A Direct Adaptive Fuzzy Control of Nonlinear Systems with Application to Robot Manipulator Tracking Control

Young-Wan Cho, Ki-Sung Seo, and Hee-Jin Lee

Abstract: In this paper, we propose a direct model reference adaptive fuzzy control (MRAFC) for MIMO nonlinear systems whose structure is represented by the Takagi-Sugeno fuzzy model. The adaptive law of the MRAFC estimates the approximation error of the fuzzy logic system so that it provides asymptotic tracking of the reference signal for the systems with uncertain or slowly time-varying parameters. The developed control law and adaptive law guarantee the boundedness of all signals in the closed-loop system. In addition, the plant state tracks the state of the reference model asymptotically with time for any bounded reference input signal. To verify the validity and effectiveness of the MRAFC scheme, the suggested analysis and design techniques are applied to the tracking control of robot manipulator and simulation studies are carried out. In the control design, the MRAFC is combined with feedforward PD control to make the actual joint trajectories of the robot manipulator with system uncertainties track the desired reference joint position trajectories asymptotically stably.

Keywords: Adaptive fuzzy control, model reference adaptive control, nonlinear system, robot manipulator, Takagi-Sugeno model, tracking control.

1. INTRODUCTION

In some control tasks, such as those in robot manipulation, the systems to be controlled have constant or slowly-time varying uncertain parameters. Unless such parameter uncertainty is gradually reduced on-line by an appropriate adaptation or estimation mechanism, it may cause inaccuracy or instability for the control systems. In many other tasks, such as those in power systems, the system dynamics may have well known dynamics at the beginning, but experience unpredictable parameter variations as the control operation goes on. Without continuous redesign of the controller, the initially appropriate controller design may not be able to control the changing plant well [1-4]. The problem of adaptation of dynamical systems having parameter uncertainty has attracted a lot of research efforts in all times. In

particular, for nonlinear systems, several approaches have been proposed to deal with this important problem [5-7].

On the other hands, as a model free design method, fuzzy logic systems have been successfully applied to complex or mathematically understandable systems. However, the fuzzy control has not been regarded as a rigorous science due to the lack of guaranteed global stability and acceptable performance. To overcome these drawbacks, during the last decade, there has been growing interest in systematic analysis and design of fuzzy control systems such as stability and robustness [8-12]. One of the motivations in this research is the success developed in [9] where the authors provided a sufficient condition for the asymptotic stability of fuzzy control system based on Takagi-Sugeno (TS) model [8] in the sense of Lyapunov through the existence of a common Lyapunov function for all subsystems.

In recent years, in order to deal with the uncertainties of nonlinear systems in the fuzzy control system literature, a lot of effort has been put to adaptive fuzzy control system such as neural network based approaches [13-14], and the TS model based approaches [15-17]. The main advantages of adaptive fuzzy control over nonadaptive fuzzy control are: (1) better performance is usually achieved because the adaptive fuzzy controller can adjust itself to the changing environment, and (2) less information about the plant is required because the adaptation law can

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help to learn the dynamics of the plant during realtime operation. However, these approaches still have some problems. The adaptive control scheme proposed by Wang [13] guarantees the uniform boundedness of all signals of the control system but it is applicable only to single-input single-output system.

In many applications, the structure of the model of the plant may be known, but its parameters may be unknown and/or change with time. Recently, the concept of incorporating fuzzy logic control into the model reference adaptive control has grown into an interesting research topic [18-20]. In this paper, a direct model reference adaptive fuzzy control (MRAFC) scheme is proposed to provide asymptotic tracking of a reference signal for the systems having uncertain or slowly time-varying parameters. This paper presents the design and analysis of on-line parameter adaptation for the plant model whose structure is represented by the Takagi-Sugeno model. The adaptation law adjusts the controller parameters on-line so that the plant output tracks the reference model output. The developed control law and adaptive law guarantee the boundedness of all signals in the closed-loop system. In addition, the plant state tracks the state of the reference model asymptotically with time for any bounded reference input signal. Tracking control of robotic manipulators is an important research topic as many robots are widely used in industry to perform the task of path tracking at high speed with the requirement of high accuracy. The proposed adaptive fuzzy control scheme is applied to tracking control of robot manipulator to verify the validity and effectiveness of the control scheme.

2. TAKAGI-SUGENO MODEL BASED FUZZY CONTROL

In the control system design, it is important and significant to select an appropriate model representing a real system. As an expression model of a real plant, we use the fuzzy implications and the fuzzy reasoning method suggested by Takagi and Sugeno [8]. The Takagi-Sugeno (TS) fuzzy model is widely accepted as a powerful tool for design and analysis of fuzzy control systems and applications of the TS models to various kinds of nonlinear systems can be found. The Takagi-Sugeno fuzzy model uses smooth aggregation of local linear mathematical models to represent dynamical systems, which are useful because they can provide description of a physical phenomenon or a process, and can be well suited to analysis, prediction and design of dynamic control systems.

Consider the continuous-time nonlinear system described by the Takagi-Sugeno fuzzy model. The *i* th rule of continuous-time TS model is of the following form:

$$R^{i}: If x_{1}(t) is M_{1}^{i} and \cdots and x_{n}(t) is M_{n}^{i}$$

 $then \dot{\mathbf{x}}(t) = A_{i}\mathbf{x}(t) + B_{i}\mathbf{u}(t),$ (1)

where $R^{i}(i=1,2,\dots,l)$ denotes the *i*th implication, l is the number of fuzzy implications, $\mathbf{x}(t) = [x_{1}(t),\dots,x_{n}(t)]^{T}$, $\mathbf{u}(t) = [u_{1}(t),\dots,u_{m}(t)]^{T}$. Given a pair of input the final output of the fuzzy system is inferred as follows:

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{l} \omega_i(t) \{ A_i \mathbf{x}(t) + B_i \mathbf{u}(t) \}}{\sum_{i=1}^{l} \omega_i(t)},$$
(2)

where $\omega_i(t) = \prod_{j=1}^n M_j^i(x_j(t))$ and $M_j^i(x_j(t))$ is the

grade of membership of $x_i(t)$ in M_i^i .

In order to design fuzzy controllers to stabilize fuzzy system (2), we utilize the concept of parallel distributed compensation (PDC) [10]. The idea is to design compensators for each subsystem of the fuzzy model. Since the PDC controller shares the same fuzzy sets with fuzzy model (2) to construct its premise part, the resulting overall controller is a fuzzy blending of each individual subsystem controller of the following form:

$$R^{i}: If x_{1}(t) is M_{1}^{i} and \cdots and x_{n}(t) is M_{n}^{i}$$

 $then \mathbf{u}(t) = -K.\mathbf{x}(t).$ (3)

Given a state feedback $\mathbf{x}(t)$, the final output of the fuzzy PDC controller (3) is inferred as follows:

$$\mathbf{u}(t) = -\frac{\sum_{i=1}^{l} \omega_i(t) K_i \mathbf{x}(t)}{\sum_{i=1}^{l} \omega_i(t)}.$$
 (4)

By substituting the controller (4) into the model (2), we can construct the closed-loop fuzzy control system as following:

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_i(t) \omega_j(t) \{A_i - B_i K_j\} \mathbf{x}(t)}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_i(t) \omega_j(t)}.$$
 (5)

A sufficient condition for ensuring the stability of the closed-loop fuzzy system (5) is given in Theorem 1, which is derived in [10].

Theorem 1: The equilibrium of a fuzzy control

system (5) is asymptotically stable in the large if there exists a common positive definite matrix P such that

$$G_{ij}^T P + P G_{ij}^T = -Q (6)$$

for all $i, j = 1, 2, \dots, l$, where $G_{ij} = A_i - B_i K_j$ and Q_{ij} is a positive definite matrix.

The design problem of TS model based fuzzy control is to select K_i ($j = 1, 2, \dots, l$) such that stability conditions (6) in Theorem 1 are satisfied. It has long been recognized that there is lack of systematic procedures to find a common positive definite matrix P to check the stability of fuzzy control system. Most of the time, a trial-and-error type of procedure has been used. In [11], the common P problem was solved efficiently via optimization techniques for LMI's (Linear Matrix Inequality). However, the fuzzy control (4) does not guarantee the stability of system in the presence of parameter uncertainties or variations. Moreover, the design of control parameters is not possible for the systems whose parameters are unknown. To overcome these drawbacks, this paper proposes an adaptive control scheme for the TS fuzzy systems whose parameters are unknown or time varying.

3. DIRECT MODEL REFERENCE ADAPTIVE FUZZY CONTROL

In this section, a direct model reference adaptive fuzzy control (MRAFC) scheme for TS system is developed. Consider again the nonlinear system represented by the TS model (1) or (2), where the state $\mathbf{x} \in R^n$ is available for measurement, $A_i \in R^{n \times n}$ and $B_i \in R^{n \times q}$ $(i = 1, 2, \dots, l)$ are unknown constant matrices and (A_i, B_i) are controllable. The control objective is to choose the input vector $\mathbf{u} \in R^q$ such that all signals in the closed-loop plant are bounded and the plant state \mathbf{x} follows the state $\mathbf{x}_m \in R^n$ of the reference model specified by the following system

$$\mathbf{x}_{m}(t) = \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t)) \mu_{j}(\mathbf{x}(t)) \{(A_{m})_{ij} \mathbf{x}_{m}(t) + (B_{m})_{ij} \mathbf{r}\}}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t)) \mu_{j}(\mathbf{x}(t))},$$
(7)

where $(A_m)_{ij} \in R^{n \times n}$ $(i, j = 1, 2, \dots, l)$ satisfy the stability condition of fuzzy system given in Theorem 1, i.e., there exists a common symmetric positive definite matrix $P = P^T > 0$ such that $(A_m)_{ij}^T P + 1$

 $P(A_m)_{ij} < -Q_{ij}$ for all $Q_{ij} = Q_{ij}^T > 0$, $(B_m)_{ij} \in R^{n \times q}$, and $\mathbf{r} \in R^q$ is a bounded reference input vector. The reference model and input \mathbf{r} are chosen so that $\mathbf{x}_m(t)$ represents a desired trajectory that \mathbf{x} has to follow.

3.1. Control law

If the matrices A_i , B_i were known, we could apply the control law

$$\mathbf{u}(t) = \frac{\sum_{j=1}^{l} \mu_j(\mathbf{x}(t)) \{-K_j^* \mathbf{x}(t) + L_j^* \mathbf{r}(t)\}}{\sum_{j=1}^{l} \mu_j(\mathbf{x}(t))},$$
 (8)

where $\mu_j(\mathbf{x}(t)) = \omega_j(\mathbf{x}(t))$, and obtain the closed-loop control system

$$\frac{\dot{\mathbf{x}}(t) =}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_i(\mathbf{x}(t)) \mu_j(\mathbf{x}(t)) \{ (A_i - B_i K_j^*) \mathbf{x}(t) + B_i L_j^* \mathbf{r}(t) \}}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_i(\mathbf{x}(t)) \mu_j(\mathbf{x}(t))}.$$
(9)

Hence, if $K_j^* = R^{q \times n}$ and $L_j^* = R^{q \times q}$ are chosen to satisfy the algebraic equations

$$A_i - B_i K_i^* = (A_m)_{ii}, \quad B_i L_i^* = (B_m)_{ii},$$
 (10)

then the transfer matrix of the closed-loop system is the same as that of the reference model and $\mathbf{x}(t) \to \mathbf{x}_m(t)$ exponentially fast for any bounded reference input signal $\mathbf{r}(t)$. We should note that given the matrices A_i , B_i , $(A_m)_{ij}$ and $(B_m)_{ij}$, no K_j^* and L_j^* may exist to satisfy the matching condition (10) indicating that the control law (8) may not have enough structural flexibility to meet the control objective. In some cases, if the structure of A_i and B_i is known, $(A_m)_{ij}$ and $(B_m)_{ij}$ may be designed so that (10) has a solution for K_j^* and L_j^* .

Let us assume that K_j^* and L_j^* in (10) exist, i.e., there is sufficient structural flexibility to meet the control objective, and propose the control law

$$\mathbf{u}(t) = \frac{\sum_{j=1}^{l} \mu_{j}(\mathbf{x}(t)) \{-K_{j}(t)\mathbf{x}(t) + L_{j}(t)\mathbf{r}(t)\}}{\sum_{j=1}^{l} \mu_{j}(\mathbf{x}(t))}, \quad (11)$$

where $K_j(t)$ and $L_j(t)$ are the estimates of K_j^* and L_j^* , respectively, to be generated by an appropriate adaptive law.

3.2. Adaptive law

By adding and subtracting the desired input term, $\mathbf{u}(t) = \sum_{j=1}^{l} \mu_j(\mathbf{x}(t)) \{ -B_i(K_j^* \mathbf{x}(t) - L_j^* \mathbf{r}(t)) \} / \sum_{j=1}^{l} \mu_j(\mathbf{x}(t))$ to the plant equation (2) and using (10), we obtain

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))(A_{m})_{ij}}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))} \mathbf{x}(t)$$

$$+ \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))(B_{m})_{ij}}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))} \mathbf{r}(t)$$

$$= \sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))B_{i}(K_{j}^{*}\mathbf{x}(t) - L_{j}^{*}\mathbf{r}(t) + \mathbf{u}(t))$$

$$+ \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))B_{i}(K_{j}^{*}\mathbf{x}(t) - L_{j}^{*}\mathbf{r}(t) + \mathbf{u}(t))}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))B_{i}(K_{j}^{*}\mathbf{x}(t) - L_{j}^{*}\mathbf{r}(t) + \mathbf{u}(t))}$$

$$+\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}\omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))B_{i}(K_{j}^{*}\mathbf{x}(t)-L_{j}^{*}\mathbf{r}(t)+\mathbf{u}(t))}{\sum_{i=1}^{l}\sum_{j=1}^{l}\omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))}.$$

Furthermore, by adding and subtracting the estimated input term multiplied by $\sum_{i=1}^{l} \omega_i(\mathbf{x}(t)) B_i / \sum_{i=1}^{l} \omega_i(\mathbf{x}(t)),$ that is,

that is,
$$\frac{\sum_{i=1}^{l} \omega_i(\mathbf{x}(t)) B_i}{\sum_{i=1}^{l} \omega_i(\mathbf{x}(t))} \left[\frac{\sum_{j=1}^{l} \mu_j(\mathbf{x}(t)) \{K_j(t) \mathbf{x}(t) - L_j(t) \mathbf{r}(t)\}}{\sum_{j=1}^{l} \omega_i(\mathbf{x}(t))} - \mathbf{u}(t) \right] \qquad \dot{\mathbf{e}}(t) = \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_i(\mathbf{x}(t)) \mu_j(\mathbf{x}(t)) (A_m)_{ij}}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_i(\mathbf{x}(t)) \mu_j(\mathbf{x}(t))} \mathbf{e}(t) \\
\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_i(\mathbf{x}(t)) \mu_j(\mathbf{x}(t)) (B_m)_{ij} L_i^{*-1}(-\tilde{K}_j(t)) (B_m)_{ij} (B_m)_{ij}$$

to the reference model (7), we obtain

$$\dot{\hat{\mathbf{x}}}_{m}(t) = \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t)) \mu_{j}(\mathbf{x}(t)) (A_{m})_{ij}}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t)) \mu_{j}(\mathbf{x}(t))} \hat{\mathbf{x}}_{m}(t)$$

$$+ \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t)) \mu_{j}(\mathbf{x}(t)) (B_{m})_{ij}}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t)) \mu_{j}(\mathbf{x}(t))} \mathbf{r}(t)$$

$$+\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}\omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))B_{i}(K_{j}(t)\mathbf{x}(t) - L_{j}(t)\mathbf{r}(t) + \mathbf{u}(t))}{\sum_{i=1}^{l}\sum_{j=1}^{l}\omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))}.$$
(13)

By using the reference model (13), we can express (12) in terms of the tracking error defined as $e = x - x_m$, i.e.,

$$\begin{split} \dot{\mathbf{e}}(t) &= \frac{\sum\limits_{i=1}^{l}\sum\limits_{j=1}^{l}\omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))(A_{m})_{ij}}{\sum\limits_{i=1}^{l}\sum\limits_{j=1}^{l}\omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))} \\ &+ \frac{\sum\limits_{i=1}^{l}\sum\limits_{j=1}^{l}\omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))B_{i}(-\tilde{K}_{j}(t)\mathbf{x}(t)+\tilde{L}_{j}(t)\mathbf{r}(t))}{\sum\limits_{i=1}^{l}\sum\limits_{j=1}^{l}\omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))}, \end{split}$$

where $\tilde{K}_j(t) = K_j(t) - K_j^*$ and $\tilde{L}_j(t) = L_j(t) - L_j^*$. In the dynamic equation (14) of tracking error, B_i are unknown. We assume that L_j^* are either positive definite or negative definite and define $\Gamma_j^{-1} = L_j^* \operatorname{sgn}(l_j)$, where $l_j = 1$ if L_j^* is positive definite and $l_j = -1$ if L_j^* is negative definite. Then $B_i = (B_m)_{ij} L_i^{*-1}$ and (14) becomes

$$\dot{\mathbf{e}}(t) = \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))(A_{m})_{ij}}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))} \mathbf{e}(t)$$

$$+ \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))(B_{m})_{ij} L_{j}^{*-1}(-\tilde{K}_{j}(t)\mathbf{x}(t) + \tilde{L}_{j}(t)\mathbf{r}(t))}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}(t))\mu_{j}(\mathbf{x}(t))}.$$
(15)

Now, by using the tracking error dynamics (15), we derive the adaptive law for updating the desired control parameters K_j^* and L_j^* so that the closed-loop system (12) follows the reference model (7). We assume that the adaptive law has the general structure

$$\dot{K}_{j}(t) = F_{j}(\mathbf{x}(t), \mathbf{x}_{m}(t), \mathbf{e}(t), \mathbf{r}(t)),$$

$$\dot{L}_{j}(t) = G_{j}(\mathbf{x}(t), \mathbf{x}_{m}(t), \mathbf{e}(t), \mathbf{r}(t)),$$
(16)

where F_j and G_j $(j=1,\cdots,l)$ are functions of known signals that are to be chosen so that the equilibrium

$$K_{je} = K_j^*, \ L_{je} = L_j^*, \ \mathbf{e}_e = 0$$
 (17)

of dynamic equations (15) and (16) has some desired stability properties.

We propose the following Lyapunov function candidate

$$V(\mathbf{e}, \tilde{K}_{j}, \tilde{L}_{j}) = \mathbf{e}^{T} P \mathbf{e} + \sum_{j=1}^{l} tr(\tilde{K}_{j}^{T} \Gamma_{j} \tilde{K}_{j} + \tilde{L}_{j}^{T} \Gamma_{j} \tilde{L}_{j}),$$
(18)

where $P = P^T > 0$ is a common positive definite matrix of the Lyapunov equations $(A_m)_{ij}^T P + P(A_m)_{ij}$ $< -Q_{ij}$ for all $Q_{ij} = Q_{ij}^T > 0$ $(i, j = 1, 2, \dots, l)$, whose existence is guaranteed by the stability assumption for A_m and Theorem 1. From here, for the convenience of notation, we omit the time notation. Then, after some straightforward mathematical manipulations, we obtain the time derivative \dot{V} of V along the trajectory of (15) and (16) as

$$\dot{V} = -\mathbf{e}^{T} \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x}) Q_{ij}}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x})} \mathbf{e}$$

$$+2tr \left\{ -\frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x}) \tilde{K}_{j}^{T} \Gamma_{j}(B_{m})_{ij}^{T} \operatorname{sgn}(l_{j})}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x})} P \mathbf{e} \mathbf{x}^{T} \right.$$

$$+ \sum_{j=1}^{l} \tilde{K}_{j}^{T} \Gamma_{j} \dot{\tilde{K}}_{j}$$

$$+2tr \left\{ \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x}) \tilde{L}_{j}^{T} \Gamma_{j}(B_{m})_{ij}^{T} \operatorname{sgn}(l_{j})}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x})} P \mathbf{e} \mathbf{r}^{T} \right.$$

$$+ \sum_{j=1}^{l} \tilde{L}_{j}^{T} \Gamma_{j} \dot{\tilde{L}}_{j}$$

In the last two terms of (19), if we let

$$\sum_{j=1}^{l} \tilde{K}_{j}^{T} \Gamma_{j} \dot{\tilde{K}}_{j} = \sum_{j=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x}) \tilde{K}_{j}^{T} \Gamma_{j} (B_{m})_{ij}^{T} \operatorname{sgn}(l_{j}) \qquad (20a)$$

$$\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x})$$

$$\sum_{j=1}^{l} \tilde{L}_{j}^{T} \Gamma_{j} \dot{\tilde{L}}_{j} = \sum_{j=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x}) \tilde{L}_{j}^{T} \Gamma_{j} (B_{m})_{ij}^{T} \operatorname{sgn}(l_{j})$$

$$\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x}) \tilde{L}_{j}^{T} \Gamma_{j} (B_{m})_{ij}^{T} \operatorname{sgn}(l_{j})$$

$$\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x})$$

we can make \dot{V} to be negative, i.e.,

$$\dot{V} = -\mathbf{e}^{T} \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x}) Q_{ij}}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x})} \mathbf{e} \le 0.$$
(21)

Hence, the obvious choice for adaptive law to make \dot{V} negative is

$$\frac{\dot{K}_{j} = \dot{K}_{j}(t) =}{\left\{ \frac{\sum_{i=1}^{l} \omega_{i}(\mathbf{x})(B_{m})_{ij}^{T}}{\sum_{i=1}^{l} \omega_{i}(\mathbf{x})} \right\}} \left\{ \frac{\mu_{j}(\mathbf{x})}{\sum_{j=1}^{l} \mu_{j}(\mathbf{x})} \right\} \operatorname{sgn}(l_{j}) P \mathbf{e} \mathbf{x}^{T}, \tag{22a}$$

$$\dot{\tilde{L}}_{j} = \dot{L}_{j}(t) = \left\{ \frac{\sum_{i=1}^{l} \omega_{i}(\mathbf{x})(B_{m})_{ij}^{T}}{\sum_{i=1}^{l} \omega_{i}(\mathbf{x})} \right\} \left\{ \frac{\mu_{j}(\mathbf{x})}{\sum_{j=1}^{l} \mu_{j}(\mathbf{x})} \right\} \operatorname{sgn}(l_{j}) P \mathbf{e} \mathbf{r}^{T}. \tag{22b}$$

3.3 Implementation and analysis

All the quantities in the right hand sides of (22a) and (22b) are known or available for measurement. Therefore, the adaptive law (22) for direct model reference adaptive control of TS fuzzy system can be implemented. Fig. 1 illustrates the configuration of the direct MRAFC system. The reference model is used to specify the ideal response that the fuzzy control system should follow. The plant is assumed to contain

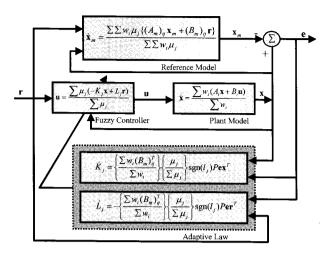


Fig. 1. The configuration of direct MRAFC.

unknown parameters, but its structure is known. The fuzzy controller is constructed from fuzzy systems whose parameters are adjustable. The adaptation law adjusts the control parameters K(t) and L(t) online such that the state \mathbf{x} of plant tracks the state \mathbf{x}_m of reference model, which allows plant output to follow the reference model output.

Using arguments previously discussed, we establish the following theorem which shows the properties of the MRAFC derived in this research. The control law (11) together with the adaptive law (22) guarantees boundedness for all signals in the closed-loop system. In addition, the plant state \mathbf{x} tracks the state of the reference model \mathbf{x}_m asymptotically with time for any bounded reference input signal \mathbf{r} .

Theorem 2 (Stability of the direct MRAFC): Consider the plant model (2) and the reference model (7) with the control law (11) and adaptive law (22). Assume that the input \mathbf{r} and the state \mathbf{x}_m of the reference model are uniformly bounded. Then the control law (11) and the adaptive law (22) guarantee that

(i) K(t), L(t), $\mathbf{e}(t)$ are bounded.

(ii)
$$\mathbf{e}(t) \to 0$$
 as $t \to \infty$.

Proof: From (18) and (21), it directly follows that V is a Lyapunov function for the system (15) and (16), which implies that the equilibrium given by (17) is uniformly stable, which, in turn, implies that the trajectories $\tilde{K}(t)$, $\tilde{L}(t)$, $\mathbf{e}(t)$ are bounded for all t>0. Because $\mathbf{e}=\mathbf{x}-\mathbf{x}_m$ and $\mathbf{x}_m\in\mathscr{L}_\infty$, we have that $\mathbf{x}\in\mathscr{L}_\infty$. From (11) and $\mathbf{r}\in\mathscr{L}_\infty$, we also have that $\mathbf{u}\in\mathscr{L}_\infty$; therefore, all signals in the closed-loop system are bounded. Now, let us show that $\mathbf{e}\in\mathscr{L}_\infty$. From (18) and (21), we conclude that because V is bounded from below and is nonincreasing with time, it has a limit, i.e.,

$$\lim_{t \to \infty} V(\mathbf{e}(t), \ \tilde{K}_j(t), \ \tilde{L}_j(t)) = V_{\infty} < \infty. \tag{23}$$

From (21) and (23), it follows that

$$\int_{0}^{\infty} \mathbf{e}^{T} \frac{\sum_{i,j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x}) Q_{ij}}{\sum_{i,j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x})} \mathbf{e} d\tau$$

$$= -\int_{0}^{\infty} \dot{V} d\tau = V_{0} - V_{\infty},$$
(24)

where $V_0 = V(\mathbf{e}(0), \tilde{K}_j(0), \tilde{L}_j(0))$. On the other hand, from $0 \le \omega_i(\mathbf{x}) \le 1$, $0 \le \mu_i(\mathbf{x}) \le 1$, and $\lambda_{\min}(Q_{ii}) \|\mathbf{e}\|^2 \le \mathbf{e}^T Q_{ii} \mathbf{e} \le \lambda_{\max}(Q_{ii}) \|\mathbf{e}\|^2$, we have

$$(\lambda_{\min}(Q_{ij}))_{\min} \|\mathbf{e}\|^{2} \leq \mathbf{e}^{T} \left(\frac{\sum_{i,j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x}) Q_{ij}}{\sum_{i,j=1}^{l} \omega_{i}(\mathbf{x}) \mu_{j}(\mathbf{x})} \right) \mathbf{e}$$

$$\leq (\lambda_{\max}(Q_{ij}))_{\max} \|\mathbf{e}\|^{2},$$
(25)

where $(\lambda_{\min}(Q_{ij}))_{\min} = \min\{\lambda_{\min}(Q_{11}), \dots, \lambda_{\min}(Q_{ll})\},$ $(\lambda_{\max}(Q_{ij}))_{\max} = \min\{\lambda_{\max}(Q_{11}), \dots, \lambda_{\max}(Q_{ll})\}.$ After inserting (25) into (24), and straightforward manipulation, we have

$$\frac{V_0 - V_{\infty}}{\left(\lambda_{\min}\left(Q_{ij}\right)\right)_{\min}} \leq \int_0^{\infty} \left\|\mathbf{e}\right\|^2 d\tau \leq \frac{V_0 - V_{\infty}}{\left(\lambda_{\max}\left(Q_{ij}\right)\right)_{\max}},$$

which implies that $\mathbf{e} \in \mathcal{L}$. Because \mathbf{e} , K_j , L_j , $\mathbf{r} \in \mathcal{L}$, it follows from (15) that $\mathbf{e} \in \mathcal{L}$, which, together with $\mathbf{e} \in \mathcal{L}$, implies that $\mathbf{e}(t) \to 0$ as $t \to \infty$.

4. TRACKING CONTROL OF A 2-LINK ROBOT MANIPULATOR

In this section, the validity and effectiveness of the proposed MRAFC scheme are examined through the simulation of tracking control for two-link robot manipulator shown in Fig. 2.

The objective of the adaptive tracking control design of a robot manipulator is to derive an adaptive control law for the actuator torque ${\bf u}$ to make the actual trajectories of the robot manipulator with system uncertainties to track the given desired trajectories ${\bf q}_d(t)$ of the joint position and velocity with desired accuracy and stability. In the simulation,

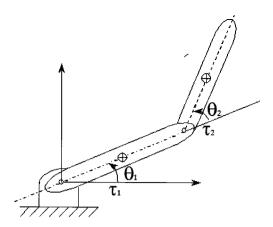


Fig. 2. An articulated 2-link robot manipulator.

we examine the effects of parametric variation caused by some internal uncertainties on behaviors of the closed-loop control system with the nonadaptive PD controller, adaptive sliding mode controller, and the proposed TS model based adaptive fuzzy control scheme, respectively.

4.1. Design of direct adaptive fuzzy tracking control system

In this section, we derive the Takagi-Sugeno model representation for the equation of motion of the two-link robot manipulator, and describe the design of direct adaptive fuzzy controller which is composed of adaptive fuzzy stabilizer incorporated with feedforward PD controller. Consider the two-link robot manipulator of Fig. 2, whose dynamics can be written explicitly as

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1 + \dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix},$$
(26)

where

$$\begin{split} &M_{11} = a_1 + 2a_3\cos q_2 + 2a_4\sin q_2,\\ &M_{12} = M_{21} = a_2 + a_3\cos q_2 + a_4\sin q_2,\\ &M_{22} = a_2, \quad h = 2a_3\sin q_2 - a_4\cos q_2 \end{split}$$

with
$$a_1 = I_1 + m_1 l_{c1}^2 + I_2 + m_2 l_{c2}^2 + m_2 l_1^2$$
, $a_2 = I_2 + m_2 l_{c2}^2$, $a_3 = m_2 l_1 l_{c2} \cos \delta_2$, $a_4 = m_2 l_1 l_{c2} \sin \delta_2$.

In order to apply the suggested direct MRAFC, we must have a fuzzy model which represents the behavior of the manipulator. By rewriting the equations of motion (26) as

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = -M^{-1}N \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + M^{-1} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \tag{27}$$

where
$$M = \begin{bmatrix} M_{11} M_{12} \\ M_{21} M_{22} \end{bmatrix}$$
 and $N = \begin{bmatrix} -h\dot{q}_2 - h(\dot{q}_1 + \dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix}$,

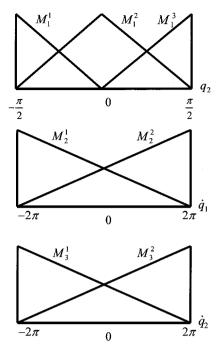


Fig. 3. Membership functions of q_2 , \dot{q}_1 , \dot{q}_2 .

we have the following nonlinear state equations:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u},\tag{28}$$

where
$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ -M^{-1}N \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \end{bmatrix}$$
, $\mathbf{g}(\mathbf{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ M^{-1} \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ M^{-1} \end{bmatrix}$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$
, and $\mathbf{x} = \begin{bmatrix} q_1 & q_2 & \dot{q}_1 & \dot{q}_2 \end{bmatrix}^T$. Then, by applying

the Lyapunov linearization method at operating points $\mathbf{x} = \mathbf{x}_i$, we obtain the affine linear model description for the two-link robot manipulator (26) around the operating point as following:

$$\dot{\mathbf{x}} = A_{\mathbf{x}_i} \mathbf{x} + B_{\mathbf{x}_i} \mathbf{u} + \mathbf{f}_{\mathbf{x}_i}, \tag{29}$$

where $A_{\mathbf{x}_i}$ and $B_{\mathbf{x}_i}$ denote the Jacobian matrices of $\mathbf{f}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ evaluated at $(\mathbf{x}_i, \mathbf{u}_i)$, and $\mathbf{f}_{\mathbf{x}_i} = \mathbf{f}(\mathbf{x}_i) - A_{\mathbf{x}_i} \mathbf{x}_i$.

The whole state space formed by state vector of the original nonlinear equations (28) is partitioned into $3 \times 2 \times 2$ different fuzzy subspaces whose center is located at the center of corresponding membership functions shown in Fig. 3.

To apply the proposed adaptive fuzzy control scheme, the reference model for the plant state \mathbf{x} to follow should be specified. In this simulation, the closed-loop eigenvalues for each subsystem are chosen to be the same, which in turn make the

reference model for each fuzzy subspace to be the same and linear one as following:

$$\dot{\mathbf{x}}_{m} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4.25 & 0 & -4 & 0 \\ 0 & -0.75 & 0 & -5 \end{bmatrix} \mathbf{x}_{m} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \mathbf{r}. (30)$$

The feedback stabilizer is combined with PD controller to generate the command for tracking the given trajectory.

$$\mathbf{u} = \mathbf{u}_f + \mathbf{r}$$
$$\mathbf{r} = -K_P \tilde{\mathbf{q}} - K_D \dot{\mathbf{q}}$$

The state feedback stabilizer \mathbf{u}_f is the PDC fuzzy controller combined with the compensator to eliminate the biased terms of the linearized plant models (29). The PDC controller shares the same fuzzy sets (Fig. 3) with fuzzy model to construct its premise part. That is, the fuzzy stabilizer is of the following form:

$$R^{i}: If \ q_{2} \ is \ M_{1}^{i} \ and \ \dot{q}_{1} \ is \ M_{2}^{i} \ and \ \dot{q}_{2} \ is \ M_{3}^{i}$$

$$then \quad \mathbf{u}_{f} = -K_{i} \left[q_{1} \ q_{2} \ \dot{q}_{1} \ \dot{q}_{2} \right]^{T} + \mathbf{u}_{a}. \tag{31}$$

The feedback control gain K_i of each stabilizer is updated by adaptive law so that the closed-loop plant follows the reference model (30). The initial values of K_i are designed from the nominal parameters of the plant model (Table 1). In this simulation, we design the initial parameters of the K_i so that the closed-loop plant including the feedback stabilizer has the same eigenvalues as reference model, and inputoutput of the closed-loop plant are decoupled.

Now by using (22), we derive the adaptive law for updating the elements of K_i so that the closed-loop plant controlled by adaptive fuzzy feedback stabilizer follows the reference model.

$$\dot{K}_{j}(t) = \frac{\mu_{j}(\mathbf{x})}{\sum_{j=1}^{12} \mu_{j}(\mathbf{x})} \operatorname{sgn}(l_{j}) B_{m}^{T} P \mathbf{e} \mathbf{x}^{T},$$
(32)

where
$$B_m^T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
 and
$$P = \begin{bmatrix} 1.13 & 0 & 0.12 & 0 \\ 0 & 1.17 & 0 & 0.07 \\ 0.12 & 0 & 0.15 & 0 \\ 0 & 0.07 & 0 & 0.11 \end{bmatrix}.$$

Table 1. The parametric variations of simulated robot manipulator.

| | | Variation of robot manipulator parameters |
|------------------|-------|---|
| 0 ≤ <i>t</i> < 2 | | $m_1 = 1, m_2 = 2,$ |
| | | $l_1 = 1, \delta_2 = 30^{\circ},$ |
| | | $I_1 = 0.12, I_2 = 0.25,$ |
| | | $l_{c1} = 0.5, l_{c1} = 0.6$ |
| 2 ≤ <i>t</i> < 4 | Case1 | $m_1 = 3, m_2 = 6,$ |
| | | $l_1 = 1, \delta_2 = 30^{\circ},$ |
| | | $I_1 = 0.12, I_2 = 0.25,$ |
| | | $l_{c1} = 0.5, l_{c1} = 0.6$ |
| | Case2 | $m_1 = 6, m_2 = 12,$ |
| | | $l_1 = 1, \delta_2 = 30^{\circ},$ |
| | | $I_1 = 0.12, I_2 = 0.25,$ |
| | | $l_{c1} = 0.5, l_{c1} = 0.6$ |

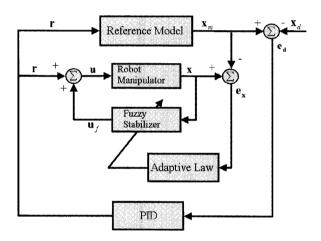


Fig. 4. The configuration of model reference adaptive fuzzy tracking control.

The configuration of overall control scheme for tracking control of robot manipulator using the proposed MRAFC is illustrated in Fig. 4. The fuzzy stabilizer incorporated with the adaptive law in the inner control loop plays the role to make robot manipulator to follow the reference model. The reference model is controlled by the outer loop control to perform the desired control activity.

The well established linear and nonlinear control techniques such as PID, sliding mode control, feedback linearization control will be helpful to design the outer loop control. Since adaptation strategy is the main issue of this paper, PID control is adopted to the outer loop control for the purpose of simplicity.

4.2. Simulation results and discussions

The adaptive fuzzy stabilizer combined with

feedforward PD control is simulated for the tracking control of the robot manipulator, and the results are discussed in this section. In this simulation, from comparative viewpoint, three control schemes were simulated on the tracking control of the two-link robot manipulator shown in Fig. 2. The first one is a conventional PD controller; the second one is the adaptive sliding mode controller [4]; and the third is the direct model reference adaptive control scheme designed in the previous section.

4.2.1 Comparative controllers

The conventional proportional-derivative (PD) controller used as the first comparative control scheme for achieving the tracking control of the manipulator has the following general form:

$$\mathbf{\tau} = -K_P \hat{\mathbf{q}} - K_D \dot{\mathbf{q}},\tag{33}$$

where the gain matrices K_P and K_D are chosen as $K_D = 100I$ and $K_P = 20K_D$. The second comparative

controller is a robust adaptive control which uses a sliding mode controller as a robust controller and has an estimation law for the unknown or time-varying parameters. The adaptive sliding mode controller takes the control law to be

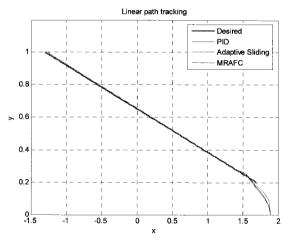
$$\mathbf{\tau} = Y\hat{\mathbf{a}} - K_D \mathbf{s},\tag{34}$$

which includes the term $Y\hat{\mathbf{a}}$ in addition to a simple PD term $K_D\mathbf{s}$, where $\mathbf{s} = \dot{\tilde{\mathbf{q}}} + \Lambda \tilde{\mathbf{q}}$. The parameter estimates $\hat{\mathbf{a}}$ is updated by the adaptive law

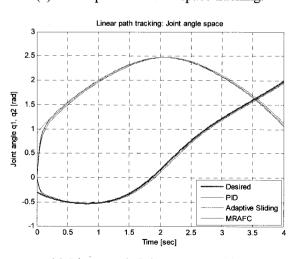
$$\dot{\hat{\mathbf{a}}} = -\Gamma Y^T \mathbf{s}. \tag{35}$$

The components of the matrix Y are written explicitly as

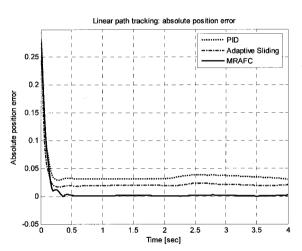
$$\begin{split} &Y_{11} = \ddot{q}_{r1}, \quad Y_{12} = \ddot{q}_{r2}, \quad Y_{21} = 0, \quad Y_{22} = \ddot{q}_{r1} + \ddot{q}_{r2}, \\ &Y_{13} = \left(2\ddot{q}_{r1} + \ddot{q}_{r2}\right)\cos q_2 - \left(\dot{q}_2\dot{q}_{r1} + \dot{q}_1\dot{q}_{r2} + \dot{q}_2\dot{q}_{r2}\right)\sin q_2, \\ &Y_{14} = \left(2\ddot{q}_{r1} + \ddot{q}_{r2}\right)\sin q_2 - \left(\dot{q}_2\dot{q}_{r1} + \dot{q}_1\dot{q}_{r2} + \dot{q}_2\dot{q}_{r2}\right)\cos q_2, \end{split}$$



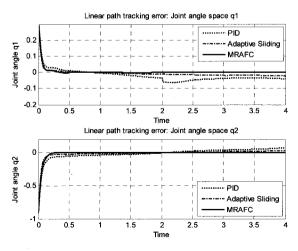
(a) Linear path-Cartesian space tracking.



(c) Linear path-Joint space tracking.



(b) Linear path-Cartesian space tracking error.



(d) Linear path-Joint space tracking error.

Fig. 5. Case1: Small parametric variation-Linear path tracking.

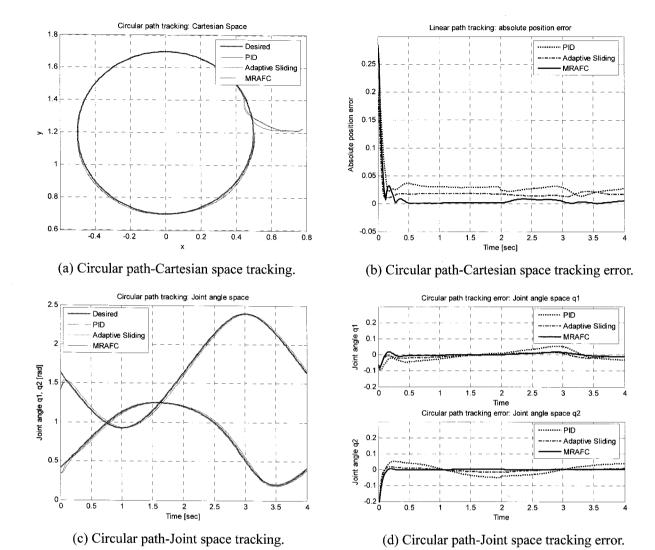


Fig. 6. Case1: Small parametric variation-Circular path tracking.

$$Y_{23} = \ddot{q}_{r1} \cos q_2 - \dot{q}_2 \dot{q}_{r1} \sin q_2,$$

$$Y_{24} = \ddot{q}_{r1} \sin q_2 - \dot{q}_1 \dot{q}_{r1} \cos q_2.$$

Slotine *et al.* [3,4] used the gain matrices and the adaptation rate matrix as $\Lambda = 20 \, \text{I}$, $K_D = 100 \, \text{I}$, $\Gamma = diag [0.03, 0.05, 0.1, 0.3]$.

4.2.2 Simulation and results

To test the adaptation abilities of three comparative controllers, the simulations were conducted for the same plants having different parametric variations. The first one is the case when the plant parameters are varied relatively small at 2 seconds after starting of the simulation. The parameters of the second simulated plant have relatively large amounts of variation at the same time as the first case. The plant parameters used in the simulations are shown in Table 1. Tracking abilities of each comparative control scheme were investigated through two kinds of path tracking; linear path tracking and circular path

tracking. Since the manipulator is simulated to track the linear path generated at initial position (1.7,0.2) and reach to the final position (-0.3,1) after 4 seconds, the velocity of the desired linear motion is (0.78m/s). The angular velocity of desired circular moving is $\pi/2\text{rad/sec}$. All these simulations were carried out using MATLAB with $\Delta=5\text{ms}$ sampling interval.

When we simulated the tracking control of the manipulator having slightly varied parameters, the PD control alone showed sensitive response to parametric variations. Figs. 5 and 6 show the simulation results for the plant whose parameters are relatively slightly varied. From the simulation results, it can be pointed out that the proposed adaptive control scheme can well cope with the parametric variations without offset error, whereas the sliding mode control has some offset error.

We have conducted computer simulations to investigate the affects of large variation of plant

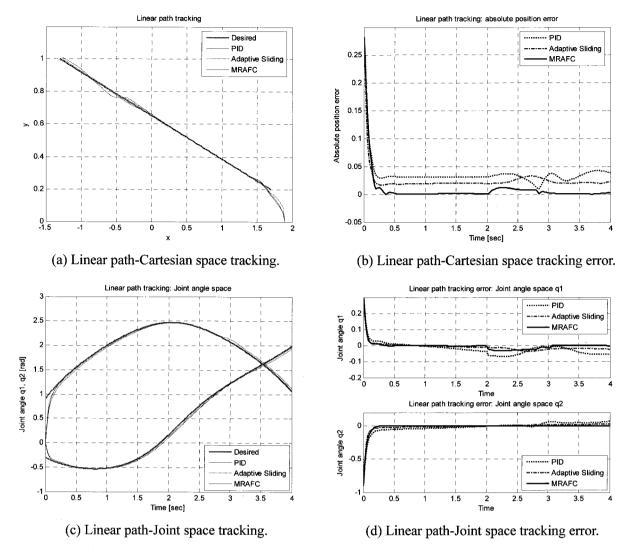


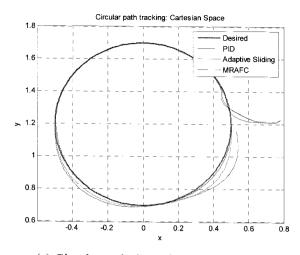
Fig. 7. Case2: Large parametric variation-Linear path tracking.

parameters on the response properties. Figs. 7 and 8 show the results of simulation for the case of large parametric variation. In the simulation results, we can see that the linear and circular tracking performance of PID control have large fluctuation after parametric variations, and the sliding mode control does not show strong adaptability under large parametric variations, whereas the developed adaptive fuzzy control scheme can stabilize the system in short time period. The following points can be pointed out from the simulation results: (1) The suggested control scheme shows excellent transient response properties such as short reaching time, small overshoot, compared with those of PD and adaptive sliding mode controller. (2) The designed adaptive fuzzy controller can effectively achieve the trajectory tracking for the plants with large amount of parametric uncertainties. (3) The suggested control scheme shows larger phase-lag and smaller transient error for all the simulated situations.

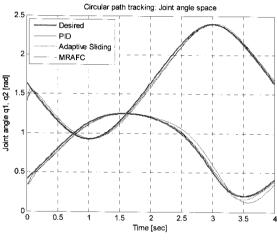
5. CONCLUSIONS

In this paper, we have developed a direct model reference adaptive fuzzy control (MRAFC) scheme for the MIMO Takagi-Sugeno model whose structure is assumed to be known but the parameters unknown. The adaptation law of the MRAFC adjusts the controller parameters on-line so that the plant output tracks the reference model output. The developed adaptive law guarantees the boundedness of all signals in the closed-loop system and ensures that the plant state tracks the state of the reference model asymptotically with time for any bounded reference input signal. In the whole design process, no strict constraints and prior knowledge of the controlled plant are required, and the asymptotic stability of the control system can be guaranteed.

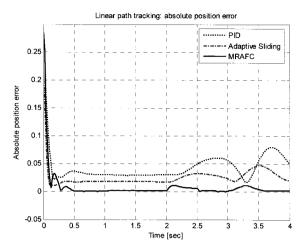
To verify the validity and effectiveness of the MRAFC scheme, the suggested analysis and design techniques were applied to the tracking control of two-link robot manipulator and simulation studies were carried out. In the control design, the MRAFC was combined with feedforward PD control and used



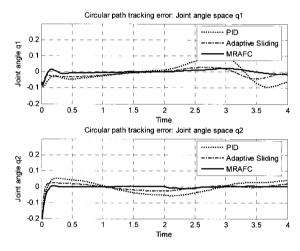
(a) Circular path-Cartesian space tracking.



(c) Circular path-Joint space tracking.



(b) Circular path-Cartesian space tracking error.



(d) Circular path-Joint space tracking error.

Fig. 8. Case2: Large parametric variation-Circular path tracking.

as the state stabilizer. From the simulation results, we conclude that the suggested control scheme can effectively achieve the trajectory tracking even for the robot manipulator with relatively large amount of parametric uncertainties.

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