S_2 = S_2^J) = n^P(S_2 = S_2^P)$. *Property 2* indicates that the buyer's modified set up reduction program is a roll-back adjusted type that is located in $S_1^R$, $S_1^0$. The intuition behind the optimal solution $S_1^J(\delta)$ is fairly simple and can be easily described. Note that the setup cost and the order quantity are proportional, except that the coefficient of proportionality is changed by the fraction of investment retrievable $A(\delta)$ when the roll-back of the setup cost takes place. Hence, (i) the higher the retrieval $A(\delta)$ or (ii) the higher the adjustment factor $\delta^J(S_1)$, the more the roll-back occurs.

We now examine a decentralized supply chain in which the supplier and the buyer maximize their own profits independently. Given the assumption of the agents' independent decisions, we derive the supplier's supply chain coordinating quantity discount contract (*SQDC*) that entices the buyer to replicate the outcome of the integrated system in choosing order size and setup costs. The reasoning is that, from its selfish perspective, it is optimal for the supplier to coordinate the supply chain for the joint system profits (Pasterneck, [1985]; Lariviere, [16]). The supplier maximizes the total system profits by coordinating the supply chain, and then takes the largest portion of the profits by arranging a profit-sharing scheme that gives the buyer slightly larger profit than the one under no coordination. Subsequently, the buyer will accept the coordination contract and enable the supplier to maximize its profit.

We assume that the supplier is a Stakelberg leader with its dominant position in the supply chain. The sequence of events are as follows:

(1) *Let $\xi$ denote the total amount of quantity discount in SQDC. The supplier determines an all unit quantity discount wholesale price* (see, for example, Monahan [1984, 1988], Lee and Rosenblatt [1986], Banerjee [1986a,b]) *with a price break point $\delta Q^R$ and $\delta \geq 1$.*

$$\textit{quantity discount wholesale price rate} = \begin{cases} W & \text{if } 0 \leq Q < \delta Q^R \\ W - \xi/D & \text{if } \delta Q^R \leq Q \end{cases}$$

*Thus, a discount wholesale price rate $W - \xi/D$ is applied to all items of an order quantity $Q \geq \delta Q^R$.*

(2) *The buyer chooses a new order quantity according to the quantity discount wholesale price rate, and adjusts the setup reduction investment according to the new order quantity.*

(3) *The supplier adjusts setup reduction investment according to the new order quantity.*

Since the buyer's profit function is not continuous at the price break point $\delta Q^R$, the buyer orders either $Q^R$ or $\delta Q^R$. Assume now that the buyer adjusts the order size from the current level of $Q^R$ to $\delta Q^R$ (a detailed description of how to induce the buyer to order $\delta Q^R$ will be given at the end of this section when we discuss the SQDC). Let $\Delta\Pi(S_1|\delta, \xi)$ denote the difference in profits before and after the order size adjustment. The buyer would now find it beneficial to modify current investment in setup reduction to a new level according to the new order size $\delta Q^R$ so as to maximize

$$\Delta\Pi(S_1|\delta, \xi) = \Pi_1(\delta Q^R, S_1|W - \xi/D) - \Pi_1(Q^R, S_1^R|W) =$$
$$\xi - \left\{D\left(S_1/\delta - S_1^R\right)/Q^R + h_1 Q^R(\delta - 1)/2 - A(\delta)B_1 \ln\left(S_1/S_1^R\right)\right\} \quad (6)$$

The buyer's optimal setup cost level maximizing (6) is given in Proposition 1.

***Proposition 1.*** *(See Appendix 1 for the proof for Proposition 1) $\Delta\Pi(S_1|\delta, \xi)$ is strictly concave in $S_1$. The optimal setup cost $S_1^*(\delta) = \delta A(\delta) S_1^R \in \left[S_1^R \bullet S_1^0\right]$ is identical with the supply chain benchmark solution $S_1^J(\delta)$ (see property 2).*

*Let* $L_1 := \delta < \left(U - \sqrt{U^2 - 4U}\right)/2$, $L_2 := \left(U - \sqrt{U^2 - 4US_1^0/S_1^R}\right)/2$,

$L_3 := \left(U + \sqrt{U^2 - 4US_1^0/S_1^R}\right)/2$, *and* $L_4 := \delta > \left(U + \sqrt{U^2 - 4U}\right)/2$.

*(i)* $S_1^*(\delta) = \delta A(\delta) S_1^R$ *when* $L_1 \le \delta \le L_2$ *or* $L_3 \le \delta \le L_4$.

*(ii)* $S_1^*(\delta) = S_1^0$ *when* $L_2 \le \delta \le L_3$

*(iii)* $S_1^*(\delta) = S_1^R$ *when* $\delta < L_1$ *or* $\delta > L_4$. □

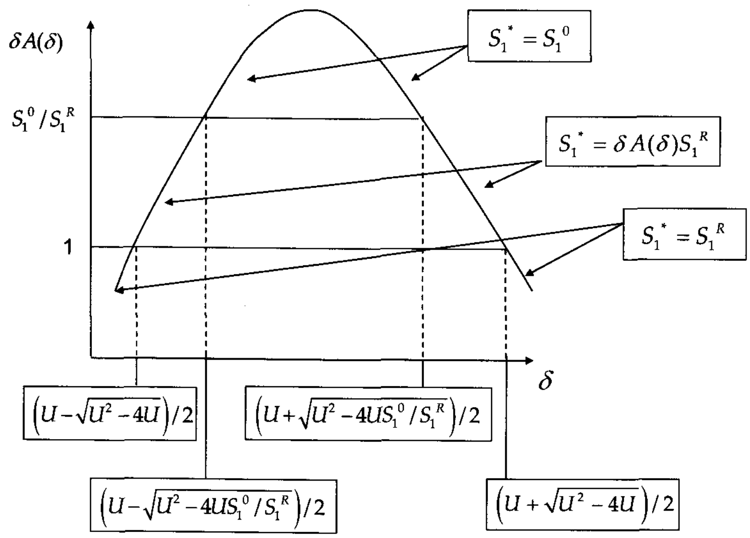



Figure 1. Buyer's setup cost

Note the buyer's optimal decision can be summarized as follows:

$$\begin{cases} (Q,\ S_1) = \{\delta Q^R,\ S_1^*(\delta)\} & \text{if} \quad \Delta\Pi\left(S_1^* \mid \delta,\ \xi\right) \ge 0 \\ (Q,\ S_1) = \{Q^R,\ S_1^R\} & \text{if} \quad \Delta\Pi\left(S_1^* \mid \delta,\ \xi\right) < 0 \end{cases}.$$

That is the buyer either accepts the coordination contract (if $\Delta\Pi\left(S_1^* \mid \delta,\ \xi\right) \ge 0$), and modifies its order and setup reduction policies to $\{\delta Q^R,\ S_1^*(\delta)\}$ (see Proposition 1), or ignores the coordination contract (if $\Delta\Pi\left(S_1^* \mid \delta,\ \xi\right) < 0$), and maintains previous order and setup reduction policies $\{Q^R,\ S_1^R\}$. Now, if the supplier designs a SQDC with a quantity discount

$$\xi(\theta)=\theta+\left\{D\left(S_1^*/\delta-S_1^R\right)/Q^R+h_1Q^R(\delta-1)/2-A(\delta)B_1\ln\left(S_1^*/S_1^R\right)\right\}$$
$$\text{and} \quad \theta\ge 0,$$

then, upon substitution, it leads to $\Delta\Pi\left(S_1^*\,|\delta,\ \xi(\theta)\right)=\theta\ge 0$ (see equation (6)). Thus, the supplier is able to induce the buyer to replicate the order policy of centralized system.

As with the buyer's case, the supplier must also adjust its setup cost according to the new order quantity $\delta Q^R$. The supplier's net profit increase is given as follows:

$$\begin{aligned}&\Delta\Pi\left(\Delta,\ \delta,\ S_2|\xi(\theta)\right)=\Pi_2\left(\Delta,\ \delta,\ S_2,\ W-\xi(\theta)/D|\delta Q^R\right)-\Pi_2\left(n^P,\ S_2^P,\ W|Q^R\right)\\&-\xi(\theta)+D\left(S_2^P/n^P-S_2/\Delta\right)/Q^R+B_2\ln\left(S_2/S_2^P\right)\\&+Q^Rh_2\left\{\left[\left(n^P-1\right)-\left(n^P-2\right)D/V\right]-\left[(\Delta-\delta)-(\Delta-2\delta)D/V\right]\right\}/2.\end{aligned} \tag{8}$$

Here, the supplier considers the setup cost $S_2$, $\Delta$, and $\delta$ to be the decision variables, and seeks to maximize the net profit increase $\Delta\Pi\left(\Delta,\ \delta,\ S_2|\xi(\theta)\right)$. The optimal ($S_2^*$, $\Delta^*$, $\delta^*$) maximizing (8) is given in Proposition 2.

**Proposition 2** (See Appendix 2 for the proof for Proposition 2)

**(2.1)** $\Delta\Pi\left(\Delta,\ \delta,\ S_2|\xi\ \right)$ is strictly concave with respect to $\left(\Delta,\ \delta,\ S_2\right)$

(i) The optimal setup cost and $\Delta$ are identical with the supply chain benchmark solution $S_2^J(\Delta)$ and $\Delta^J\left(S_2\right)$ (see Property 1 ).

$$S_2^*(\Delta)=\begin{cases}S_2^0 & \textit{if}\ \ \Delta>S_2^0B_1/S_1^RB_2\\ \Delta S_1^RB_2/B_1 & \textit{if}\ \ \Delta\le S_2^0B_1/S_1^RB_2\end{cases},\ \textit{and}$$
$$\Delta^*\left(S_2\right)=\sqrt{S_2h_1/S_1^Rh_2\left(1-D/V\right)}.$$

Substituting the optimal solutions $S_2^*$ results in the following simultaneous solutions :

(i) $\Delta^*\left(S_2^*\right)$ = $\sqrt{S_2^0h_1/S_1^Rh_2\left(1-D/V\right)}$ $\quad$ *when* $B_2/B_1>\sqrt{S_2^0h_2\left(1-D/V\right)/S_1^Rh_1}$

(ii) $\Delta^*(S_2^*) = B_2h_1/B_1h_2(1-D/V)$ *when* $B_2/B_1 \le \sqrt{S_2^0 h_2(1-D/V)/S_1^R h_1}$

(ii) The optimal adjustment factor $\delta^*(S_1^*)$ derived for a given buyer's optimal setup cost $S_1^*$ is identical with the supply chain benchmark solution $\delta^J$ (see Property 2).

$$\delta^*(S_1^*) = \sqrt{S_1^* h_1/S_1^R\left[h_1 - h_2(1-2D/V) + h_1 ln(S_1^*/S_1^R)/U\right]}.$$

Substituting the optimal solution $S_1^*$ given in Proposition 1 results in the following simultaneous solutions:

(i) $\delta^*(S_1^*) = \bar{\delta}$ satisfying $\bar{\delta}\left\{1 - h_2(1-2D/V)/h_1 + \ln(\bar{\delta}A(\bar{\delta}))/h_1U\right\} - A(\bar{\delta}) = 0$, when $L_1 \le \bar{\delta} \le L_2$ or $L_3 \le \bar{\delta} \le L_4$

(ii) $\delta^*(S_1^* = S_1^0) = \sqrt{S_1^0 h_1/S_1^R\left[h_1 - h_2(1-2D/V) + h_1 ln(S_1^0/S_1^R)/U\right]}$, when $L_2 < \bar{\delta} < L_3$.

(iii) $\delta^*(S_1^* = S_1^R) = \sqrt{1/(1-h_2(1-2D/V)/h_1)}$ when $\bar{\delta} < L_1$ or $\bar{\delta} > L_4$. □

**Proposition 2.1** reveals that the supplier invests in setup reduction, $S_2^* = \Delta S_1^R B_2/B_1$, when (i) $B_2/B_1$ is relatively small (the supplier's setup reduction investment cost being relatively low), or when (ii) $S_2^0/S_1^R$ is relatively large (the buyer's optimal order size is relatively smaller). Proposition 2.2 shows that the buyer roll-back adjusts its setup cost, $S_1^*(\delta) =$, $\delta A(\delta)S_1^R$, when (iii) $S_1^0/S_1^R$ is relatively large (leading to $S_1^R \le \delta A(\delta)S_1^R \le S_1^0$). We see that $(\Delta^J, \delta^J, S_2^J, S_1^J) = (\Delta^*, \delta^*, S_2^*, S_1^*)$ (see Properties 1 and 2; Propositions 1 and 2); thus, SQDC effectively coordinates the supply chain, and induces the buyer to replicate the benchmark order and setup reduction policies given in properties 1 and 2.

Let $\Pi(n^P, S_2^P, S_1^R) = \Pi_1(Q^R, S_1^R|W) + \Pi_2(n^P, S_2^P, W|Q^R)$ denote the supply chain joint profit of the passive policy. The supplier and the buyer will design a mutually agreeable $\theta$ for dividing the system net gains $\Pi(\Delta^J, \delta^J, S_2^J, S_1^J) - \Pi(n^P, S_2^P, S_1^R)$. Assume now that the supplier uses a profit sharing arrangement in which the buyer shares $0 \le \lambda \le 1$ (the supplier shares $1-\lambda$) portion of the system net profit (see Dada and Srikanth [8]). Weng (1997) has proposed a non-decreasing share profit sharing arrangement in which it is assumed that members of the supply chain are willing to adopt the jointly optimal policy, provided that their share of the joint profits will not decrease. Let $1-\lambda := \Pi_2\left(n^P,\, S_2^P,\, W\middle|Q^R\right)\big/\Pi\left(n^P,\, S_2^P,\, S_1^R\right)$ denote the shares of the supplier's individual profit to the joint profits when both parties do not coordinate. The scenario postulates that the supplier and the buyer will go with the coordination model, provided that the supplier's (buyer's) share of the joint profits generated from the coordination equals $1-\lambda$ ($\lambda$). Let Profit Increase Ratio $\Lambda = \Pi(\Delta^J,\, \delta^J,\, S_2^J,\, S_1^J)/\Pi(n^P,\, S_2^P,\, S_1^R)$. We see that $(1-\lambda)\Pi\left(\Delta^J,\, \delta^J,\, S_2^J,\, S_1^J\right) = \Lambda\Pi_2\left(n^P,\, S_2^P,\, W\middle|Q^R\right)$ and $\lambda\Pi\left(\Delta^J,\, Q^J,\, S_2^J,\, S_1^J\right) = \Lambda\Pi_1\left(Q^R,\, S_1^R\middle|W\right)$; hence, by following the non-decreasing profit sharing arrangement, both the supplier's and the buyer's profits are exactly increased by the additional Profit Increase Ratio. The supplier will then designs a SQDC with $\theta = (\Lambda-1)\Pi_1\left(Q^R,\, S_1^R|W\right)$.

## 4. Discussions, Extensions, and Numerical Examples

Thus far, our analysis is performed under a produce-to-stock policy. Let us now consider an order-for-order policy (see, e.g., Monahan [21, 22] ; Banerjee [2, 3]). If the supplier applies an order-for-order policy ($n$=1), then the optimal policies are $S_1^J(\delta) = \delta A(\delta)S_1^R \in \left[S_1^R, S_1^0\right]$, $S_2^J(\delta) = \min\left[S_2^0,\, \delta S_1^R B_2/B_1\right]$, and $\delta^J(S_2,\, S_2) = \sqrt{(S_2+S_2)h_1/S_1^R\{h_1\ [1+ln(S_1/S_1^R)/U]+h_2\}}$. Here, the expression of $S_2^J(\delta)$ reveals that (i) the higher the buyer's setup cost ($S_1^R$), or (ii) the greater the buyer's order size adjustment ($\delta$), or (iii) the more expensive the supplier's setup reduction investment cost relative to that of the buyer ($B_2/B_1$), the less will be invested in reducing the supplier's setup cost. Upon comparison with $S_2^P = S_1^R B_2/B_1$, we see that the supplier's modified setup cost is also a roll-back adjustment type, where the higher the adjustment factor $\delta$, the more the roll-back adjustment. Substituting the optimal solutions $\left(S_2^J,\, S_1^J\right)$ into $\delta^J\left(S_1^J,\, S_2^J\right)$ yields the following simultaneously optimal solutions. Let Cases CXY denote:

(a) X = A, B : CA : $=\delta < S_2^0 B_1/S_1^R B_2$ and CB : $=\delta \geq S_2^0 B_1/S_1^R B_2$.

(b) Y = 1, 2, 3, 4, and 5 : C2 := $L_1 < \delta < L_2$, C4 := $L_3 < \delta < L_4$, C3 := $L_2 \le \delta \le L_3$, C1 := $\delta \le L_1$, and C5 := $\delta \ge L_4$.

| Simultaneously optimal $\delta^J(S_1^J, S_2^J)$ | |
|---|---|
| *CA2; CA4:*<br>$\delta^J = \delta_A$ *satisfying*<br>$\delta_A\{1+h_2/h_1+\ln[\delta_A A(\delta_A)]/h_1U\} - B_2/B_1 - A(\delta_A) = 0$. | *Case CA2 or CA4 apply when*<br>*CA2:* $=\{\delta_A \ge S_2^0B_1/S_1^RB_2, L_1 \le \delta_A \le L_2\}$ *or*<br>*CA4:* $=\{\delta_A \ge S_2^0B_1/S_1^RB_2, L_3 \le \delta_A \le L_4\}$. |
| *CB2; CB4:*<br>$\delta^J = \delta_B$ *satisfying*<br>$\delta_B^2\{1+h_2/h_1+\ln[\delta_B A(\delta_B)]/h_1U\} - S_2^0/S_1^R - \delta_B A(\delta_B) = 0$. | *Case CB2 or CB4 apply when*<br>*CB2:* $=\{\delta_A \ge S_2^0B_1/S_1^RB_2, L_1 \le \delta_B \le L_2\}$ *or*<br>*CB4:* $=\{\delta_A \ge S_2^0B_1/S_1^RB_2, L_3 \le \delta_B \le L_4\}$. |
| *CA3:*<br>$\delta^J = (B_2/B_1)/2(1+h_2/h_1) + \sqrt{(B_2/B_1)^2 + 4(S_1^0/S_1^R)(1+h_2/h_1+\ln(S_1^0/S_1^R)/h_1U)}\Big/ 2(1+h_2/h_1)$ | *Case CA3 apply when*<br>$\{(S_1^0/S_1^R)(1+h_2/h_1) <$<br>$[(S_2^0B_1/S_1^RB_2)(1+h_2/h_1+\ln(S_1^0/S_1^R)/U)]^2$<br>$-(S_2^0/S_1^R)[1+h_2/h_1+\ln(S_1^0/S_1^R)/U], L_2 < \delta_A < L_3\}$ |
| *CB3:*<br>$\delta^J = \sqrt{(S_1^0/S_1^R + S_2^0/S_1^R)/(1+h_2/h_1+\ln(S_1^0/S_1^R)/h_1U)}$ | *Case CB3 apply when*<br>$\{(S_1^0/S_1^R)(1+h_2/h_1) \ge$<br>$[(S_2^0B_1/S_1^RB_2)(1+h_2/h_1+\ln(S_1^0/S_1^R)/U)]^2$<br>$-(S_2^0/S_1^R)[1+h_2/h_1+\ln(S_1^0/S_1^R)/U], L_2 < \delta_B < L_3\}$ |
| *CB1 or CB5:*<br>$\delta^J = \sqrt{(1+S_2^0/S_1^R)/(1+h_2/h_1)}$ | *Case CB1 or CB5 apply when*<br>*CA1:* $=\{1+h_2/h_1 < [(1+h_2/h_1)(S_2^0B_1/S_1^RB_2)]^2$<br>$-(S_2^0/S_1^R)(1+h_2/h_1), \delta_A < L_1\}$ *or*<br>*CA5:* $=\{1+h_2/h_1 < [(1+h_2/h_1)(S_2^0B_1/S_1^RB_2)]^2$<br>$-(S_2^0/S_1^R)(1+h_2/h_1), \delta_A > L_4\}$ |
| *CA1 or CA5:*<br>$\delta^J = (B_2/B_1)/2(1+h_2/h_1) + \sqrt{(B_2/B_1)^2 + 4(1+h_2/h_1)}\Big/2(1+h_2/h_1)$ | *Case CB1 or CB5 apply when*<br>*CB1:* $=\{1+h_2/h_1 < [(1+h_2/h_1)(S_2^0B_1/S_1^RB_2)]^2$<br>$-(S_2^0/S_1^R)(1+h_2/h_1), \delta_B < L_1\}$ *or*<br>*CB5:* $=\{1+h_2/h_1 < [(1+h_2/h_1)(S_2^0B_1/S_1^RB_2)]^2$<br>$-(S_2^0/S_1^R)(1+h_2/h_1), \delta_B > L_4\}$ *or* |

Figure 2 illustrates that the possibility of both the buyer and the supplier making the investment in reducing the setup increases as the areas *C1+C 2+C4 +C5* and *CA* increase. Figure 2 reveals that case *CA* (the supplier invests in setup reduction) applies when (i) $B_1/B_2$ is relatively large (the supplier's setup reduction investment cost being relatively low), or when (ii) $S_2^0/S_1^R$ is relatively large (the buyer's optimal order size is relatively smaller). Cases *C2 and C4* (the buyer roll-back adjusts her setup cost) apply when (iv) $S_1^0/S_1^R$ is relatively large (leading to $S_1^R \le \delta A(\delta) S_1^R \le S_1^0$ ).

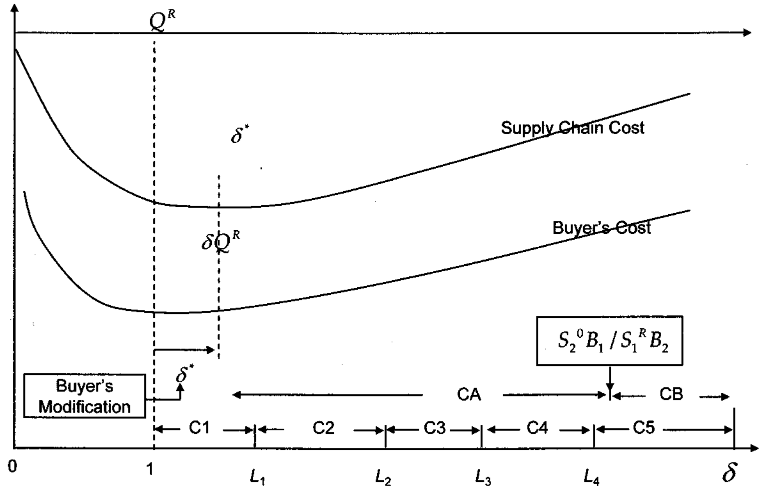


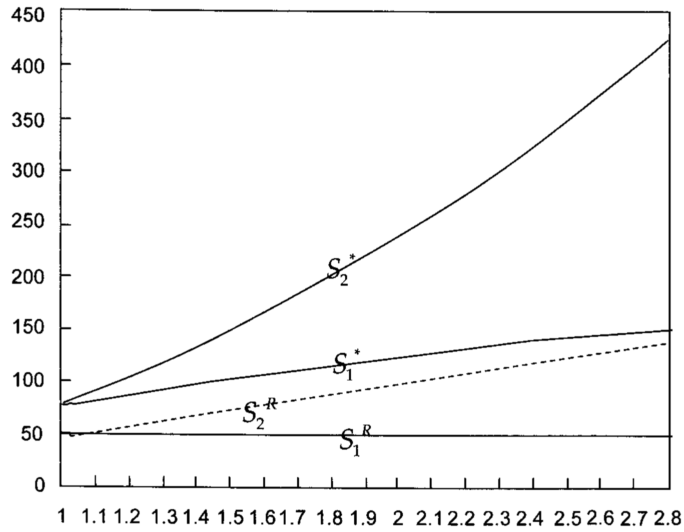

Figure 2. Buyer's and supplier's setup cost region

**Numerical Examples**: We present a numerical example to highlights *SQDC*. We assume that the supplier applies an order-for-order policy; thus, $n = 1$. Parameters take the following base values: $D = 6{,}000$, $h_2 = 1$, $B_1 = 100$, $A(\delta) \equiv 1$, $S_1^0 = 500$, $S_2^0 = 800$, $W = 20$, $P = 25$, and $m = 1$, and $\theta = 0$. The model experiments with different levels $h_1$ = 4, 7, and 3, and increases $B_2/B_1$ from 1 to 2.8 in increment of 0.1. We derive the optimal setup costs in each case and compute the Percentage Increase Ratio (PIR),

$$\text{PIR}_{\,SupplyChain} = \left[\, \Pi(Q^J,\, S_2^J,\, S_1^J)/\Pi\left(Q^R,\, S_2^P,\, S_1^R\right) - 1 \right] \times 100 \text{ and}$$

$$\text{PIR}_{\,Supplier} = \left[\, \Pi_2\left(S_2^J,\, W - \xi/D \middle| \delta Q^R\right)/\Pi_2\left(S_2^P,\, W \middle| Q^R\right) - 1 \right] \times 100$$

Positive PIR measures the relative advantage of *SQDC* compared to *passive policy*. Figures 3, 4, and 5 summarize the numerical experiment. Figures 3, 4, and 5 verify that compared to "passively" maintaining order size $Q^R$ (as in the passive policy) and investing in a rather expensive setup reduction program (as $B_2 / B_1$ increases), more savings are generated by actively inducing the buyer to increase its order size as




Figures 3−1. Setp costs as a function of $B_2 / B_1$ Case $h_1 = 4$

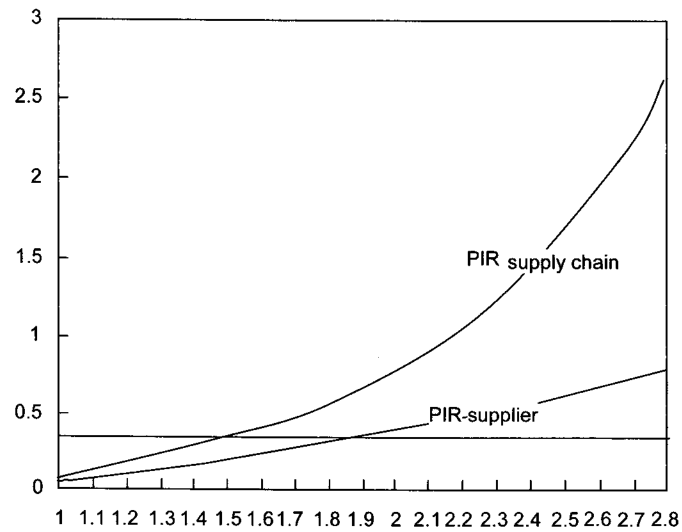



Figure 3−2. Supply chain and supplier's PIR as a function of $B_2 / B_1$ Case $h_1 = 4$

in *SQDC*. As we predicted with propositions 1 and 2, figures 3.1, 4.1, and 5.1 show that $S_2^*$ and $S_1^*$ are both roll-back adjustment types, where the higher the adjustment factor $\delta$, the more the roll-back adjustment. We see that *SQDC* outperforms the passive policy. However, the relative advantage of the *SQDC* increases as $B_2/B_1$ or $h_1/h_2$ increases.

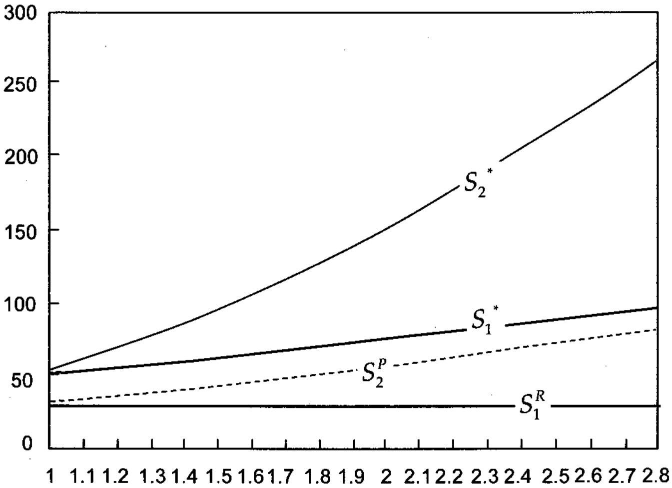



Figure 4-1. Setp costs as a function of $B_2/B_1$ Case $h_1=7$

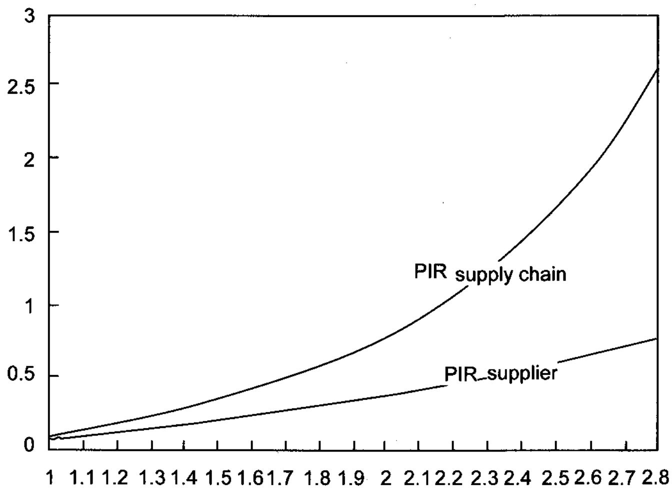



Figure 4-2. Supply chain and supplier's PIR as a function of $B_2/B_1$ Case $h_1=7$

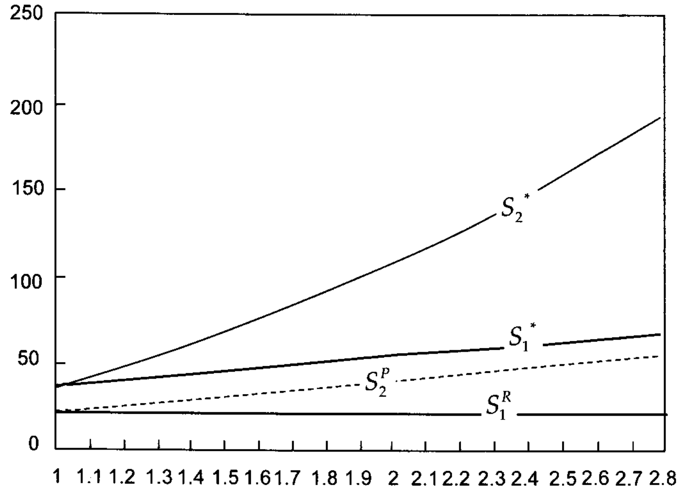



Figure 5-1. Setp costs as a function of $B_2 / B_1$ Case $h_1 = 10$

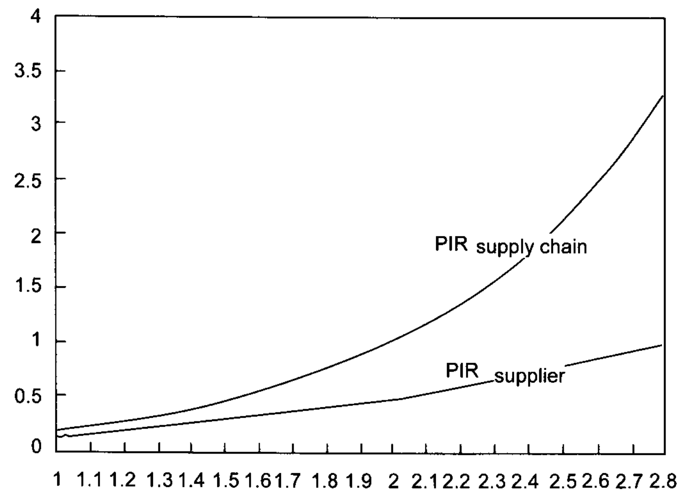



Figure 5-2. Supply chain and supplier's PIR as a function of $B_2 / B_1$ Case $h_1 = 10$

## 5. Conclusion

This paper contributes by integrating works on supply chain coordination and investments in setup reduction programs. We consider a hypothetical two-echelon, EOQ-like inventory system consisting of a supplier and a buyer. The buyer has independently implemented a setup reduction program. This event, however, leads to a situation in which the supplier suffers a cost penalty from excessively frequent orders. A supply chain coordinating quantity discount contract (*SQDC*) is offered to the buyer, the objective is to entice the buyer to alter its order frequency to achieve a mutually agreeable delivery schedule. An analysis of the model reveals that both the supplier's and the buyer's optimally adjusted setup costs are a roll-back adjustment type. We also show that the order quantity will be adjusted more when (i) the ratio between the supplier's and the buyer's holding costs is relatively small (leading to a larger supplier's (smaller buyer's) order quantity), (ii) the ratio between the supplier's and the buyer's setup reduction investment costs is relatively large (leading to a higher supplier's (lower buyer's) setup cost), or (iii) lesser roll-back adjustment of buyer's setup cost is performed. Our most important findings are that (i) *SQDC* is a supply chain coordinating contract, and (ii) by offering a *SQDC*, the supplier captures as high a percentage as possible of the jointly maximized supply chain profit, and earns an profit higher than that earned from any other contracts that are acceptable to the buyer (buyer's profit to be no less than those in the decentralized case).

In the present study, we set out to analyze the possibility of implementing a setup reduction program in a supply chain so as to achieve a system-wide improvement in procurement-delivery functions. In our view, the analysis has some limitations. First of all, our research is done in a much simplified setting by considering a supplier with a buyer case. In discussing the topic, including multiple-heterogeneous customers might provide more meaningful results. Second, we assume that the demand is independent of retail price. An extension of the work to include retail price sensitivity might be more appropriate. These limitations indicate some of the possible extensions to future study.

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# Appendix

*Appendix 1. Proof of Proposition 1:*

$d\Delta\Pi(S_1|\delta, \xi)/dS_1 = -D/\delta Q^R + A(\delta)B_1/S_1$, which has the same sign as $-DS_1/\delta Q^R + A(\delta)B_1$. It is positive when $S_1 = 0$, negative when $S_1 = \infty$, and strictly decreasing in $S_1$. Thus, the objective function is concave. $d\Delta\Pi(S_1|\delta,\xi)/dS_1 = 0$ leads to $S_1^*(\delta) = \delta A(\delta)Q^R B_1/D$. Equation (2) $\Rightarrow Q^R/D = S_1^R/B_1$. Hence, we obtain $S_1^*(\delta) = \delta A(\delta)S_1^R$ by substitution. Consider two extreme cases. (1) If $S_1^*(\delta) = \delta A(\delta)S_1^R \le S_1^R$, then $\delta A(\delta) = 1$; thus, $S_1^* = S_1^R$. (2) $\delta A(\delta)S_1^R > S_1^0 \Rightarrow S_1^* = S_1^0$, since $S_1^0$ is the initial unreduced setup cost. Upon substitution of $A(\delta) = 1 - \delta/U$ the buyer's optimal setup cost satisfies :

(i) If $S_1^R \le S_1^R\delta(1-\delta/U) \le S_1^0$ $(S_1^* = \delta A(\delta)S_1^R) \Rightarrow$ either $L_1 \le \delta \le L_2$ or $L_3 \le \delta \le L_4$.

(ii) If $S_1^R\delta(1-\delta/U) > S_1^0$ $(S_1^* = S_1^0) \Rightarrow$ $L_2 < \delta < L_3$.

(iii) If $S_1^R > S_1^R\delta(1-\delta/U)$ $(S_1^* = S_1^R) \Rightarrow$ either $\delta < L_1$ or $\delta > L_4$. □

*Appendix 2. Proof of Proposition 2:*

**Proposition 2.1:**

(i) $\partial\Delta\Pi(\Delta, \delta, S_2|\xi)/\partial S_2 = 0 \Rightarrow S_2^* = \Delta B_2 S_1^R/B_1$, and $\partial\Delta\Pi(\Delta, \delta, S_2|\xi)/\partial\Delta = 0 \Rightarrow$

$$\Delta^*(S_2) = \sqrt{S_2 h_1/S_1^R h_2(1-D/V)}.$$

(ii) $\partial\Delta\Pi(\Delta, \delta, S_2|\xi)/\partial\delta = 0 \Rightarrow$

$$-D/Q^R \times \frac{\partial(S_1^*(\delta)/\delta)}{\partial\delta} + \frac{h_1 - h_2(1-2D/V)}{2} + \frac{B_1 \ln(S_1^*(\delta)/S_1^R)}{U}$$

$$-A(\delta)B_1\frac{\partial \ln(S_1^*(\delta)/S_1^R)}{\partial\delta} = 0$$ Upon substituting $S_1^*(\delta) = (S_1^0, S_1^R, \delta A(\delta)S_1^R)$

$$\Rightarrow \frac{DS_1^*(\delta)}{Q^R\delta^2} - \frac{h_1 - h_2(1-2D/V)}{2} - \frac{B_1\ln(S_1^*(\delta)/S_1^R)}{U} = 0$$

$$\Rightarrow \delta^*(S_1^*) = \sqrt{S_1^* h_1/S_1^R\left[h_1 - h_2(1-2D/V) + h_1 ln(S_1^*/S_1^R)/U\right]},$$

Let $f_x := \partial f/\partial x$ denote the partial derivative with respect to $x$. The second derivatives are : $\Delta\Pi(\Delta, \delta, S_2|\xi)_{S_2S_2} = -B_2/S_2^2 < 0$, $\Delta\Pi(\Delta, \delta, S_2|\xi)_{\Delta\Delta} = -2S_2/\Delta^3 Q^R < 0$, and

$\Delta\Pi(\Delta,\ \delta,\ S_2|\xi)_{\Delta S_2} = 1/\Delta^2 Q^R > 0$. $\Delta\Pi(\Delta,\delta,S_2|\xi)_{\delta\delta}$ can have three different values depending on $S_1^* = (S_1^0,\ S_1^R,\ \delta A(\delta)S_1^R)$

(1) When $S_1^* = \delta A(\delta)S_1^R$ : $\Delta\Pi(\Delta,\delta,S_2|\xi)_{\delta\delta} = -B_1(U - 2\delta + 2\delta^2/U)/\delta^2 UA(\delta)$. Here, $U - 2\delta + 2\delta^2/U$ is convex in $\delta$, and minimum $\arg\min_\delta U - 2\delta + 2\delta^2/U$ is $\delta = U/2$. Substituting $\delta = U/2$ yields $U - 2\delta + 2\delta^2/U = U/2 \geq 0$.

(2) When $S_1^* = S_1^R$ or $S_1^0$: $\Delta\Pi(\Delta,\ \delta,\ S_2|\xi)_{\delta\delta} = -DS_1^*/\delta^3 Q^R \leq 0$.

Thus, by (1) and (2) $\Delta\Pi(\Delta,\ \delta,\ S_2|\xi)_{\delta\delta} \leq 0$, and $\Delta\Pi(\Delta,\ \delta,\ S_2|\xi)$ is concave in $\delta$.

The Hessian matrix H of $\Delta\Pi(\Delta,\ \delta,\ S_2|\xi)$ is given a follows:

$$\mathrm{H} = \begin{pmatrix} \Delta\Pi_{\delta\delta} & \Delta\Pi_{\delta S_2} & \Delta\Pi_{\delta\Delta} \\ \Delta\Pi_{\delta S_2} & \Delta\Pi_{S_2 S_2} & \Delta\Pi_{\Delta S_2} \\ \Delta\Pi_{\delta\Delta} & \Delta\Pi_{\Delta S_2} & \Delta\Pi_{\Delta\Delta} \end{pmatrix} = \begin{pmatrix} \Delta\Pi_{\delta\delta} & 0 & 0 \\ 0 & -B_2/S_2^{\ 2} & 1/\Delta^2 Q^R \\ 0 & 1/\Delta^2 Q^R & -2S_2/\Delta^3 Q^R \end{pmatrix}$$

It is seen that H is negative definite, since

$|\mathrm{H}_1| = \Delta\Pi(\Delta,\ \delta,\ S_2|\xi)_{\delta\delta} < 0$, and

$|\mathrm{H}_2| = \Delta\Pi(\Delta,\ \delta,\ S_2|\xi)_{\delta\delta} \times (-B_2/S_2^{\ 2}) - 0 > 0$, and

$$|\mathrm{H}_2| = \Delta\Pi(\Delta,\ \delta,\ S_2|\xi)_{\delta\delta} \times \{(2S_2/\Delta^3 Q^R)(B_2/S_2^{\ 2}) - (1/\Delta^2 Q^R)\}$$
$$= \Delta\Pi(\Delta,\ \delta,\ S_2|\xi)_{\delta\delta} \times Dh_1/2B_1 < 0.$$