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Vibration Suppression Control for Mechanical Transfer Systems by Jerk Reduction

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Abstract: This paper considers vibration suppression of a mechanical transfer system, where the work is connected with the hand flexibly. We adopt the idea of jerk reduction of the hand. From the equation of motion, we first derive a state equation including the jerk and acceleration of the hand, but excluding the displacement and velocity of the work. Then, we design optimal state feedback for a suitable cost function, and show by simulation that jerk reduction of the hand is effective for vibration suppression of the work and improvement of the settling time. Since state feedback including the jerk and acceleration is not practical, we propose a computation method for optimal feedback using displacements and velocities in the state only.

Keywords: Jerk reduction, mechanical transfer system, motion control, optimal control, vibration control.

1. INTRODUCTION

In manufacturing processes, improvement of productivity is commonly required. A way to achieve this requirement, large-scale products such as liquid crystal panels of 8G (the eight generation) are desired, for which we need to run big manufacturing machines fast. However, fast movement of big machines usually generates vibration and then deteriorate the positioning and settling time. Therefore, for improvement of productivity, we need to develop a control strategy for vibration suppression.

In this paper, we treat a transfer machine which is widely used in manufacturing processes. We represent the machine and a work on it as a multi-mass system connected by springs and dampers. It is composed of a driven part, intermediate parts, a hand, and a work. We assume that the hand holds the work by a spring and a damper. We adopt the idea of jerk reduction [1,2] of the hand for vibration suppression of the work and improvement of the settling time.

To design a control law for jerk reduction of the hand, we first derive a state equation including the jerk and acceleration of the hand, but excluding the displacement and velocity of the work, from the equation of motion. Then, we apply the Linear Quadratic Control [3] to the state equation with a performance index evaluating the jerk of the hand, and obtain optimal state feedback. By simulation, we show that in this way, we can suppress the vibration of the work and improve the settling time.

Although the optimal control law can be obtained, it is not practical since the state feedback contains the jerk of the hand, which cannot be measured directly. It may be also desired not to use an accelerometer for cost reduction. For these reasons, we consider a feedback control law excluding the jerk, or jerk and acceleration, of the hand and propose an iterative method for computing such control laws. In this case as well, we show that by considering a performance index evaluating the jerk of the hand, we can suppress the vibration of the work and improve the settling time.

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2. SYSTEM DESCRIPTION

Let us consider a mechanical system of Fig. 1. The system is composed of masses $m_1,\cdots,m_p$, m_h, which are connected by springs $k_1,\cdots,k_p$ and dampers $d_1,\cdots,d_p$, and a mass m_w connected to

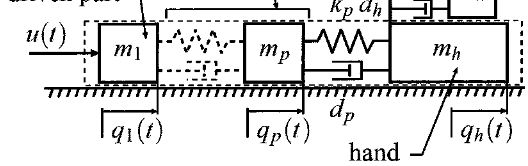

Fig. 1. A mechanical transfer system.

the mass m_h by the spring k_h and damper d_h. This multi-mass system is a model of a transfer machine and a work on it, where the force u drives the mass m_1 and the hand m_h holds the work m_w flexibly.

2.1. Equation of motion

To describe this system by the equation of motion, we denote the displacement of the mass m_* by q_*, where $*$ means the subscript $1,\cdots,p$, h, or w. For individual masses, the displacements $q_*=0$ are defined as stationary points where the springs are neither stretched nor compressed. Then, the system is described by

$$\begin{aligned}
&M\ddot{q}(t)+D\dot{q}(t)+Kq(t)+gd_p\{g^{\mathrm{T}}\dot{q}(t)-\dot{q}_h(t)\}\\
&\quad+gk_p\{g^{\mathrm{T}}q(t)-q_h(t)\}=\ell u(t),\\
&m_h\ddot{q}_h(t)+d_p\{\dot{q}_h(t)-g^{\mathrm{T}}\dot{q}(t)\}+d_h\{\dot{q}_h(t)-\dot{q}_w(t)\}\\
&\quad+k_p\{q_h(t)-g^{\mathrm{T}}q(t)\}+k_h\{q_h(t)-q_w(t)\}=0,\\
&m_w\ddot{q}_w(t)+d_h\{\dot{q}_w(t)-\dot{q}_h(t)\}+k_h\{q_w(t)-q_h(t)\}=0,
\end{aligned} \tag{1}$$

where

$$q(t)=\begin{bmatrix}q_1(t) & \cdots & q_p(t)\end{bmatrix}^{\mathrm{T}},\ M=\begin{bmatrix}m_1 & & 0\\ & \ddots & \\ 0 & & m_p\end{bmatrix},$$

$$D=\begin{bmatrix}d_1 & -d_1 & & & 0\\ -d_1 & d_1+d_2 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & d_{p-2}+d_{p-1} & -d_{p-1}\\ 0 & & & -d_{p-1} & d_{p-1}\end{bmatrix},$$

$$K=\begin{bmatrix}k_1 & -k_1 & & & 0\\ -k_1 & k_1+k_2 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & k_{p-2}+k_{p-1} & -k_{p-1}\\ 0 & & & -k_{p-1} & k_{p-1}\end{bmatrix},$$

$$g=[0\ \cdots\ 0\ \ 1]^{\mathrm{T}}\in\mathbb{R}^p,$$

$$\ell=[1\ \ 0\ \cdots\ 0]^{\mathrm{T}}\in\mathbb{R}^p.$$

This model can be used also in the case where the work is a vessel for liquid [4-6].

To see the mechanism of vibration of the work, we compute the Laplace transform of (1) as

$$\begin{bmatrix}Z_{11}(s) & z_{12}(s) & 0\\ z_{12}^{\mathrm{T}}(s) & z_{22}(s) & z_{23}(s)\\ 0 & z_{23}(s) & z_{33}(s)\end{bmatrix}\begin{bmatrix}\hat{q}(s)\\ \hat{q}_h(s)\\ \hat{q}_w(s)\end{bmatrix}=\begin{bmatrix}\ell\\ 0\\ 0\end{bmatrix}\hat{u}(s), \tag{2}$$

where $\hat{q}(s)$, $\hat{q}_*(s)$, $\hat{u}(s)$ are the Laplace transforms of $q(t)$, $q_*(t)$, $u(t)$, and

$$\begin{aligned}
&Z_{11}(s)=Ms^2+\overline{D}s+\overline{K},z_{12}(s)=-d_pgs-k_pg,\\
&z_{22}(s)=m_hs^2+(d_p+d_h)s+(k_p+k_h),\\
&z_{23}(s)=-d_hs-k_h,z_{33}(s)=m_ws^2+d_hs+k_h,\\
&\overline{D}=D+gd_pg^{\mathrm{T}},\overline{K}=K+gk_pg^{\mathrm{T}}.
\end{aligned}$$

By multiply this equation by

$$\begin{bmatrix}I & 0 & 0\\ 0 & z_{33} & -z_{23}\\ 0 & 0 & z_{33}^{-1}\end{bmatrix} \tag{3}$$

from the left, we obtain

$$\begin{bmatrix}Z_{11} & z_{12} & 0\\ z_{33}z_{12}^{\mathrm{T}} & z_{22}z_{33}-z_{23}^2 & 0\\ 0 & z_{23}z_{33}^{-1} & 1\end{bmatrix}\begin{bmatrix}\hat{q}\\ \hat{q}_h\\ \hat{q}_w\end{bmatrix}=\begin{bmatrix}\ell\\ 0\\ 0\end{bmatrix}\hat{u}, \tag{4}$$

where "(s)" has been omitted for simplicity. We see from the bottom equation

$$\hat{q}_w=-z_{23}z_{33}^{-1}\hat{q}_h \tag{5}$$

that the displacement q_w of the work is the output of a second order vibration system whose input is the displacement q_h of the hand. Therefore, rapid changes of q_h, even if the magnitude would be small, contain frequency components of a wide range and stimulate the vibration mode of q_w. To avoid this phenomenon, we suppose that reduction of the jerk $q_h^{(3)}$ of the hand is effective.

2.2. State equation

In designing a control system, it is common to transform (1) to a state equation with the state variable composed of the displacements and velocities of the $p+2$ masses in the system. In this paper, we derive another state equation to evaluate the jerk of the hand, in which the state variable contains the jerk $q_h^{(3)}$ and acceleration $\ddot{q}_h$ of the hand instead of the displacement q_w and velocity $\dot{q}_w$ of the work. In this way, we describe the system dynamics using the variables of the transfer machine only, that is, without the variables of the work.

The behavior of the variables of the transfer machine is written by the top two equations of (4) as

$$\begin{bmatrix}Z_{11} & z_{12}\\ z_{33}z_{12}^{\mathrm{T}} & z_{22}z_{33}-z_{23}^2\end{bmatrix}\begin{bmatrix}\hat{q}\\ \hat{q}_h\end{bmatrix}=\begin{bmatrix}\ell\\ 0\end{bmatrix}\hat{u} \tag{6}$$

and the coefficient matrix of the left side is expanded as

$$\begin{bmatrix} 0 & 0 \\ 0 & m_w m_h \end{bmatrix} s^4 + \begin{bmatrix} 0 & 0 \\ -m_w d_p g^{\mathrm{T}} & m_w d_h + m_w d_p + m_h d_h \end{bmatrix} s^3 + \begin{bmatrix} M & 0 \\ -m_w k_p g^{\mathrm{T}} - d_h d_p g^{\mathrm{T}} & m_w k_h + m_w k_p + m_h k_h + d_h d_p \end{bmatrix} s^2 + \begin{bmatrix} \overline{D} & -d_p g \\ -d_h k_p g^{\mathrm{T}} - d_p k_h g^{\mathrm{T}} & d_h k_p + d_p k_h \end{bmatrix} s + \begin{bmatrix} \overline{K} & -k_p g \\ -k_h k_p g^{\mathrm{T}} & k_h k_p \end{bmatrix}. \tag{7}$$

To derive the unorthodox state equation, we multiply (7) by

$$\begin{aligned} T &= \begin{bmatrix} I & 0 \\ \alpha_1 s + \alpha_2 & 1 \end{bmatrix}, \\ \alpha_1 &= m_w d_p g^{\mathrm{T}} M^{-1}, \\ \alpha_2 &= (m_w k_p g^{\mathrm{T}} + d_h d_p g^{\mathrm{T}} - \alpha_1 \overline{D}) M^{-1} \end{aligned} \tag{8}$$

from the left to obtain

$$\begin{bmatrix} 0 & 0 \\ 0 & m_w m_h \end{bmatrix} s^4 + \begin{bmatrix} 0 & 0 \\ 0 & m_w d_h + m_w d_p + m_h d_h \end{bmatrix} s^3 + \begin{bmatrix} M & 0 \\ 0 & m_w k_h + m_w k_p + m_h k_h + d_h d_p - \alpha_1 d_p g \end{bmatrix} s^2 + \begin{bmatrix} \overline{D} & -d_p g \\ -d_h k_p g^{\mathrm{T}} - d_p k_h g^{\mathrm{T}} + \alpha_1 \overline{K} + \alpha_2 \overline{D} & d_h k_p + d_p k_h - \alpha_1 k_p g - \alpha_2 d_p g \end{bmatrix} s + \begin{bmatrix} \overline{K} & -k_p g \\ -k_h k_p g^{\mathrm{T}} + \alpha_2 \overline{K} & k_h k_p - \alpha_2 k_p g \end{bmatrix}, \tag{9}$$

where the $(2,1)$ blocks of the coefficient matrices for s^3 and s^2 are zero. We also multiply the coefficient matrix of the right side of (6) by T of (8) to obtain

$$T\begin{bmatrix} \ell \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha_1 \ell \end{bmatrix} s + \begin{bmatrix} \ell \\ \alpha_2 \ell \end{bmatrix}. \tag{10}$$

These equations in s domain imply that dynamics of the system can be described by two differential equations, where the first one is with q, $\dot{q}$, and $\ddot{q}$, and the second one is with q_h, $\dot{q}_h$, $\ddot{q}_h$, $q_h^{(3)}$, $q_h^{(4)}$ and q, $\dot{q}$. Then, we can describe the dynamics in time domain by a state equation with the state variable

$$x = \begin{bmatrix} q^{\mathrm{T}} & q_h & \dot{q}^{\mathrm{T}} & \dot{q}_h & \ddot{q}_h & q_h^{(3)} \end{bmatrix}^{\mathrm{T}} \tag{11}$$

as

$$\dot{x} = Ax + b_1 u + b_2 \dot{u}, \tag{12}$$

where

$$A = \begin{bmatrix} 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ A_{31} & a_{32} & A_{33} & a_{34} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix},$$

$$\begin{aligned} b_1 &= \begin{bmatrix} 0 & 0 & b_{13}^{\mathrm{T}} & 0 & 0 & b_{16} \end{bmatrix}^{\mathrm{T}}, \\ b_2 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & b_{26} \end{bmatrix}^{\mathrm{T}}, \\ A_{31} &= -M^{-1}\overline{K}, \quad a_{32} = M^{-1} k_p g, \\ A_{33} &= -M^{-1}\overline{D}, \quad a_{34} = M^{-1} d_p g, \\ a_{61} &= -m_w^{-1} m_h^{-1}(-k_h k_p g^{\mathrm{T}} + \alpha_2 \overline{K}), \\ a_{62} &= -m_w^{-1} m_h^{-1}(k_h k_p - \alpha_2 k_p g), \\ a_{63} &= -m_w^{-1} m_h^{-1}(-d_h k_p g^{\mathrm{T}} - d_p k_h g^{\mathrm{T}} + \alpha_1 \overline{K} + \alpha_2 \overline{D}), \\ a_{64} &= -m_w^{-1} m_h^{-1}(d_h k_p + d_p k_h - \alpha_1 k_p g - \alpha_2 d_p g), \\ a_{65} &= -m_w^{-1} m_h^{-1}(m_w k_h + m_w k_p + m_h k_h + d_h d_p - \alpha_1 d_p g), \\ a_{66} &= -m_h^{-1}(d_h + d_p) - m_w^{-1} d_h, \\ b_{13} &= M^{-1}\ell, \quad b_{16} = m_w^{-1} m_h^{-1} \alpha_2 \ell, \\ b_{26} &= m_w^{-1} m_h^{-1} \alpha_1 \ell. \end{aligned}$$

This state equation may look unusual since it contains a term of the input derivative $\dot{u}$. However, when $p \geq 2$, that is, when there is at least one intermediate mass between the driven part and the hand, $b_{26} = 0$ and then $b_2 = 0$. Therefore, it is a standard state equation. In the present paper, we consider this case only. The case $p = 1$ has been dealt with in [7].

3. OPTIMAL CONTROL LAW

To show that reduction of the jerk of the hand is effective for vibration suppression of the work and improvement of the settling time, we apply optimal regulator theory [3] to the state equation

$$\dot{x} = Ax + b_1 u \tag{13}$$

with several performance indices. In this section, we consider an example of $p = 2$ for simplicity of discussions. The cases $p \geq 3$ can be treated in the same way.

When $p = 2$, the state is an eight dimensional vector as

$$x = \begin{bmatrix} q_1 & q_2 & q_h & \dot{q}_1 & \dot{q}_2 & \dot{q}_h & \ddot{q}_h & q_h^{(3)} \end{bmatrix}^{\mathrm{T}}. \tag{14}$$

For computation, we use the parameters $m_1 = m_2 = m_h = 1.0$, $m_w = 0.1$, $d_1 = d_2 = 0.1$, $d_h = 0.05$, $k_1 = k_2 = 5.0$, and $k_h = 1.0$.

We consider a typical behavior of transfer systems. That is, the initial state is a stationary one as

$$x_0 = \begin{bmatrix} -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \tag{15}$$

and the terminal state is

$$x_e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \tag{16}$$

where the moving distance is normalized as 1.

3.1. Performance indices

We consider a performance index

$$J = \int_0^\infty \left(y^{\mathrm{T}} Q y + r u^2 \right) dt, \tag{17}$$

where

$$y = Cx \tag{18}$$

is the output of the system. The coefficient matrix C is determined by the variables which we want to evaluate. In (17), Q and r are weights to take the balance of evaluation of state and input variables.

We consider the following four cases, where the output variables are indicated, which are evaluated together with the input.

Case 1: displacement of hand

$$C_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad Q_1 = 1.0, \quad r_1 = 0.0056. \tag{19}$$

Case 2: displacement and velocity of hand

$$C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \tag{20}$$

$$Q_2 = \begin{bmatrix} 1.0 & 0 \\ 0 & 0.204 \end{bmatrix}, \quad r_2 = 0.025.$$

Case 3: displacement and acceleration of hand

$$C_3 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 1.0 & 0 \\ 0 & 0.31 \end{bmatrix}, \quad r_3 = 0.077. \tag{21}$$

Case 4: displacement and jerk of hand

$$C_4 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad Q_4 = \begin{bmatrix} 1.0 & 0 \\ 0 & 0.69 \end{bmatrix}, \quad r_4 = 0.227. \tag{22}$$

The choices of the weights Q_i and r_i are explained later.

3.2. Optimal control law

Since the pair (A, b_1) is stabilizable and the pairs (C_i, A), $i = 1, \cdots, 4$, are observable, the optimal control law is given by the positive definite solution P_i of the Riccati equation

$$A^{\mathrm{T}} P_i + P_i A - P_i b_1 r_i^{-1} b_1^{\mathrm{T}} P_i + C_i^{\mathrm{T}} Q_i C_i = 0 \tag{23}$$

as

$$u = -r_i^{-1} b_1^{\mathrm{T}} P_i x. \tag{24}$$

The computed optimal gains

$$f_i = -r_i^{-1} b_1^{\mathrm{T}} P_i \tag{25}$$

are:

Case 1: displacement of hand

$f_1 = -[10.1 \quad 3.11 \quad 0.179 \quad 4.59 \quad 1.26 \quad 10.8 \quad 0.162 \quad 0.832]$.

Case 2: displacement and velocity of hand

$f_2 = -[6.32 \quad 2.45 \quad -2.44 \quad 3.60 \quad 1.14 \quad 5.32 \quad -0.322 \quad 0.412]$.

Case 3: displacement and acceleration of hand

$f_3 = -[7.15 \quad 1.84 \quad -5.39 \quad 3.77 \quad 0.815 \quad 2.34 \quad -1.44 \quad 0.275]$.

Case 4: displacement and jerk of hand

$f_4 = -[14.2 \quad 1.38 \quad -13.5 \quad 5.29 \quad 0.803 \quad -0.857 \quad -5.96 \quad 0.327]$.

Now, we denote the total cost and the contribution of the input in the performance index by c_i and c_i^u, respectively, in cases $i=1,2,3,4$. We also denote contributions of displacement, velocity, acceleration and jerk of the hand by c_2^v, c_3^a, and c_4^j in cases $i=2,3,4$. When we apply the above feedback gains for the initial state x_0 of (15), the ratios of the contributions are

Case 1: $c_1^u/c_1 = 8.42\times10^{-2}$,

Case 2: $c_2^u/c_2 = 8.42\times10^{-2}$, $c_2^v/c_2 = 8.42\times10^{-2}$,

Case 3: $c_3^u/c_3 = 8.42\times10^{-2}$, $c_3^a/c_3 = 8.41\times10^{-2}$,

Case 4: $c_4^u/c_4 = 8.43\times10^{-2}$, $c_4^j/c_4 = 8.42\times10^{-2}$,

which mean that the weights Q_i and r_i in (19)-(22) are chosen so that the ratio of the input contribution to the total cost is approximately 8.4×10^{-2} in every case and the velocity, acceleration, or jerk contributions to the total cost are approximately 8.4×10^{-2} in cases $i=2,3,4$.

3.3. Simulation results

Figs. 2 and 3 respectively show the responses of the hand and work in the four cases. In Fig. 2, the

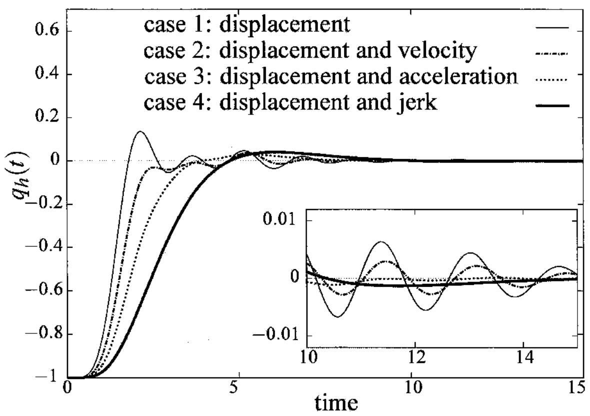

Fig. 2. Responses of the displacement q_h of the hand.

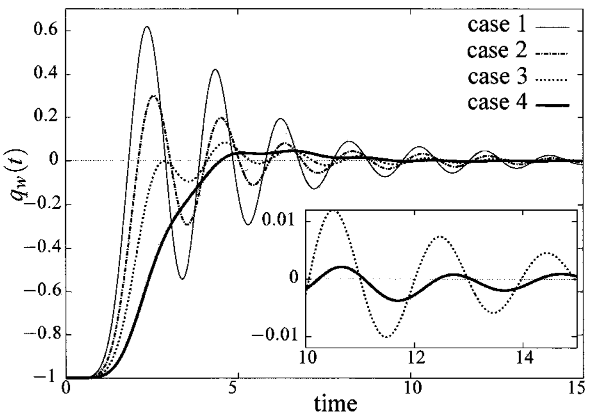

Fig. 3. Responses of the displacement q_w of the work.

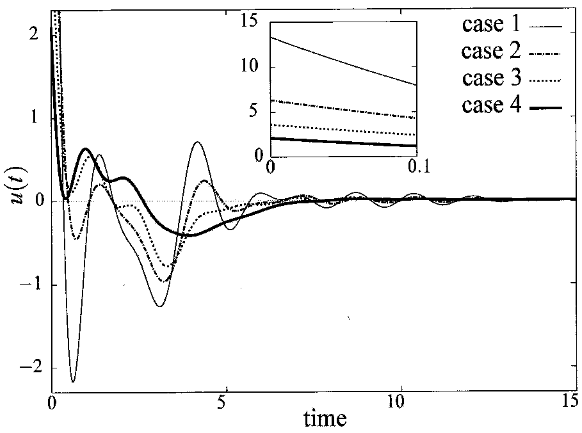

Fig. 4. Responses of the input u.

response of case 3 is the best such that the overshoot is small, no vibration exists, and the settling time is the shortest. However, in Fig. 3, the response of case 3 is not so. The response of case 4 is the best in all aspects. Then, we see that inclusion of the jerk of the hand in the performance index is effective for suppression of vibration of the work and improvement of the settling time.

Moreover, we see from Fig. 4 that the input in case 4 is relatively small and smooth. This is good for the actuator.

4. PRACTICAL CONTROL LAW

The optimal control law obtained in the previous section is state feedback, which needs the information of jerk and acceleration of the hand. However, there is no sensor which can measure the jerk and, in actual situations, it may be desired not to use an accelerometer for cost reduction. For these reasons, we propose feedback control laws which use only the displacements and velocities of the masses in the transfer machine or those plus the acceleration of the hand. This means that we consider state feedback gains $\tilde{f}$ under the structural constraints where the 7th and 8th elements are 0 or the 8th element is 0.

Optimal gains with such structural constraints cannot be obtained analytically and depend on the initial state. So, we present an iterative computation method. We consider the performance index with C_4, Q_4 and r_4 of (22) for case 4, by which we can expect reduction of the jerk of the hand and, as a result, vibration suppression of the work. We deal with two structural constraints as

Case 4a: the 8th element is 0 in the feedback gain $\tilde{f}$.

Case 4b: the 7th and 8th elements are 0 in the feedback gain $\tilde{f}$.

When we apply stabilizing feedback control

$$u = \tilde{f}x \quad (26)$$

to the system (13) with the initial state x_0, the value of the performance index is

$$J = x_0^{\mathrm{T}} \tilde{P} x_0, \quad (27)$$

where $\tilde{P}$ is the positive definite solution of the Liapunov equation

$$(A + b_1\tilde{f})^{\mathrm{T}} \tilde{P} + \tilde{P}(A + b_1\tilde{f}) + C^{\mathrm{T}}QC + r\tilde{f}^{\mathrm{T}}\tilde{f} = 0. \quad (28)$$

Therefore, the problem we should consider is

$$\min_{\tilde{f}} x_0^{\mathrm{T}} \tilde{P} x_0. \quad (29)$$

To solve this problem using an existing software which is good at treating matrix inequalities rather than equalities, we use the Liapunov inequality

$$(A + b_1\tilde{f})^{\mathrm{T}} \tilde{P} + \tilde{P}(A + b_1\tilde{f}) + C^{\mathrm{T}}QC + r\tilde{f}^{\mathrm{T}}\tilde{f} \le 0 \quad (30)$$

the minimum solution of which is the solution of the Liapunov equation (28) [8]. Then, we solve

$$\min_{\tilde{f},\tilde{P}} x_0^{\mathrm{T}} \tilde{P} x_0 \quad (31)$$

under (30) instead of (29) under (28).

Now, we consider small perturbations $\tilde{f}_\delta$, satisfying the same structural constraint as $\tilde{f}$, and $\tilde{P}_\Delta$ for $\tilde{f}$ and $\tilde{P}$, respectively, so that we write (30) as

$$\begin{aligned}&\{A + b_1(\tilde{f} + \tilde{f}_\delta)\}^{\mathrm{T}}(\tilde{P} + \tilde{P}_\Delta) \\ &\quad + (\tilde{P} + \tilde{P}_\Delta)\{A + b_1(\tilde{f} + \tilde{f}_\delta)\} \\ &\quad + C^{\mathrm{T}}QC + r(\tilde{f} + \tilde{f}_\delta)^{\mathrm{T}}(\tilde{f} + \tilde{f}_\delta) \le 0.\end{aligned} \quad (32)$$

We assume that $\tilde{f}_\delta$ and $\tilde{P}_\Delta$ are small enough and we can ignore their squares and products. Then, (32) becomes a linear matrix inequality

$$\tilde{A}^{\mathrm{T}}\tilde{P}_\Delta + \tilde{P}_\Delta\tilde{A} + \tilde{b}\tilde{f}_\delta + (\tilde{b}\tilde{f}_\delta)^{\mathrm{T}} + \tilde{Q} < 0, \quad (33)$$

where

$$\tilde{A} = A + b_1\tilde{f}, \quad \tilde{b} = \tilde{P}b_1 + r\tilde{f}^{\mathrm{T}},$$
$$\tilde{Q} = \tilde{A}^{\mathrm{T}}\tilde{P} + \tilde{P}\tilde{A} + C^{\mathrm{T}}QC + r\tilde{f}^{\mathrm{T}}\tilde{f}.$$

We note that "$\le$" in (32) is replaced with "$<$" in (33) so that a solution set $(\tilde{f}_\delta, \tilde{P}_\Delta)$ of (33) satisfies (32) if $\tilde{f}_\delta$ and $\tilde{P}_\Delta$ are sufficiently small. Thus, if (33) has a solution set $(\tilde{f}_\delta, \tilde{P}_\Delta)$ such that $x_0^{\mathrm{T}}\tilde{P}_\Delta x_0 < 0$ for x_0 of (15), we can expect that a stabilizing feedback gain $\tilde{f} + \tilde{f}_\delta$ exists which reduces $x_0^{\mathrm{T}}\tilde{P}x_0$. To compute such a solution set, we introduce an upper bound λ of the change rate as

$$\begin{bmatrix} \tilde{f}\tilde{f}^{\mathrm{T}} & \tilde{f}_\delta \\ \tilde{f}_\delta^{\mathrm{T}} & \lambda^2 I \end{bmatrix} > 0, \quad -\lambda\tilde{P} < \tilde{P}_\Delta < \lambda\tilde{P} \quad (34)$$

and consider the optimization problem

$$\min_{\tilde{f}_\delta, \tilde{P}_\Delta} x_0^{\mathrm{T}} \tilde{P}_\Delta x_0. \quad (35)$$

To start the iteration, we need an initial stabilizing feedback gain $\tilde{f}_0$. Since the optimal solution $\tilde{f}$ to (31) depends on the initial value $\tilde{f}_0$, we propose to use many $\tilde{f}_0$. That is, we randomly generate many $\tilde{f}$ under the structural constraints of cases 4a or 4b, and use them as $\tilde{f}_0$ if the closed-loop system matrices $A + b_1\tilde{f}$ are stable. We note that the unstable mode of the mechanical system of Fig. 1 is only the rigid mode and can be stabilized by negative feedback of the displacement and velocity of the driven mass m_1. Such feedback makes the damping and stiffness matrices positive definite in the closed-loop system and this fact ensures stability of the mechanical system [9]. Therefore, the existence of a stabilizing gain is guaranteed.

The best gains obtained so far for the present example are:

Case 4a: the 8th element is 0 in the feedback gain $\tilde{f}$

$$\tilde{f}_{4\mathrm{a}} = -[16.3 \quad 0.939 \quad -14.9 \quad 6.29 \quad 0.908 \quad -1.55 \quad -6.47 \quad 0].$$

Case 4b: the 7th and 8th elements are 0 in $\tilde{f}$

$$\tilde{f}_{4\mathrm{b}} = [-23.1 \;\; 17.5 \;\; 0.944 \;\; -19.2 \;\; 4.42 \;\; 2.31 \;\; 0 \;\; 0].$$

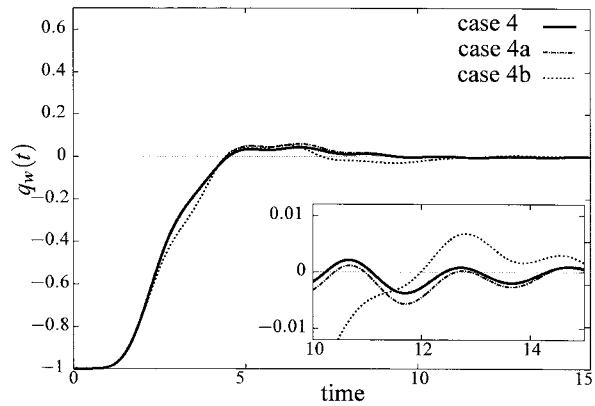

Fig. 5. Responses of the displacement q_w of the work.

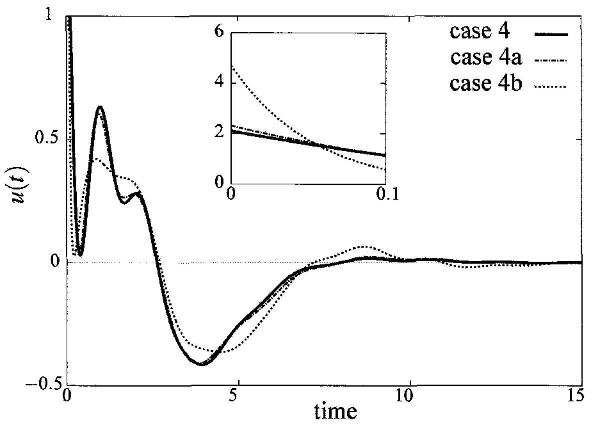

Fig. 6. Responses of the input u.

When we apply these gains, the values of the performance index are: case 4a: 2.500, case 4b: 2.600 while it is 2.496 in case 4. The responses of the displacement of the work and the input are shown in Figs. 5 and 6, respectively. We see that a good response has been obtained even in case 4b and conclude that sufficient reduction of the jerk of the hand has been achieved by feedback of displacements and velocities in the state only.

5. CONCLUSIONS

This paper has considered a vibration suppression problem for a mechanical transfer system, where the work is connected with the hand of a transfer machine by a spring and a damper. A feedback control law for reduction of the jerk of the hand has been proposed to suppress the vibration of the work. A state equation including the jerk and acceleration of the hand has been introduced to compute a state feedback gain using the Linear Quadratic Control theory. For practical applications, an iterative computation method has been presented for optimal feedback using displacements and velocities of the transfer machine only. Simulation results have shown that reduction of the jerk of the hand is effective for vibration suppression of the work and improvement of the settling time.

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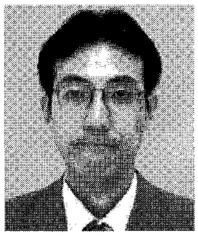

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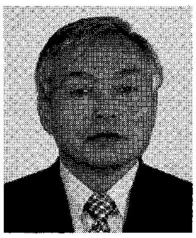

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