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A Study on the Combination of Deductible System with Bonus-Malus System[1)]

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Abstract

Bonus-Malus system in automobile insurance rewards claim-free policyholders by premium discounts and penalizes policyholders with claims by premium surcharges. The purpose of adopting bonus-malus system is to alleviate differences in risk propensity. A well-known side-effect of bonus-malus system is the tendency of policyholders to pay small claims themselves and not report them to their, in order to avoid future premium increases. This phenomenon is called hunger for bonus.

In this paper, we introduces an alternative approach to the Bonus-Malus system in automobile insurance - the approach is based on a deductible theory; and then search for a proper way combining both of them. Also, we construct a new algorithm to determine the optimal strategy of the policyholder based on the proposed model.

Keywords : Automobile Insurance, Bonus-Malus System, Deductible, Optimal Bonus-Malus System

1. Introduction

A well-known side-effect of bonus-malus system is the tendency of policyholders to pay small claims themselves and not report them to their carrier, in order to avoid future premium increases. This phenomenon is called "hunger for bonus". The aim of this paper is to introduce an alternative bonus-malus approach which eliminates this disadvantage.

A deductible is another common policy provision. A deductible is a provision by which a specified amount is deducted from the total loss payment that otherwise

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would be payable. Deductible provisions typically are found in property and automobile insurance contracts. These are not used in life insurance because the insured's death is always a total loss, not reducible on a normal life policy. For the same reason, these are also not applicable to personal liability insurance.

The deductible provision has several important purposes as following: Firstly, it eliminates small claims that are expensive to handle and process, so that the insurer's loss-investigating expenses would be reduced. Secondly, it has a positive effect of reducing premiums on the side of policyholders. As a result of small losses being eliminated, more of the premium could be used for the larger claims that might cause serious financial insecurity. Insurance is not an appropriate technique for paying small losses that can be better budgeted out of personal or business income. That is, insurance is orientated toward covering large catastrophic events. Insurance contracts that protect against a catastrophic loss can be taken out more economically in the case that the deductible provision is employed. This concept of using insurance premiums to pay for large losses rather than for small losses is often called the large-loss principle. Finally, the deductible provision would lead to reducing moral hazard because some dishonest insureds may deliberately cause a loss in order to make a profit from insurance. This provision encourages persons to be more careful with respect to the protection of their property and prevention of a loss.

The following deductible provision are commonly found in property insurance contracts. With a straight deductible provision the insured must pay a certain amount of loss before the insured is required to make a payment. Such a deductible typically applies to each loss. In some commercial property insurance contracts, an aggregate deductible may be used, by which all the losses occurred during the year are added together until they reach a certain level defined in a contract. If total losses occurred are below the aggregate deductible, the insurer pays nothing. Once total losses aggregated during the year exceed the defined level, all losses thereafter are paid in full. A franchise deductible provision is used in the sector of ocean marine insurance. With a franchise deductible provision, the insurer has no liability if the loss is under a certain amount defined in a contract, but once this amount is exceeded, the entire loss is paid in full.

In this paper, we will investigate the effect of introducing a deductible provision in the sector of automobile insurance so that we combine two major provisions, deductible and Bonus-Malus systems on the grounds that these two system has a similar effect on preventing losses and thus drivers are encouraged to drive more carefully. Moreover, bad drivers are penalized, since over a lifetime they are expected to pay more deductible than good drivers. So it may be argued that the Bonus-Malus system could be more reasonably modified through the deductible system.

2. Construction of an Optimal Bonus-Malus System Using a Deductible System

When a deductible is put into effect or an existing deductible is altered we are interested in both of the distribution of the amount paid and the severity. For a deductible amount of d the payment is described by the truncated and shifted random variable, W, given by

$$W = \begin{cases} X-d & \text{if} \quad X > d \\ 0 & \text{if} \quad X \leqq d \end{cases}.$$

The deductible has two effects. It eliminates some losses $(X \leq d)$ and reduces to $X-d$ the rest. For continuous loss distribution the c.d.f. and p.d.f. of W are respectively,

$$F_W(x) = \begin{cases} 0 & \text{if} \quad x \leqq 0 \\ \dfrac{F_X(x+d) - F_X(d)}{1 - F_X(d)} & \text{if} \quad x > 0 \end{cases},$$

and

$$f_W(x) = \begin{cases} 0 & \text{if} \quad x \leqq 0 \\ \dfrac{f_X(x+d)}{1 - F_X(d)} & \text{if} \quad x > 0 \end{cases}.$$

If the frequency of a loss (prior to imposing the deductible) is λ, then with a deductible amount of d the frequency will be $\lambda^* = \lambda[1 - F_X(d)]$.

As a consequence of these assumptions, the distribution of the number of accident during a given period can be written as

$$p_k(\lambda^*) = \frac{e^{-\lambda^* k}}{k!} \quad (\lambda^* > 0),$$

with a Γ-structure function

$$dU(\lambda^*) = \frac{\beta^{*a} e^{-\beta^* \lambda^*} \lambda^{*a-1}}{\Gamma(a)} d\lambda^*,$$

where $\lambda^* = \lambda[1 - F_X(d)]$ and $\beta^* = \dfrac{\beta}{(1 - F_X(d))}$. It is well-known(see for instance Haehling von Lanzenauer et al., 1973)) that in this case the distribution of the number of claims in the portfolio is a negative binomial with parameter $(\alpha, \beta^*/(1+\beta^*))$.

Suppose the risk class has been observed for t years and let $k_i(i=1,\cdots,t)$ be the number of claims declared during year i. These $k_i's$ are the realizations of random variable K_i, assumed to be independent and identically distributed. To each set of observations $(k_1,\cdots,k_t)$, we must associate a premium $P_{t+1}=P_{t+1}(k_1,\cdots,k_t)$.

Considering the assumptions of the model, we have

$$\begin{aligned} P(k_1,\cdots,k_t|\lambda^*) &= P(k_1|\lambda^*)\cdots P(k_t|\lambda^*) \\ &= \frac{\lambda^{*k_1}e^{-\lambda^*}}{k_1!}\cdots\frac{\lambda^{*k_t}e^{-\lambda^*}}{k_t!} \\ &= \frac{\lambda^{*k}e^{-t\lambda^*}}{\Pi_{j=1}^{t}(k_j!)}. \end{aligned}$$

If $P(k_1,\cdots,k_t|\lambda^*)$ denotes the probability that a policyholder with given parameter λ^* will produce a sequence $(k_1,\cdots,k_t)$, the posteriori distribution of λ^* is

$$dU(\lambda^*|k_1,\cdots,k_t)=\frac{P(k_1,\cdots,k_t|\lambda^*)dU(\lambda^*)}{\int_0^\infty P(k_1,\cdots,k_t|\lambda^*)dU(\lambda^*)}.$$

The negative binomial model has the interesting property that the posteriori distribution of the claim frequency λ^* also admits Γ-distribution

$$dU(\lambda^*|k_1,\cdots,k_t)=\frac{\beta^{*\alpha+k}\lambda^{*\alpha+k-1}e^{-\beta^*\lambda^*}}{\Gamma(\alpha+k)}d\lambda^*,$$

with parameters $\alpha+k$ and $\beta^*=\beta/[1-F_X(d)]+t$, where $k=\sum_{i=1}^{t}k_i$ is the total number of claims.

The expected value principle defines the premium $P_{t+1}(k_1,\cdots,k_t)$ by

$$P_{t+1} = (1+\delta)\int_0^\infty \lambda^* dU(\lambda^* | k_1, \cdots, k_t),$$

where δ is a safety loading.
It is easier to define a bonus-malus system by the relativities

$$\frac{\int_0^\infty \lambda^* dU(\lambda^* | k_1, \cdots, k_t)}{\int_0^\infty \lambda^* dU(\lambda^*)} \times 100,$$

i. e. the premium the policyholder has to pay if its initial premium (t=0) is 100. The relativities are in this case

$$P_{t+1}'(k_1, \cdots, k_t) = \frac{\alpha + k}{\beta + [1 - F_X(d)] \cdot t} \frac{\beta}{\alpha} \times 100.$$

Applied to our example, they provide the following bonus-malus table. Table 3.1. ~ Table 3.3. gives the result for P^*_{t+1} for various t, k and deductible amount d. We limited ourselves to $k=6$ since $k>6$ accidents is most unlikely to occur. In Table 3.1. ~ Table 3.3., we observe that three variable may change the level of insurance premium, i.e. time, the number of accumulated accidents and deductible amount. Also, if the amount of the deductible is increasing, then the bonus-malus rates is increasing. We can easily check from Table 3.1. that each insured always pays a premium proportional to the estimation of his frequency of accidents, according to the information accumulated during t years. For example, if the insured had 1 accident during his first year, he would have a surcharge of 80.95%((180.95-100)/100). But if he had no accident during that first year, he is entitled to a reduction of 9.2%((100-90.79)/100). Later, if he has 1 accident during the second year (and none in his first), he will be penalized 82.50%((165.69-90.79)/90.79). Similarly, if he has no accident during the second year (and 1 in the first year), he will be granted a bonus of 8.4%((180.95-165.69)/180.95).

<Table 3.1. Deductible Bonus-Malus Rate Table : d=50,000>

	k						
t	0	1	2	3	4	5	6
0	100						
1	90.79	180.95	271.11	361.26	451.42	541.57	631.73
2	83.14	165.69	248.25	330.81	413.36	495.92	578.47
3	76.67	152.81	228.95	305.09	381.22	457.36	533.50
4	71.14	141.79	212.43	283.08	353.72	424.37	495.01
5	66.36	132.25	198.14	264.03	329.92	395.81	461.70
6	62.17	123.91	185.65	247.38	309.12	370.86	432.59
7	58.49	116.56	174.64	232.71	290.79	348.86	406.94

<Table 3.2. Deductible Bonus-Malus Rate Table : d=250,000>

	k						
t	0	1	2	3	4	5	6
0	100						
1	91.94	183.24	274.53	365.83	457.12	548.42	639.71
2	85.08	169.57	254.06	338.54	423.03	507.51	592.00
3	79.18	157.80	236.42	315.04	393.67	472.29	550.91
4	74.04	147.56	221.08	294.60	368.12	441.64	515.16
5	69.53	138.57	207.60	276.64	345.68	414.72	483.76
6	65.53	130.61	195.68	260.75	325.82	390.90	455.97
7	61.97	123.51	185.05	246.59	308.12	369.66	431.20

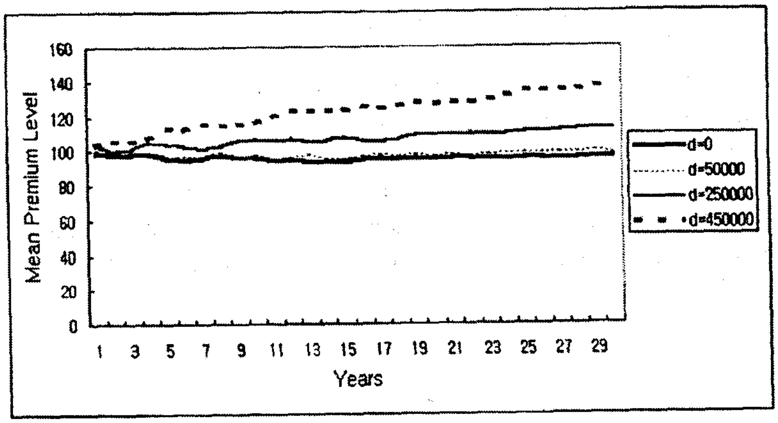
<Table 3.3. Deductible Bonus-Malus Rate Table : d=450,000>

	k						
t	0	1	2	3	4	5	6
0	100						
1	93.67	186.68	279.70	372.71	465.73	558.74	651.75
2	88.10	175.57	263.05	350.53	438.00	525.48	612.96
3	83.15	165.71	248.27	330.83	413.40	495.96	578.52
4	78.72	156.89	235.07	313.24	391.41	469.58	547.75
5	74.75	148.97	223.19	297.42	371.64	445.86	520.09
6	71.15	141.81	212.46	283.12	353.77	424.43	495.08
7	67.89	135.30	202.72	270.13	337.55	404.96	472.37

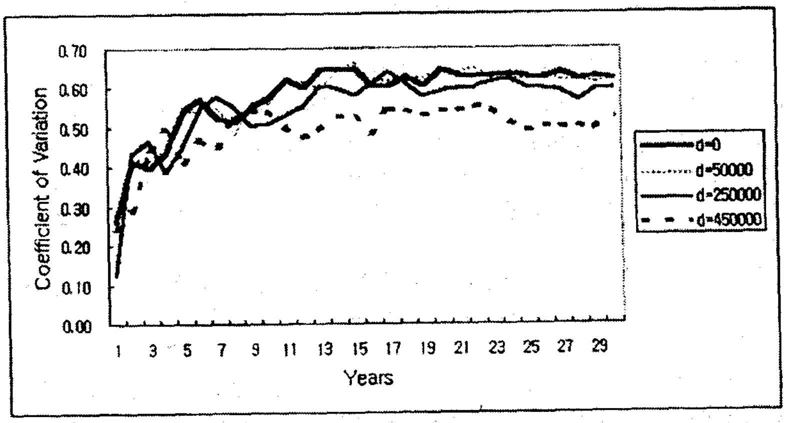
3. Simulation Study

We simulated a portfolio of 50,000 policyholders with the characteristics of the observed distribution, i. e. a mean of 0.10485 and a variance of 0.11576.

Figure 3.1 shows the evolution of the mean premium level for some specified values of deductible amount. Figure 3.2 shows the evolution of the coefficient of variation with time. As the result of the simulation, the mean premium level has a tendency of increasing as time goes under the condition of $d > 0$. At each time, the mean premium level is as large as the deductible amount d. There is no relationship between coefficient of variation and deductible amount.



<Figure 3.1 The Evolution of Mean Premium Level : Deductible Case>



<Figure 3.2 The Evolution of Coefficient of Variation : Deductible Case>

4. CONCLUSIONS AND FUTURE WORK

Bonus-Malus system in automobile insurance rewards claim-free policyholders by premium discounts and penalizes policyholders with claims by premium surcharges. The purpose of adopting bonus-malus system is to alleviate differences in risk propensity. Bonus-Malus system is generally constructed based on claim frequency and Bayesian credibility model is used to represent claim frequency distribution.

However, there is a problem with traditionally used credibility model for the purpose of constructing bonus-malus system. In this paper, we introduce an alternative approach to the Bonus-Malus system in automobile insurance. This approach is based on a deductible theory; an then search for a proper way combining both of them.

Many efforts have recently been focused on the study of discrete time series. In developing such models the integer-valued first-order autoregressive(INAR(1)) process, introduced independently by McKenzie(1986) and Al-Osh and Alzaid(1987), has received considerable attention. We plan to study integer-valued first-order autoregressive process for the construction of bonus-malus system. Finally, an effort will be made to rationalize the forms of our models in which the coefficients vary stochastically.

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