

Double-Circuit Transmission Lines Fault location Algorithm for Single Line-to-Ground Fault

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Abstract – This paper proposes a fault location algorithm for double-circuit transmission lines in the case of single line-to-ground fault. The proposed algorithm requires the voltage and current from the sending end of the transmission line. The fault distance is simply determined by solving a second order polynomial equation which is achieved directly by the analysis of the circuit. In order to testify the performance of the proposed algorithm, several other conventional approaches have been taken out to compare with it. The test results corroborate its superior effectiveness.

Keywords : Double-Circuit Transmission Line, Fault Location, Single Line-to-Ground Fault

1. Introduction

Parallel transmission lines have been widely adopted in modern power systems to increase transmission capacity and enhance dependability and security. However, certain incidents can cause unexpected failures. Thus, the fault location technique would become more difficult and complex than for single lines, because double-circuit lines are characterized by a significant increase in the mutual coupling effects, which result in the change of the total line impedance. When the fault location algorithm applicable to single lines is directly used for double circuit lines, location accuracy cannot be ensured because of the mutual coupling between parallel lines.

Double circuit transmission lines are frequently subjected to a variety of technical problems from the perspective of protection engineering. The most popular method is recording the voltage and/or current signals at the ends of the line. This can be classified into two categories: double-ends method [1]-[2] and single-terminal fault location method [3]-[8].

Various fault location algorithms on parallel transmission lines have been put forward. In the double-ends method, a distributed parameter model based fault location algorithm for double-circuit transmission lines can be executed independently of the source impedance and fault resistance by using two terminal voltages and currents [1]. Another research presents a novel time-domain fault location algorithm for parallel transmission lines using two terminal currents based on differential component net [2].

Although two ends algorithms may present a better performance, single end algorithms have advantages from the commercial viewpoint. This is mainly due to the extra-complexity associated with two ends algorithms including

communication and synchronization between both ends as well as the cost increase. Thus, the importance of improving the performance of single end algorithms becomes significantly greater.

Therefore, there have been more researches focused on the application of the single end method. They are as follows. A practical fault location approach depending on modal transformation uses single end data of the parallel transmission lines [3]. A least error squares method for locating fault on coupled double-circuit HV transmission line uses one terminal data [4]. One group of researchers have developed some accurate fault location algorithms on two-parallel transmission lines for both single phase-to-ground fault [5] and non-earth fault utilizing one terminal data [6]. Furthermore, a new approach is based on artificial neural networks using the fundamental components of the fault and pre-fault voltage and current magnitudes of the reference end [7]. An accurate fault location algorithm for double-circuit transmission systems uses the voltage and current collected at only the local end of a single-circuit, while the fault distance is determined by solving the forth-order equation [8].

This paper proposes a simple approach for fault location on double-circuit transmission lines in the case of single line-to-ground fault. The proposed algorithm belongs to the single-terminal method since it requires voltage and current at the relaying point in the sending-end. Its effectiveness has been testified in a simple double-circuit transmission system through simulations by PSCAD/EMTDC. And the comparison results have demonstrated that the proposed algorithm is more accurate than other conventional algorithms.

2. Proposed Algorithm

The proposed algorithm not only requires voltage and current at the relaying point, but also requires the zero-

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sequence current of the adjacent parallel line. Furthermore, an assumption has been pointed out that the mutual impedance between two circuits is the same as that between phases in a single circuit, and the mutual impedance is equalized for distribution along the line. In addition, the shunt capacitance of the system is not taken into account in order to simplify the analysis.

2.1 Circuit Model

Fig. 1 shows a single line diagram of a simple double-circuit transmission system in the case of a single line-to-ground fault.

- Z: line impedance;
- Z012: sequence impedance of lines;
- Zm: mutual impedance between circuits/lines;
- Zm0: zero-sequence mutual impedance;
- Is: current at the local end of faulted circuit;
- Ir: current at the remote end of faulted circuit;
- It: current at the healthy circuit;
- Rf : fault impedance;
- If : fault current;
- p: fractional fault distance from the local end.

Even though the proposed algorithm requires the zero-sequence current of the adjacent parallel line, it does not need the source impedances on both ends during the circuit analysis.

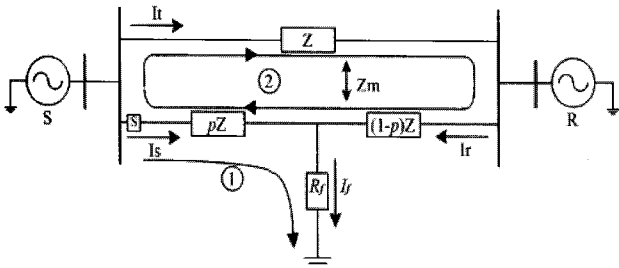


Fig. 1. A Simple Double-Circuit Transmission System

2.2 Circuit Analysis

In terms of the superposition principle in linear networks, the faulted network is decomposed into sequence networks which are positive, negative, and zero-sequence networks. Thus, the relationship between the a-phase voltage Vsa and its sequence components Vs0, Vs1, Vs2 can be expressed as

$$V_{sa} = V_{s0} + V_{s1} + V_{s2} \quad (1)$$

In the circuit, phase a to ground fault is assumed. The voltage at the relaying point is achieved through the analysis of loop 1 based on KVL.

$$V_{sa} = p(I_{s0}Z_0 + I_{s1}Z_1 + I_{s2}Z_2) + I_f R_f + pI_{t0}Z_{m0} \quad (2)$$

where,

Is012: sequence current at the local end of faulted circuit;

It0: zero-sequence current at the healthy circuit.

For a transmission line in a three-phase power system, the positive and negative sequence impedances Z1 and Z2 are always equal.

$$Z_1 = Z_2 \quad (3)$$

Substituting (3) into (2), (4) can be obtained below.

$$V_{sa} = pZ_1(I_{s0}(\frac{Z_0}{Z_1} - 1) + I_{sa}) + I_f R_f + pI_{t0}Z_{m0} \quad (4)$$

where,

I_{sa}: a-phase current at the local end of faulted circuit.

Define,

$$I_A \equiv I_{s0}(Z_0/Z_1 - 1) + I_{sa} \quad (5)$$

Thus,

$$V_{sa} = pZ_1 I_A + I_f R_f + pI_{t0}Z_{m0} \quad (6)$$

In (6), all impedances except for fault resistance Rf are known constants; Vsa, I_{sa}, Is0, and It0 can be obtained at the measuring point. On the other hand, fault current If cannot be calculated with only the local end relaying signals of the faulted circuit. Hence, KVL based loop 2 is taken into account in a zero-sequence circuit.

$$-pZ_0 I_{s0} - pZ_m I_{t0} + (1-p)Z_0 I_{r0} - (1-p)Z_m I_{t0} + Z_0 I_{t0} + pZ_m I_{s0} - (1-p)Z_m I_{r0} = 0 \quad (7)$$

where,

Ir0: zero-sequence current at remote end of faulted circuit.

Thus,

$$I_{r0} = \frac{pI_{s0} - I_{t0}}{(1-p)} \quad (8)$$

In the case of a single line-to-ground fault, the relationship between zero-sequence If0 and the currents Ifa, Ifb, Ifc is considered, where, $I_{fb} = I_{fc} = 0$, then,

$$I_f = 3I_{f0} \quad (9) \quad (a_r + ja_i)p^2 + (b_r + jb_i)p + (c_r + jc_i) + (d_r + jd_i)R_f = 0 \quad (18)$$

Furthermore,

$$I_{f0} = I_{s0} + I_{r0} \quad (10)$$

In order to eliminate the unknown I_f , substituting (8), (9) and (10) into (6), then to achieve below,

$$V_{sa} = pZ_1I_A + 3\left(\frac{I_{s0} - I_{t0}}{1-p}\right)R_f + pI_{t0}Z_{m0} \quad (11)$$

Define,

$$I_B \equiv I_{s0} - I_{t0} \quad (12)$$

Thus,

$$V_{sa} = pZ_1I_A + \frac{3I_B R_f}{1-p} + pI_{t0}Z_{m0} \quad (13)$$

By transforming (13), the second order polynomial equation of fractional fault distance p is achieved below.

$$p^2(Z_1I_A + I_{t0}Z_{m0}) - p(V_{sa} + Z_1I_A + I_{t0}Z_{m0}) + V_{sa} - 3I_B R_f = 0 \quad (14)$$

And considering,

$$Z_{m0} = 3Z_m \quad (15)$$

Substituting (15) into (14),

$$p^2(Z_1I_A + 3I_{t0}Z_m) - p(V_{sa} + Z_1I_A + 3I_{t0}Z_m) + V_{sa} - 3I_B R_f = 0 \quad (16)$$

Define,

$$a \equiv (Z_1I_A + 3I_{t0}Z_m); \quad c \equiv V_{sa}; \\ b \equiv -(V_{sa} + Z_1I_A + 3I_{t0}Z_m); \quad d = -3I_B.$$

Thus,

$$ap^2 + bp + c + dR_f = 0 \quad (17)$$

In order to eliminate R_f , (17) would be separated into the real and imaginary parts shown in (18).

It means,

$$a_r p^2 + b_r p + c_r + d_r R_f = 0 \quad (19)$$

$$a_i p^2 + b_i p + c_i + d_i R_f = 0 \quad (20)$$

Fault resistance R_f is achieved due to (20).

$$R_f = -\frac{a_i}{d_i} p^2 - \frac{b_i}{d_i} p - \frac{c_i}{d_i} \quad (21)$$

Substituting (21) into (19),

$$(a_r - \frac{a_i}{d_i} d_r) p^2 + (b_r - \frac{b_i}{d_i} d_r) p + (c_r - \frac{c_i}{d_i} d_r) = 0 \quad (22)$$

Define,

$$A = (a_r - \frac{a_i}{d_i} d_r), \quad B = (b_r - \frac{b_i}{d_i} d_r), \quad C = (c_r - \frac{c_i}{d_i} d_r)$$

Thus, the final second order polynomial equation is attained.

$$Ap^2 + Bp + C = 0 \quad (23)$$

The roots of (23) are

$$p = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (24)$$

Finally, the fractional fault distance p is between 0 and 1.

To sum up, there are two steps: at first, establishing two KVL equations around the parallel line loops; secondly, applying these to the voltage equation at the relaying point and separating the real and imaginary parts, the fault resistance can be eliminated. Finally, the second order polynomial equation that is a function having only one variable showing the fault location can be obtained.

3. Conventional Algorithms

3.1 Case 1: An Algorithm with an Assumption Based on Single Lines

This fault location algorithm is applicable to single lines, so the coupling is not taken into account in (6) which is the equation of voltage drop in the faulted circuit. Hence,

$$V_{sa} = pZ_1 I_A + I_f R_f \quad (25)$$

Further transformation,

$$\frac{V_{sa}}{I_A} - pZ_1 - \frac{I_f}{I_A} R_f = 0 \quad (26)$$

It is assumed here that when I_f and I_A are in phase, there is only a real component about I_f/I_A . However, R_f must be a real number, so that it can obtain,

$$\text{imag}(V_{sa}/I_A - pZ_1) = 0 \quad (27)$$

The solution is,

$$p = \text{imag}\left(\frac{V_{sa}}{I_A}\right) / \text{imag}(Z_1) \quad (28)$$

Normally, when the fault location algorithm applicable to single lines is directly used for double-circuit lines, location accuracy cannot be ensured by reason of the influence of the mutual coupling between parallel lines.

3.2 Case 2: An Algorithm with an Assumption Based on Double-Circuit Lines

This conventional fault location algorithm is similar to the proposed one. The only difference is that the conventional algorithm has an assumption.

As same as in (4) above, it is transformed into (29) below.

$$V_{sa} = pZ_1 \left(I_{s0} \left(\frac{Z_0}{Z_1} - 1 \right) + I_{sa} + I_{t0} \frac{Z_{m0}}{Z_1} \right) + I_f R_f \quad (29)$$

Define,

$$I_{fA} = I_{s0} \left(\frac{Z_0}{Z_1} - 1 \right) + I_{sa} + I_{t0} \frac{Z_{m0}}{Z_1} \quad (30)$$

Thus,

$$V_{sa} = pZ_1 I_{fA} - I_f R_f \quad (31)$$

Further transformation,

$$\frac{V_{sa}}{I_{fA}} - pZ_1 - \frac{I_f}{I_{fA}} R_f = 0 \quad (32)$$

On the assumption that I_f and I_{fA} are in phase, then it would achieve,

$$\text{imag}\left(\frac{V_{sa}}{I_{fA}} - pZ_1\right) = 0 \quad (33)$$

The solution is,

$$p = \text{imag}\left(\frac{V_{sa}}{I_{fA}}\right) / \text{imag}(Z_1) \quad (34)$$

3.3 Case 3: An Algorithm using Source Impedances

This conventional algorithm requires the source impedances on both the local and the remote side as shown in Fig. 2. It is different from the proposed one in which the source impedances are not required.

- Zs: impedance for source S;
- ZR: impedance for source R;
- Zs012: sequence impedance for source S;
- ZR012: sequence impedance for source R;

Both of them are based on the same analysis of the double-circuit transmission system. So the same equation has been achieved as in (4). However, this conventional algorithm requires source impedance but does not require the zero-sequence current of the adjacent parallel line. And it can get (35) through the analysis of current distribution factors on both positive-sequence and zero-sequence.

$$V_{sa} = p[Z_1 I_{sa} + (Z_0 - Z_1) I_{s0}] + pZ_{m0} \frac{I_{s0}}{CDF_{TS}} + R_f \frac{3I_{s1}}{CDF_{Saf}} \quad (35)$$

CDF_{TS} is the zero-sequence current distribution ratio between the healthy and faulted circuits.

$$CDF_{TS} = \frac{I_{s0}}{I_{t0}} = \frac{pA_{ST} + B_{ST}}{pC_{ST} + D_{ST}} \quad (36)$$

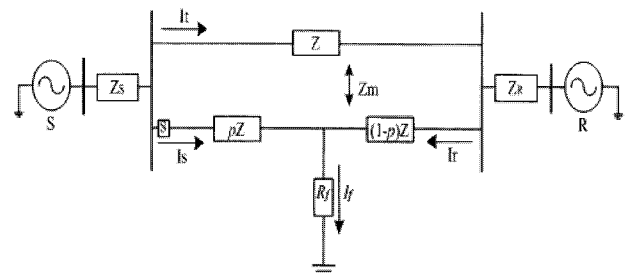


Fig. 2. A System Model for An Algorithm using Source Impedance

where,

$$\begin{aligned} A_{ST} &= (Z_m - Z_0)(Z_{s0} + Z_{R0} + Z_m) - (Z_0 - Z_m)Z_0 \\ B_{ST} &= (Z_0 - Z_m)(Z_{s0} - Z_{R0} + Z_m) + (Z_0 - Z_m)(Z_{R0} + Z_0) \\ C_{ST} &= (Z_0 - Z_m)(Z_{s0} + Z_{R0}) \\ D_{ST} &= (Z_m - Z_0)Z_{s0} \end{aligned}$$

And CDF_{Sa1} is the ratio of the positive-sequence current at the local end of the faulted circuit to the positive-sequence fault current.

$$CDF_{Sa1} = \frac{I_{s1}}{I_{f1}} = \frac{pB_{Sa1} + C_{Sa1}}{A_{Sa1}} \quad (37)$$

where,

$$\begin{aligned} A_{Sa1} &= Z_1(Z_{s1} + Z_{R1}) + Z_1(Z_{s1} + Z_{R1} + Z_1) \\ B_{Sa1} &= -Z_1(Z_{s1} + Z_{R1} + Z_1) \\ C_{Sa1} &= Z_1(Z_{s1} + Z_{R1} + Z_1) + Z_1Z_{R1} \end{aligned}$$

Then eliminating the fault resistance, R_f , a 4th-order nonlinear equation having only one unknown variable is obtained.

$$p^4 + k_1p^3 + k_2p^2 + k_3p + k_4 = 0 \quad (38)$$

where, k_1, k_2, k_3, k_4 are known coefficients.

The fault distance, p , can be obtained by solving (38) through the iterative Newton-Raphson method.

4. Case Study

A performance test of the proposed algorithm on accuracy has been carried out with variations of fault resistance and fault distance; and the comparisons with other conventional algorithms have been demonstrated as well.

4.1 Accuracy

Simulations by PSCAD/EMTDC have been performed in a simple double-circuit transmission system as presented in Fig. 3. The voltage level is 154 [kV], and the total length is 100 [km]. The system data are listed in Table 1.

In this case, phase-a to ground fault is assumed. The fault distance is varying from 10 [km] to 90 [km] with variation of fault resistance from 10 [Ω] to 100 [Ω]. An estimation error is defined as follows.

$$Error(\%) = \frac{\text{Estimated Distance} - \text{Actual Distance}}{\text{Total Line Distance}} \times 100 \quad (39)$$

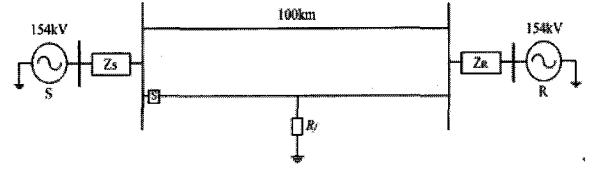


Fig. 3. A simple simulation model system

Table 1. System data

	Positive-sequence impedance	Zero-sequence impedance	
		Self	Mutual
Line [Ω /km]	0.0357 +j0.4828	0.3610 +j1.3790	0.3252 +j0.8963
Source [Ω]		10.261 \angle 79.5 $^\circ$	
	S	4.145 \angle 82.6106 $^\circ$	
	R	13.4187 \angle 80.2905 $^\circ$	
		49.0618 \angle 68.9051 $^\circ$	

The estimated fault distance is shown in Table 2.

Table 2. Estimated fault distance

Rf [Ω]	10	30	50	100
d [km]				
10	9.9838	9.9713	9.9686	9.9668
20	19.9936	19.9673	19.9603	19.9580
30	29.9944	29.9559	29.9426	29.9367
40	39.9779	39.9279	39.9075	39.8957
50	50.0093	49.9465	49.9177	49.8959
60	60.0492	59.9716	59.9334	59.8985
70	70.0805	69.9865	69.9398	69.8926
80	80.1511	80.0399	79.9871	79.9314
90	90.2156	90.1050	90.0672	90.0513

Fig. 4 shows the estimation error of fault distance with variation of fault resistance.

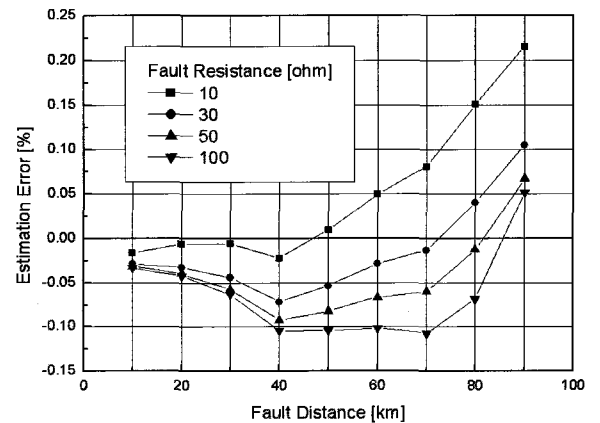


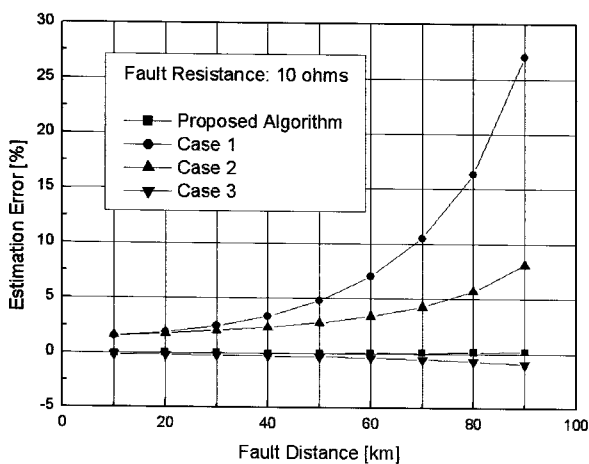
Fig. 4. Estimation error with variation of fault resistance

4.2 Comparisons

The comparisons between the proposed algorithm and those conventional algorithms have been carried out in the same simulation model system as presented in Fig. 3. The results are listed in Table 3. When fault resistance is 10

Table 3. Estimated fault distance

d[km]	10 [Ω]			
	Proposed	Case 1	Case 2	Case 3
10	9.98	11.55	11.59	9.86
20	19.99	21.88	21.80	19.83
30	29.99	32.46	32.05	29.79
40	39.98	43.36	42.34	39.73
50	50.00	54.81	52.79	49.68
60	60.05	67.03	63.41	59.61
70	70.08	80.52	74.29	69.49
80	80.15	96.33	85.69	79.32
90	90.22	117.03	98.11	89.07

**Fig. 5.** Comparison results

[ohm], the conventional algorithms could not achieve estimation results as accurate as the proposed one. Moreover, Fig. 5 indicates the comparison results among the proposed algorithm and the three conventional algorithms in the case of fault resistance with 10 [ohm].

In case 1, the error is very large. This demonstrates that the algorithm applicable for single lines cannot be efficient to apply to double-circuit lines since the mutual impedance is not considered. The error in case 2 is still greater than the proposed one even though the coupling between parallel circuits is taken into account. In fact, the assumption inside still inevitably causes an error. In case 3, it achieves a small error, but this conventional algorithm requires both the local and remote source impedances. As illustrated in Fig. 5, the proposed algorithm achieves more precise results than any other conventional algorithms.

5. Conclusion

The proposed fault location algorithm for double-circuit transmission lines is achieved through the circuit analysis combined with the voltage and current of the sending-end. The test results demonstrate a high accuracy almost not influenced by the variations of the fault resistance. And the

comparison results have also verified that the proposed algorithm is more accurate than the other three conventional algorithms.

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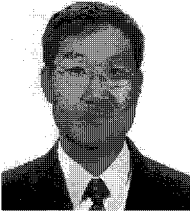
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