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Reliability Estimation in an Exponentiated Logistic Distribution under Multiply Type-II Censoring

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Abstract

In this paper, we derive the approximate maximum likelihood estimators of the scale parameter and location parameter in an exponentiated logistic distribution based on multiply Type-II censored samples. We compare the proposed estimators in the sense of the mean squared error for various censored samples.

We also propose and compare the estimators of the reliability function by using the proposed estimators of the parameters.

Keywords : Approximate Maximum Likelihood Estimator, Exponentiated Logistic Distribution, Multiply Type-II Censored Sample, Reliability

1. Introduction

A random variable X is said to have an exponentiated logistic distribution if X has the probability density function (pdf) of the form

$$f(x;\theta,\sigma) = \frac{\lambda}{\sigma}\left[1+\exp\left\{-\frac{x-\theta}{\sigma}\right\}\right]^{-\lambda-1}\exp\left\{-\frac{x-\theta}{\sigma}\right\},\quad \sigma,\lambda>0,\ -\infty<\theta<\infty. \qquad (1.1)$$

In the special case, when $\lambda=1$, this distribution is the logistic distribution.

Two-parameter exponentiated Weibull distribution was introduced by Mudholkar

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and Srivastava (1993), which is an extension of the well-known Weibull distribution. Shao (2002) provided a full solution for the maximum likelihood estimates of the Type-I generalized logistic distribution.

The approximated maximum likelihood estimating method was first developed by Balakrishnan (1989) for the purpose of providing the explicit estimators of the scale parameter in the Rayleigh distribution. Balakrishnan et al. (2004) discussed point and interval estimation for the extreme value distribution under progressively Type-II censoring. Kang and Park (2005) derived the approximate maximum likelihood estimators (AMLEs) of the scale parameter in the half-logistic distribution based on multiply Type-II censored samples. Chiou (1987) proposed a preliminary test estimator for the reliability of an exponential life-testing model.

Recently, Han and Kang (2007) obtained the AMLEs of the scale parameter and the location parameter in a generalized extreme value distribution under multiply Type-II censored sample. Ali et al. (2007) studied the properties of some new exponentiated distributions which include the two-parameter exponentiated logistic distribution. They also obtained the moments of the distributions.

In this paper, we derive the AMLEs of the scale parameter σ and the location parameter θ under multiply Type-II censoring when the parameter λ is known. The scale parameter is estimated by approximate maximum likelihood estimation method using two different Taylor series expansion types. We also propose the estimators of the reliability function by using the proposed estimators of the parameters and compare the proposed estimators in the sense of the mean squared error (MSE).

2. Estimators of the Two Parameters

Let

$$X_{a_1:n} \le X_{a_2:n} \le \dots \le X_{a_s:n} \tag{2.1}$$

be the available multiply Type-II censored sample from the exponentiated logistic distribution with pdf (1.1), where

$$1 \le a_1 < a_2 < \dots < a_s \le n$$

and $X_{a_i:n}$ is the a_ith order statistic.

Let $a_0 = 0$, $a_{s+1} = n+1$, $F(x_{a_0:n}) = 0$, $F(x_{a_{s+1}:n}) = 1$, then the likelihood function based on the multiply Type-II censored sample (2.1) can be written as

$$L(\theta,\sigma;\boldsymbol{x}) = n!\prod_{j=1}^{s} f(x_{a_j:n})\prod_{j=1}^{s+1} \frac{\left[F(x_{a_j:n}) - F(x_{a_{j-1}:n})\right]^{a_j-a_{j-1}-1}}{(a_j-a_{j-1}-1)!}. \tag{2.2}$$

The random variable $Z_{i:n} = (X_{i:n}-\theta)/\sigma$ has a standard exponentiated logistic distribution with pdf and cumulative distribution function (cdf);

$$f(z) = \lambda\left[1+e^{-z}\right]^{-\lambda-1}e^{-z}, \quad F(z) = \left[1+e^{-z}\right]^{-\lambda}, \ -\infty < z < \infty, \lambda > 0.$$

Since

$$\frac{f(z)}{F(z)} = \frac{\lambda e^{-z}}{1+e^{-z}}, \quad \frac{f'(z)}{f(z)} = \frac{1+\lambda}{1+e^{-z}}e^{-z}-1,$$

we can obtain the likelihood equations as follows;

$$\begin{aligned}\frac{\partial \ln L}{\partial\sigma} = -\frac{1}{\sigma}\Bigg[& s+(a_1-1)\lambda\frac{e^{-Z_{a_1:n}}}{1+e^{-Z_{a_1:n}}}Z_{a_1:n} - (n-a_s)\frac{f(Z_{a_s:n})}{1-F(Z_{a_s:n})}Z_{a_s:n} - \sum_{j=1}^{s} Z_{a_j:n} \\ &+ (1+\lambda)\sum_{j=1}^{s}\frac{e^{-Z_{a_j:n}}}{1+e^{-Z_{a_j:n}}}Z_{a_j:n} + \sum_{j=2}^{s}(a_j-a_{j-1}-1)\frac{f(Z_{a_j:n})Z_{a_j:n} - f(Z_{a_{j-1}:n})Z_{a_{j-1}:n}}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})}\Bigg] \\ =0 &\end{aligned} \tag{2.3}$$

and

$$\begin{aligned}\frac{\partial \ln L}{\partial\theta} = -\frac{1}{\sigma}\Bigg[& (a_1-1)\lambda\frac{e^{-Z_{a_1:n}}}{1+e^{-Z_{a_1:n}}} - (n-a_s)\frac{f(Z_{a_s:n})}{1-F(Z_{a_s:n})} + (1+\lambda)\sum_{j=1}^{s}\frac{e^{-Z_{a_j:n}}}{1+e^{-Z_{a_j:n}}} - s \\ &+ \sum_{j=2}^{s}(a_j-a_{j-1}-1)\frac{f(Z_{a_j:n}) - f(Z_{a_{j-1}:n})}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})}\Bigg] \\ =0. &\end{aligned} \tag{2.4}$$

Since the likelihood equations are very complicated, the equations (2.3) and (2.4) do not admit explicit solutions for σ and θ, respectively.

Let

$$\xi_i = F^{-1}(p_i) = -\ln\left[p_i^{-1/\lambda}-1\right] \text{ where } p_i = \frac{i}{n+1}, \ q_i = 1-p_i, \ \lambda \text{ is known.}$$

First, we can approximate the following functions by

$$\frac{e^{-Z_{a_j:n}}}{1+e^{-Z_{a_j:n}}}Z_{a_j:n} \approx \kappa_{1j} + \delta_{1j}Z_{a_j:n} \tag{2.5}$$

$$\frac{f(Z_{a_s:n})}{1-F(Z_{a_s:n})}Z_{a_s:n} \approx \alpha_1+\beta_1 Z_{a_s:n} \tag{2.6}$$

and

$$\frac{f(Z_{a_j:n})Z_{a_j:n}-f(Z_{a_{j-1}:n})Z_{a_{j-1}:n}}{F(Z_{a_j:n})-F(Z_{a_{j-1}:n})} \approx \alpha_{1j}+\beta_{1j}Z_{a_j:n}+\gamma_{1j}Z_{a_{j-1}:n} \tag{2.7}$$

where

$$\kappa_{1j}=p_{a_j}^{1/\lambda}\left[1-p_{a_j}^{1/\lambda}\right]\xi_{a_j}^2, \qquad \delta_{1j}=\left[1-p_{a_j}^{1/\lambda}\xi_{a_j}\right]\left[1-p_{a_j}^{1/\lambda}\right]$$

$$\alpha_1=-\frac{\xi_{a_s}^2}{q_{a_s}}\left[f'(\xi_{a_s})+\frac{f^2(\xi_{a_s})}{q_{a_s}}\right], \qquad \beta_1=\frac{1}{q_{a_s}}\left[f(\xi_{a_s})+f'(\xi_{a_s})\xi_{a_s}+\frac{f^2(\xi_{a_s})}{q_{a_s}}\xi_{a_s}\right]$$

$$\alpha_{1j}=K^2-\frac{f'(\xi_{a_j})\xi_{a_j}^2-f'(\xi_{a_{j-1}})\xi_{a_{j-1}}^2}{p_{a_j}-p_{a_{j-1}}}, \quad \beta_{1j}=\frac{1}{p_{a_j}-p_{a_{j-1}}}\left[(1-K)f(\xi_{a_j})+f'(\xi_j)\xi_{a_j}\right]$$

$$\gamma_{1j}=-\frac{1}{p_{a_j}-p_{a_{j-1}}}\left[(1-K)f(\xi_{a_{j-1}})+f'(\xi_{j-1})\xi_{a_{j-1}}\right], \quad K=\frac{f(\xi_{a_j})\xi_{a_j}-f(\xi_{a_{j-1}})\xi_{a_{j-1}}}{p_{a_j}-p_{a_{j-1}}}.$$

By substituting the equations (2.5), (2.6), and (2.7) into the equation (2.3), we can derive an estimator of σ as follows;

$$\hat{\sigma}_1=\frac{B_1+C_1\hat{\theta}}{A_1} \tag{2.8}$$

where

$$A_1=s+(a_1-1)\lambda\kappa_{11}-(n-a_s)\alpha_1+(1+\lambda)\sum_{j=1}^{s}\kappa_{1j}+\sum_{j=2}^{s}(a_j-a_{j-1}-1)\alpha_{1j}$$

$$B_1=(a_1-1)\lambda\delta_{11}X_{a_1:n}-(n-a_s)\beta_1X_{a_s:n}+(1+\lambda)\sum_{j=1}^{s}\delta_{1j}X_{a_j:n}-\sum_{j=1}^{s}X_{a_j:n}$$
$$+\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\beta_{1j}X_{a_j:n}+\gamma_{1j}X_{a_{j-1}:n})$$

$$C_1=(a_1-1)\lambda\delta_{11}-(n-a_s)\beta_1+(1+\lambda)\sum_{j=1}^{s}\delta_{1j}-s+\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\beta_{1j}+\gamma_{1j}).$$

Second, we can approximate the following functions by

$$\frac{e^{-Z_{a_j:n}}}{1+e^{-Z_{a_j:n}}} \approx \kappa_{2j}+\delta_{2j}Z_{a_j:n} \tag{2.9}$$

$$\frac{f(Z_{a_s:n})}{1-F(Z_{a_s:n})} \approx \alpha_2+\beta_2 Z_{a_s:n} \tag{2.10}$$

$$\frac{f(Z_{a_j:n})}{F(Z_{a_j:n})-F(Z_{a_{j-1}:n})} \approx \alpha_{2j}+\beta_{2j} Z_{a_j:n}+\gamma_{2j} Z_{a_{j-1}:n} \tag{2.11}$$

and

$$\frac{f(Z_{a_{j-1}:n})}{F(Z_{a_j:n})-F(Z_{a_{j-1}:n})} \approx \alpha_{3j}+\beta_{3j} Z_{a_j:n}+\gamma_{3j} Z_{a_{j-1}:n} \tag{2.12}$$

where

$$\kappa_{2j}=\left[1+p_{a_j}^{1/\lambda}\xi_{a_j}\right]\left[1-p_{a_j}^{1/\lambda}\right], \qquad \delta_{2j}=-p_{a_j}^{1/\lambda}\left[1-p_{a_j}^{1/\lambda}\right]$$

$$\alpha_2=\frac{1}{q_{a_s}}\left[f(\xi_{a_s})-f'(\xi_{a_s})\xi_{a_s}-\frac{f^2(\xi_{a_s})}{q_{a_s}}\xi_{a_s}\right], \qquad \beta_2=\frac{1}{q_{a_s}}\left[f'(\xi_{a_s})+\frac{f^2(\xi_{a_s})}{q_{a_s}}\right]$$

$$\alpha_{2j}=\frac{1}{p_{a_j}-p_{a_{j-1}}}\left[(1+K)f(\xi_{a_j})-f'(\xi_j)\xi_{a_j}\right], \qquad \beta_{2j}=\frac{1}{p_{a_j}-p_{a_{j-1}}}\left[f'(\xi_{a_j})-\frac{f^2(\xi_{a_j})}{p_{a_j}-p_{a_{j-1}}}\right]$$

$$\gamma_{2j}=\frac{f(\xi_{a_j})f(\xi_{a_{j-1}})}{\left[p_{a_j}-p_{a_{j-1}}\right]^2}, \qquad \alpha_{3j}=\frac{1}{p_{a_j}-p_{a_{j-1}}}\left[(1+K)f(\xi_{a_{j-1}})-f'(\xi_{j-1})\xi_{a_{j-1}}\right]$$

$$\beta_{3j}=-\frac{f(\xi_{a_j})f(\xi_{a_{j-1}})}{\left[p_{a_j}-p_{a_{j-1}}\right]^2}, \qquad \gamma_{3j}=\frac{1}{p_{a_j}-p_{a_{j-1}}}\left[f'(\xi_{a_{j-1}})+\frac{f^2(\xi_{a_{j-1}})}{p_{a_j}-p_{a_{j-1}}}\right].$$

By substituting the equations (2.9), (2.10), (2.11), and (2.12) into the equation (2.3), we can also derive the other estimator of σ as follows;

$$\hat{\sigma}_2=\frac{-B_2+\sqrt{B_2^2-4sC_2}}{2s} \tag{2.13}$$

where

$$\begin{aligned}B_2 &= (a_1-1)\lambda\kappa_{21}X_{a_1:n}-(n-a_s)\alpha_2 X_{a_s:n}+(1+\lambda)\sum_{j=1}^{s}\kappa_{2j}X_{a_j:n}-\sum_{j=1}^{s}X_{a_j:n}\\ &\quad+\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\alpha_{2j}X_{a_j:n}-\alpha_{3j}X_{a_{j-1}:n})\\ &\quad-\left\{(a_1-1)\lambda\kappa_{21}-(n-a_s)\alpha_2+(1+\lambda)\sum_{j=1}^{s}\kappa_{2j}-s+\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\alpha_{2j}-\alpha_{3j})\right\}\hat{\theta}\end{aligned}$$

$$C_2 = (a_1-1)\lambda\delta_{21}(X_{a_1:n}-\hat{\theta})^2-(n-a_s)\beta_2(X_{a_s:n}-\hat{\theta})^2+(1+\lambda)\sum_{j=1}^{s}\delta_{2j}(X_{a_j:n}-\hat{\theta})^2$$
$$+\sum_{j=2}^{s}(a_j-a_{j-1}-1)\left\{\beta_{2j}(X_{a_j:n}-\hat{\theta})^2+2\gamma_{2j}(X_{a_j:n}-\hat{\theta})(X_{a_{j-1}:n}-\hat{\theta})-\gamma_{3j}(X_{a_{j-1}:n}-\hat{\theta})^2\right\}.$$

Next, Equation (2.4) does not admit an explicit solution for θ. But we can expand the following function as follows;

$$\frac{f(Z_{a_j:n})-f(Z_{a_{j-1}:n})}{F(Z_{a_j:n})-F(Z_{a_{j-1}:n})}\approx \alpha_{4j}+\beta_{4j}Z_{a_j:n}+\gamma_{4j}Z_{a_{j-1}:n} \tag{2.14}$$

where

$$\alpha_{4j}=\alpha_{2j}-\alpha_{3j},\ \beta_{4j}=\beta_{2j}-\beta_{3j},\ \text{and}\ \gamma_{4j}=\gamma_{2j}-\gamma_{3j}.$$

By substituting the equations (2.9), (2.10), and (2.14) into the equation (2.4), we can derive an estimator of θ as follows;

$$\hat{\theta}=\frac{E}{D} \tag{2.15}$$

where

$$E=A_\theta B_1-A_1B_\theta,\ D=A_\theta C_1-A_1C_\theta,$$
$$A_\theta=(a_1-1)\lambda\kappa_{21}-(n-a_s)\alpha_2+(1+\lambda)\sum_{j=1}^{s}\kappa_{2j}-s+\sum_{j=2}^{s}(a_j-a_{j-1}-1)\alpha_{4j}$$
$$B_\theta=(a_1-1)\lambda\delta_{21}X_{a_1:n}-(n-a_s)\beta_2X_{a_s:n}+(1+\lambda)\sum_{j=1}^{s}\delta_{2j}X_{a_j:n}$$
$$+\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\beta_{4j}X_{a_j:n}+\gamma_{4j}X_{a_{j-1}:n})$$
$$C_\theta=(a_1-1)\lambda\delta_{21}-(n-a_s)\beta_2+(1+\lambda)\sum_{j=1}^{s}\delta_{2j}+\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\beta_{4j}+\gamma_{4j}).$$

From the above formulas, the mean squared errors of the proposed estimators are simulated by Monte Carlo method for sample size $n=20, 50$, and various choices of censoring ($m=n-s$ is the number of unobserved or missing data) under multiply Type-II censored sample. These values are given in Table 1, Table 2 and Table 3.

The estimator $\hat{\sigma}_1$ and $\hat{\theta}$ are linear functions of available order statistics and $\hat{\sigma}_1$ is more efficient than $\hat{\sigma}_2$ in the sense of the MSE.

As expected, the MSEs of all estimators decrease as sample size n increases. For fixed sample size, the MSEs increase generally as the number of unobserved or missing data m increases and the MSEs decrease generally as λ increases.

3. Estimation of the Reliability

The probability of survival of an item until time t (called the reliability at time t) in the exponentiated logistic distribution with pdf (1.1) is

$$R(t) = 1 - F(t) = P[X > t] = 1 - \left[1 + \exp\left\{-\frac{x-\theta}{\sigma}\right\}\right]^{-\lambda}, \quad t > \theta, \ \sigma, \lambda > 0. \qquad (3.1)$$

Now, we consider the estimation of the reliability function $R(t)$ in the exponentiated exponential distribution based on the multiply Type-II censored sample (2.1).

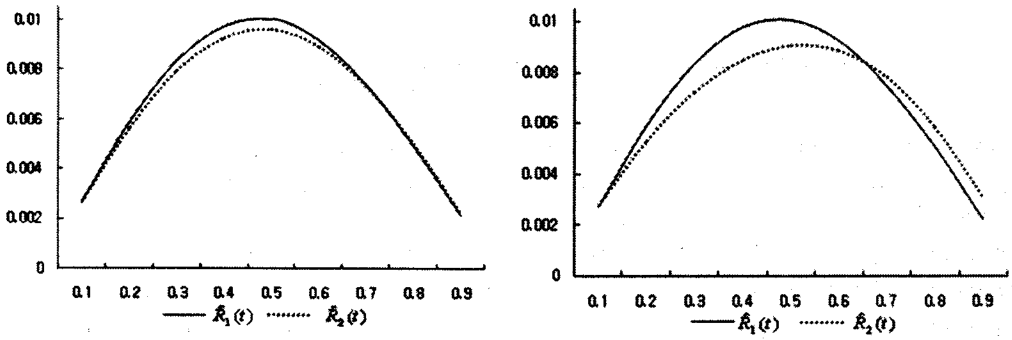
We now propose the estimators of the reliability function $R(t)$ by using the proposed estimators $\hat{\sigma}_i, i = 1,2$ and $\hat{\theta}$ that can be used for multiply Type-II censored sample as follows;

$$\hat{R}_i(t) = 1 - \left[1 + \exp\left\{-\frac{x-\hat{\theta}}{\hat{\sigma}_i}\right\}\right]^{-\lambda}, \quad i = 1, 2. \qquad (3.2)$$

From the equation (3.2), the mean squared errors of these estimators are simulated by Monte Carlo method for sample size $n = 20$, $m = 0$ and $m = 6$ (a_j=1, 2, 6~9, 12~15, 17~20) (see Fig 1 and 2). The MSEs of all estimators increase and then decrease as $R(t)$ increases.

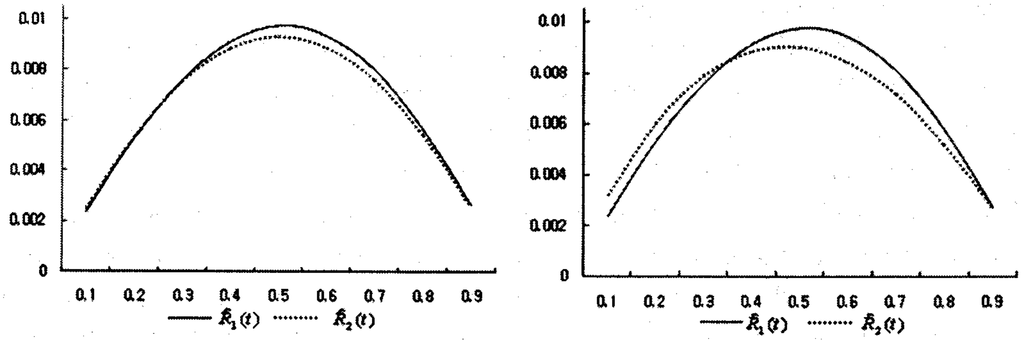
For $m = 0$, the estimator $\hat{R}_2(t)$ is generally a little more efficient than $\hat{R}_1(t)$. For multiply Type-II censored sample ($m = 6$), the estimator $\hat{R}_2(t)$ is generally more efficient than $\hat{R}_1(t)$ when $\lambda = 0.5$ and $R(t) < 0.6$. But the estimator $\hat{R}_1(t)$ is generally more efficient than $\hat{R}_2(t)$ when $\lambda = 0.5$ and $R(t) \geq 0.6$.

For $\lambda = 2.0$ and $m = 6$, the estimator $\hat{R}_1(t)$ is generally more efficient than $\hat{R}_2(t)$ when $R(t) < 0.3$, but the estimator $\hat{R}_2(t)$ is generally more efficient than $\hat{R}_1(t)$ when $R(t) \geq 0.3$. That is, the estimator $\hat{R}_1(t)$ is generally more efficient than $\hat{R}_2(t)$ for small $R(t)$, but the estimator $\hat{R}_2(t)$ is generally more efficient than $\hat{R}_1(t)$ for large $R(t)$.



(a) $m=0$ (b) $m=6$

<Fig. 1> The relative mean squared errors of the reliability function $R(t)$ ($\lambda=0.5$).



(a) $m=0$ (b) $m=6$

<Fig. 2> The relative mean squared errors of the reliability function $R(t)$ ($\lambda=2.0$).

<Table 1> The relative mean squared errors for the proposed estimators of the location parameter θ and scale parameter σ ($\lambda = 0.5$).

n	m	a_j	MSE			n	m	a_j	MSE		
			$\hat{\theta}$	$\hat{\sigma}_1$	$\hat{\sigma}_2$				$\hat{\theta}$	$\hat{\sigma}_1$	$\hat{\sigma}_2$
20	0	1~20	0.259435	0.037337	0.038480	50	0	1~50	0.102291	0.015413	0.015593
	1	1~19	0.261315	0.039486	0.040072		1	1~49	0.102415	0.015728	0.015854
		2~20	0.259617	0.039023	0.040131			2~50	0.102292	0.015697	0.015877
	2	1~18	0.263866	0.041848	0.042227		2	1~48	0.102617	0.016019	0.016122
		3~20	0.259719	0.040872	0.042000			3~50	0.102319	0.016003	0.016181
		2~19	0.261545	0.041354	0.041870			2~49	0.102415	0.016012	0.016136
	3	1~17	0.271862	0.045019	0.045256		3	1~47	0.102864	0.016347	0.016424
		4~20	0.259868	0.042901	0.043993			4~50	0.102375	0.016245	0.016425
		2~18	0.264215	0.044019	0.044327			2~48	0.102618	0.016311	0.016412
	4	2~17	0.272454	0.047544	0.047702		4	2~47	0.102867	0.016654	0.016727
		4~19	0.261851	0.045639	0.046070			4~49	0.102501	0.016576	0.016697
		3~18	0.264397	0.046349	0.046630			3~48	0.102647	0.016633	0.016729
		2~4 7~14 16~20	0.259954	0.039196	0.045466			2~4 7~14 16~50	0.102290	0.015732	0.017262
	5	3~17	0.272858	0.050324	0.050427		5	3~47	0.102899	0.016988	0.017058
		4~18	0.264706	0.048989	0.049188			4~48	0.102707	0.016888	0.016984
		2~6 10~19	0.262083	0.041698	0.044795			2~6 10~19 21~50	0.102303	0.015743	0.017252
	6	4~17	0.273467	0.053486	0.053502		6	4~47	0.102965	0.017257	0.017324
		1 2 6~9 12~15 17~20	0.260377	0.038329	0.050209			1 2 6~9 12~15 17~50	0.102308	0.015559	0.018472

<Table 2> The relative mean squared errors for the proposed estimators of the location parameter θ and scale parameter σ ($\lambda=1.0$).

n	m	a_j	MSE			n	m	a_j	MSE		
			$\hat{\theta}$	$\hat{\sigma}_1$	$\hat{\sigma}_2$				$\hat{\theta}$	$\hat{\sigma}_1$	$\hat{\sigma}_2$
20	0	1~20	0.150310	0.034477	0.035482	50	0	1~50	0.059502	0.014201	0.014367
	1	1~19	0.150353	0.036253	0.036907		1	1~49	0.059497	0.014464	0.014595
		2~20	0.150419	0.036240	0.036918			2~50	0.059521	0.014464	0.014607
	2	1~18	0.150398	0.038034	0.038530		2	1~48	0.059509	0.014703	0.014821
		3~20	0.150792	0.038340	0.038907			3~50	0.059542	0.014768	0.014896
		2~19	0.150421	0.038188	0.038481			2~49	0.059515	0.014727	0.014834
	3	1~17	0.151298	0.040604	0.040991		3	1~47	0.059519	0.014958	0.015057
		4~20	0.151318	0.040868	0.041327			4~50	0.059540	0.015043	0.015159
		2~18	0.150417	0.040233	0.040364			2~48	0.059526	0.014973	0.015066
	4	2~17	0.151255	0.043119	0.043129		4	2~47	0.059534	0.015239	0.015313
		4~19	0.151234	0.043259	0.043288			4~49	0.059532	0.015322	0.015401
		3~18	0.150701	0.042800	0.042788			3~48	0.059545	0.015290	0.015368
		2~4 7~14 16~20	0.150953	0.036376	0.040896			2~4 7~14 16~50	0.059543	0.014502	0.015705
	5	3~17	0.151439	0.046123	0.045958		5	3~47	0.059552	0.015567	0.015626
		4~18	0.151121	0.045974	0.045816			4~48	0.059541	0.015579	0.015644
		2~6 10~19	0.151673	0.038298	0.040396			2~6 10~19 21~50	0.059574	0.014507	0.015708
	6	4~17	0.151739	0.049834	0.049519		6	4~47	0.059546	0.015869	0.015913
		1 2 6~9 12~15 17~20	0.151785	0.035482	0.045178			1 2 6~9 12~15 17~50	0.059559	0.014375	0.016831

<Table 3> The relative mean squared errors for the proposed estimators of the location parameter θ and scale parameter σ ($\lambda=2.0$).

n	m	a_j	MSE			n	m	a_j	MSE		
			$\hat{\theta}$	$\hat{\sigma}_1$	$\hat{\sigma}_2$				$\hat{\theta}$	$\hat{\sigma}_1$	$\hat{\sigma}_2$
20	0	1~20	0.108670	0.032837	0.033119	50	0	1~50	0.043183	0.013382	0.013462
	1	1~19	0.108868	0.034411	0.034452		1	1~49	0.043240	0.013613	0.013671
		2~20	0.111277	0.035128	0.035130			2~50	0.043517	0.013713	0.013752
	2	1~18	0.108997	0.035891	0.035789		2	1~48	0.043263	0.013828	0.013878
		3~20	0.115359	0.037763	0.037732			3~50	0.043924	0.014102	0.014130
		2~19	0.111549	0.036904	0.036646			2~49	0.043575	0.013945	0.013961
	3	1~17	0.108998	0.038123	0.037900		3	1~47	0.043291	0.014045	0.014081
		4~20	0.120840	0.040881	0.040795			4~50	0.044253	0.014481	0.014501
		2~18	0.111769	0.038676	0.038287			2~48	0.043600	0.014167	0.014175
	4	2~17	0.111814	0.041259	0.040755		4	2~47	0.043632	0.014397	0.014390
		4~19	0.121448	0.043188	0.042839			4~49	0.044322	0.014733	0.014728
		3~18	0.116145	0.041852	0.041430			3~48	0.044014	0.014574	0.014570
		2~4 7~14 16~20	0.111821	0.035169	0.037582			2~4 7~14 16~50	0.043603	0.013744	0.014423
	5	3~17	0.116326	0.044915	0.044365		5	3~47	0.044052	0.014817	0.014798
		4~18	0.122092	0.045698	0.045217			4~48	0.044351	0.014972	0.014959
		2~6 10~19	0.112461	0.036880	0.037725			2~6 10~19 21~50	0.043629	0.013742	0.014461
	6	4~17	0.122501	0.049347	0.048746		6	4~47	0.044394	0.015229	0.015201
		1 2 6~9 12~15 17~20	0.111167	0.033433	0.039407			1 2 6~9 12~15 17~50	0.043489	0.013547	0.015088

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