

Eigenstructure Assignment Considering Probability of Instability with Flight Control Application

Young Bong Seo and Jae Weon Choi*

Abstract: Eigenstructure assignment provides the advantage of allowing great flexibility in shaping the closed-loop system responses by allowing specification of closed-loop eigenvalues and corresponding eigenvectors. But, the general eigenstructure assignment methodologies cannot guarantee stability robustness to parameter variations of a system. In this paper, we present a novel method that has the capability of exact assignment of an eigenstructure which can consider the probability of instability for LTI (Linear Time-Invariant) systems. The probability of instability of an LTI system is determined by the probability distributions of the closed-loop eigenvalues. The stability region for the system is made probabilistically based upon the Monte Carlo evaluations. The proposed control design method is applied to design a flight control system with probabilistic parameter variations to confirm the usefulness of the method.

Keywords: Eigenstructure assignment, flight control, monte carlo evaluation, probability distribution, probability of instability.

1. INTRODUCTION

Eigenstructure assignment provides the advantage of allowing great flexibility in shaping the closed-loop system responses of a linear system by allowing specification of the closed-loop eigenvalues and eigenvectors of the system [1], and has been shown to be a useful tool for flight control design [2]. Eigenstructure assignment technique is used to design flight control laws for aircrafts with many control requirements, and the technique together with a suitable feedforward design can achieve static decoupling of modes with internal stability, which is an important requirement in many flight control systems [3-6].

In direct eigenstructure assignment techniques, the design parameters are the desired closed-loop eigenvalues and specified elements of the corresponding closed-loop eigenvectors. Once the design parameters are specified, the feedback control gains are uniquely determined. Therefore, given a set of specifications, the feedback control gains will

provide the desired closed-loop transient response (or come as close to it as possible within the system constraints), but the feedback control gains might result in a system with poor stability robustness [7], i.e., a small change in a plant dynamics may destabilize the closed-loop system. The designer is then faced with the dilemma of changing the design specifications such that the resulting feedback system will also provide adequate stability robustness. Note that, in general, the designer does have a certain amount of freedom in choosing the design specifications. The designer rarely wants an exact value for a closed-loop eigenvalue or an exact shape for the corresponding eigenvector. The specifications are rather in terms of desired regions for the closed-loop eigenvalues and acceptable sets of eigenvector shapes. The general eigenstructure assignment methodologies cannot guarantee stability robustness to parameter variations of a system. This problem is still unsolved, and thus, it is worthwhile to explore the extension of the conventional robust right eigenstructure assignment technique, which can cope with a robustness problem.

Practical compensators are required to control plants satisfactorily in the presence of plant parameter uncertainties. Measures of robustness used are directly connected to the design objectives and must be consistent with the assumed system model and the structure of the parameters. These requirements are best achieved when measures of robustness is expressed as the probability that certain properties of the system will fall within acceptable bounds. For example, in reference [8], a specific degree of stability

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may be required, or settling time and actuator usage must not exceed imposed limitations. In addition, it should be possible to verify the robustness of plant operation under the most likely parameter variations [8]. With computers, singular-value analysis has extended the frequency-domain approach to multi-input/multi-output systems (e.g., [9,10]); however, guaranteed stability-bound estimates are, often, unduly conservative, and have a weak relationship to parameter variations in the physical system. Structured singular-value analysis reduces this conservatism somewhat [11], and alternate treatments of structured parameter variations have been proposed (e.g., [12-14]), although these approaches remain deterministic. The notion of "probability of instability" was introduced in [15] with its application to the robustness analysis of the Space Shuttle's flight control system. The robustness properties are analyzed by the probability distributions of closed-loop eigenvalues, given the statistics of the variable parameters in the plant's dynamic model [16,17]. The probability that all these eigenvalues lie in the open left-half s -plane is considered as a scalar measure of robustness. Based on the concept of "probability of instability", we propose a method that has the capability of exact eigenstructure assignment under the probabilistic robustness for linear time-invariant systems. The probabilistic stability region is obtained by using the relationship between the eigenvalues of the probabilistically perturbed system and those of the nominal closed-loop system. Monte Carlo simulations are employed to evaluate the eigenvalues of the perturbed system, and thus, Monte Carlo simulations can also be used to derive the concept of the probability of instability. This analysis treats not only Gaussian parameter uncertainties but also non-Gaussian cases, including uncertain-but-bounded variations. The eigenstructure assignment for a system with probabilistic parameter variations is characterized in terms of the minimization of the eigenvector sensitivity via a newly proposed performance index. Finally, the developed control strategy is applied to the design of a flight control system with probabilistic parameter variations, and evaluated by computer simulations.

2. PROBABILITY OF INSTABILITY

Consider the following linear time-invariant system subject to parameter uncertainty

$$\dot{x}(t) = A(p)x(t) + B(p)u(t), \quad (1)$$

$$u(t) = -Kx(t), \quad (2)$$

where $x(t)$, $u(t)$ and p are state, control, and parameter vectors of dimensions n , m , and r , respectively.

The matrices $A(p)$ and $B(p)$ are system and input matrices that may be arbitrary functions of p , respectively. The gain matrix K is designed using some *mean* dynamic model represented by $A(p)$ and $B(p)$ evaluated for nominal parameters. The actual system depends on the actual (unknown) value of p . p is assumed to have a known or estimated probability density function, denoted by $\text{pr}(p)$, which expresses the parameter uncertainty statistically [16].

Each eigenvalue of the closed-loop system can be represented by the sum of the nominal value (i.e., mean values) and the amount of the perturbed one of each eigenvalue shown below

$$\begin{aligned} \lambda_i &= \sigma_i + j\omega_i, \\ \hat{\lambda} &= (\sigma_i + \Delta\sigma_i) + j(\omega_i + \Delta\omega_i), \end{aligned} \quad (3)$$

where $i=1, \dots, n$. The stability of a closed-loop system is determined by the eigenvalues of the system. Since stability requires all the roots to be in the open left-half s -plane, while the probability of instability results from even a single right-half s -plane root, as follows:

$$\begin{aligned} \mathbf{P} &\equiv \text{Pr}(\text{instability}) = 1 - \text{Pr}(\text{stability}) \\ &= 1 - \int_{-\infty}^0 \text{pr}(\sigma + \Delta\sigma) d\sigma, \end{aligned} \quad (4)$$

where $\sigma + \Delta\sigma$ is an n -vector, which is composed of the real parts of the system's eigenvalues, $\text{pr}(\sigma + \Delta\sigma)$ is the joint probability density function of $\sigma + \Delta\sigma$ (unknown analytically), and the integral that defines the probability of stability is evaluated over the space of individual components of $\sigma + \Delta\sigma$.

Denoting the probability density function of p as $\text{pr}(p)$, (4) is evaluated ε times with each element of p_j , $j=1$ to ε , specified by a random-number generator whose individual outputs are shaped by $\text{pr}(\sigma + \Delta\sigma)$. This Monte Carlo evaluation of the probability of stability becomes increasingly precise as ε becomes large, then

$$\int_{-\infty}^0 \text{pr}(\sigma + \Delta\sigma) d\sigma = \lim_{\varepsilon \rightarrow \infty} \frac{N(\max(\sigma + \Delta\sigma) \leq 0)}{\varepsilon}, \quad (5)$$

where $N(\cdot)$ is the number of cases for which all elements of $\sigma + \Delta\sigma$ are less than or equal to zero, that is, for which $\sigma_{\max} \leq 0$, where σ_{\max} is the maximum real eigenvalue component in $\sigma + \Delta\sigma$.

A controller design strategy, which can reduce the probability of instability by minimizing \mathbf{P} in (1), should be utilized. Since the eigenstructure assignment technique can place eigenvalues arbitrarily and exactly at desired locations, design specifications

of the proposed controller can be employed to render $\mathbf{P} = \mathbf{0}$.

3. EIGENSTRUCTURE ASSIGNMENT CONTROL

3.1. The probabilistic stability region

In order to analyze the stability, probabilistically, of LTI Systems with probabilistic parameter variations, we separate the given system (1) into the nominal and perturbed terms as follows:

$$\dot{x}(t) = (A + \Delta A(p))x(t) + (B + \Delta B(p))u(t), \quad (6)$$

where $A = E[A(p)]$, $B = E[B(p)]$, and $E[\cdot]$ denotes the element-wise expected values of variations for each matrix ' \cdot '.

If the state feedback (2) is applied to (6), the resulting closed-loop system becomes

$$\begin{aligned} \dot{x}(t) &= \{(A + \Delta A(p)) - (B + \Delta B(p))K\} x(t) \\ &= (A_c + E(p))x(t), \end{aligned} \quad (7)$$

where $A_c = A - BK$ and $E(p) = \Delta A(p) - \Delta B(p)K$ are the nominal closed-loop system and the perturbed term of the closed-loop system, respectively.

Let A_c be diagonalizable with $A_c = \Phi^{-1}\Lambda\Phi$ where $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_n)$ [18]. Let λ_i be the i -th eigenvalue of A_c , then the generalized eigenvalue problem of the nominal closed-loop system becomes as follows:

$$(\lambda_i I - A_c)\Phi = 0. \quad (8)$$

Furthermore, $\Phi^{-1}(A_c + E(p))\Phi = \Lambda + \Phi^{-1}E(p)\Phi$. If $\hat{\lambda}$ is an eigenvalue of $A_c + E(p)$, then $\hat{\lambda}I - \Lambda - \Phi^{-1}E(p)\Phi$ is singular.

Suppose $\hat{\lambda}I - \Lambda$ is nonsingular, the following matrix is singular.

$$\begin{aligned} &(\hat{\lambda}I - \Lambda)^{-1}(\hat{\lambda}I - \Lambda - \Phi^{-1}E(p)\Phi) \\ &= I - (\hat{\lambda}I - \Lambda)^{-1}\Phi^{-1}E(p)\Phi \end{aligned} \quad (9)$$

The bound of matrix norms can be stated by the following theorem [18] for the invertibility of matrix norms.

Theorem 1: A matrix $A \in M_n$ is invertible if there is a matrix norm $\|\cdot\|$ such that $\|I - A\| < 1$. If this condition is satisfied,

$$A^{-1} = \sum_{k=0}^{\infty} (I - A)^k. \quad \square$$

Using matrix norm $\|\Phi\| = \left(\sum_{i,j=1}^n |\phi_{ij}|^2 \right)^{1/2}$ and from

the Theorem 1, if $\|(\hat{\lambda}I - \Lambda)^{-1}\Phi^{-1}E(p)\Phi\| < 1$ then $(\hat{\lambda}I - \Lambda)^{-1}\Phi^{-1}E(p)\Phi$ is invertible. But, $\|(\hat{\lambda}I - \Lambda)^{-1}\Phi^{-1}E(p)\Phi\| \geq 1$ because the matrix in (9) is not invertible. Thus, using submultiplicative inequality and the property of matrix norms, we have the following relations.

$$\begin{aligned} 1 &\leq \|(\hat{\lambda}I - \Lambda)^{-1}\Phi^{-1}E(p)\Phi\| \\ &\leq \|\Phi^{-1}E(p)\Phi\| \|(\hat{\lambda}I - \Lambda)^{-1}\| \\ &= \|\Phi^{-1}E(p)\Phi\| \max_{1 \leq i \leq n} |\hat{\lambda} - \lambda_i|^{-1} \\ &= \frac{\|\Phi^{-1}E(p)\Phi\|}{\max_{1 \leq i \leq n} |\hat{\lambda} - \lambda_i|}. \end{aligned}$$

Suppose $k(\cdot)$ is the condition number for the matrix (\cdot) , and hence

$$\begin{aligned} \max_{1 \leq i \leq n} |\hat{\lambda} - \lambda_i| &\leq \|\Phi^{-1}E(p)\Phi\| \\ &\leq \|\Phi^{-1}\| \|\Phi\| \|E(p)\| \\ &= k(\Phi) \|E(p)\|. \end{aligned} \quad (10)$$

Thus, a relationship between the perturbed eigenvalues and the nominal closed-loop eigenvalues can be obtained by

$$|\hat{\lambda} - \lambda_i| \leq \|\Phi^{-1}\| \|\Phi\| \|E(p)\| = k(\Phi) \|E(p)\|. \quad (11)$$

Let λ_i be the center of the circle in the s -plane. The perturbed eigenvalues $\hat{\lambda}$ of the closed-loop system $A_c + E(p)$ are contained in the circle described by (11). If the closed-loop system is unstable for given probabilistic uncertainties, we can move further the probabilistic stability region into the left-half s -plane by using the feedback gain, and then the system can be more robust against probabilistic uncertainties. Since the radius of the probabilistic stability region depends on the condition number, we could design the system to be more robust by minimizing the condition number.

3.2. Eigenvector sensitivity

A stable closed-loop system could be unstable due to undesired probabilistic parameter variations. Since the sensitivity of eigenvectors depends strongly upon

the probability of instability, the sensitivity minimization problem is of great importance.

First, in order to describe the problem of eigenstructure assignment, the relation between the the right modal matrix(Φ) in (8) and the left modal matrix(Ψ) is defined by

$$\Phi = [\phi_1, \dots, \phi_n], \Psi = [\psi_1, \dots, \psi_n] \equiv \Phi^{-H},$$

where ' H ' denotes the transpose and conjugate operation.

From the relation of $\Psi^H \Phi = I$, the unit matrix, one has $\psi_i^T \phi_i = 1$. It then follows that $\psi_i^T A_c \phi_i = \lambda_i$. Let a_{ij} be the (i, j) -th element of A_c , then we have [19]

$$\frac{\partial \lambda_i}{\partial a_{ij}} = \frac{\partial \psi_i^H}{\partial a_{ij}} A_c \phi_i + \psi_i^H \frac{\partial A_c}{\partial a_{ij}} \phi_i + \psi_i^H A_c \frac{\partial \phi_i}{\partial a_{ij}}, \quad (12)$$

$\forall i.$

In addition, from the relation $\psi_i^T \phi_i = 1$, we have

$$\frac{\partial \psi_i^H}{\partial a_{ij}} \phi_i + \psi_i^H \frac{\partial \phi_i}{\partial a_{ij}} = 0. \quad \text{It then follows from the fact}$$

$A_c \phi_i = \lambda_i \phi_i$ and $\psi_i^H A_c = \lambda_i \psi_i^H$ that

$$\frac{\partial \lambda_i}{\partial a_{ij}} = \psi_i^H \frac{\partial A_c}{\partial a_{ij}} \phi_i, \quad \forall i. \quad (13)$$

Hence, the governing equation on the eigenvector sensitivity can be achieved as follows:

$$\left(\frac{\partial \lambda_i}{\partial A_c} \right)^T = \Phi \Psi^H. \quad (14)$$

Using the matrix norm, the eigenvector sensitivity can be stated by

$$S(\lambda) = \|\Phi\| \|\Psi^H\| = \|\Phi\| \|\Phi^{-1}\| = k(\Phi). \quad (15)$$

From (11) and (15), the right modal matrix of the closed-loop system, which minimizes both the condition number and the eigenvector sensitivity, can be obtained from the following performance index:

$$J = \left\| (\Phi^d) \Phi_{aug}^a \bar{P} - I \right\|^2$$

$$= \left\| U^{-1} \Phi_{aug}^a \bar{P} - I \right\|^2, \quad (16)$$

where Φ^d , Φ_{aug}^a , \bar{P} and U are the desired right modal matrix, the augmented achievable right modal matrix, a linear combination coefficient matrix, and a unitary matrix, respectively. Φ_{aug}^a should lie in

certain column spaces, which consist of the columns of matrix N_{λ_i} , which is generated by the null space of the matrix $[\lambda_i I - A|B]$. By the above constraint, the achievable right eigenvectors are assigned to the best possible set of eigenvectors in the least square sense according to the relation between m , and n , where $m = \text{rank}[B]$, $n = \text{rank}[A]$.

The following theorem gives the necessary and sufficient condition for the existence [2] which yields the right eigenstructure.

Theorem 2: Let $\{\lambda_1, \lambda_2, \dots, \lambda_n\}$ be a self-conjugate set of distinct complex numbers. There exists a real $(m \times n)$ matrix K such that

$$(A - BK)\phi_i = \lambda_i \phi_i, \quad i = 1, 2, \dots, n, \quad (17)$$

if and only if, for each i ,

- 1) $\{\phi_1, \phi_2, \dots, \phi_n\}$ is a linearly independent set in C^n (\cdot) the space of complex n -vectors.
- 2) $\phi_i = \phi_j^*$ when $\lambda_i = \lambda_j^*$

In the following, the superscript $(\cdot)^*$ denotes the conjugate of a given complex vector or scalar (\cdot) .

- 3) $\phi_i = \text{span}\{N_{\lambda_i}\}$.

Also, if K exists and $\text{rank}[B] = m$, then K is unique and computed by using the obtained submatrices N_{λ_i} and M_{λ_i} . \square

From Theorem 2, the gain matrix K is given by

$$K = W(\Phi^a)^{-1}, \quad (18)$$

where $W = [w_1, \dots, w_n]$, $w_i = M_{\lambda_i} \bar{P}_i$, $\phi_i^a = N_{\lambda_i} \bar{P}_i$, and

$$[\lambda_i I - A|B] \begin{bmatrix} N_{\lambda_i}^T \\ M_{\lambda_i}^T \end{bmatrix}^T = 0.$$

3.3. Determination and update of desired eigenvalues

If the achievable right eigenvectors are obtained by using the eigenvector sensitivity minimization procedure, then the probabilistic stability radius is determined to be the minimum. However, because the imaginary axis of the s -plane would be included in the probabilistic stability region, the probability of instability is most likely to increase.

Using the relationship between the probabilistic stability region and the closed-loop eigenvalues, the eigenvalue-update equation can be obtained by

$$\delta s = -\min |k(\Phi) \|E(p)\| - \text{re}(\hat{\lambda})|, \quad (19)$$

where $|\cdot|$ is the minimum absolute value of ' \cdot ', $\text{re}(\cdot)$ is the real value of ' \cdot ', and δs is an update variable to determine the new desired eigenvalues. If

$\text{re}(\cdot)$ is smaller than $k(\Phi)\|E(p)\|$, the neighboring eigenvalues in imaginary axis can be updated by the following equation

$$\lambda^{nd} = \lambda^{od} + \delta s, \quad (20)$$

where λ^{nd} and λ^{od} are the new desired eigenvalue and the old desired eigenvalue, respectively.

The probabilistic stability region moves into the left-half plane as δs until the probability of instability becomes zero.

If the closed-loop eigenvalues λ_i are assigned to $-\infty$, the tremendous variations of the probabilistic uncertainties may not affect the stability of the system. Note that the feedback gain is increased to ∞ in this case and the norm of the perturbed term becomes ∞ . Thus, we have to address the relationship between the bound of the closed-loop eigenvalues and the norm of the feedback gain matrix for a reasonable tradeoff.

If the norm of the feedback gain is given, the movable bound of the closed-loop eigenvalues can be obtained by using the result described in Section 3.1 as follows:

$$\begin{aligned} |\lambda_i - \lambda_o| &\leq \|\Phi_o\| \|\Phi_o^{-1}\| \|BK\| \\ &= k(\Phi_o) \|BK\| \\ &\leq k(\Phi_o) \|B\| \|K\|, \end{aligned} \quad (21)$$

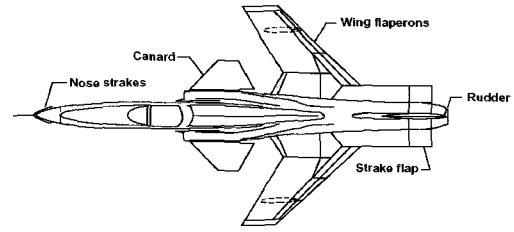
where λ_o and Φ_o are the open-loop eigenvalues and the open-loop right modal matrix, respectively.

The eigenvalues of the closed-loop system are contained in the circle given in (21). Since the radius of the movable bound is constrained by $k(\Phi_o)\|B\|\|K\|$, λ_i can be assigned a value in the neighborhood of λ_o under the norm bound of the feedback gain.

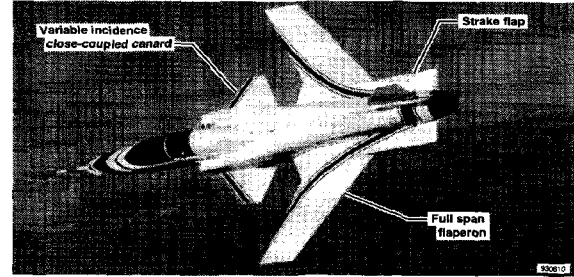
Remark 1: If the condition number of the open-loop system is very large itself, the circle becomes very large and thus, the robustness of the open-loop system may be much weakened.

4. APPLICATION TO A FLIGHT CONTROL DESIGN

An application of probabilistic robustness based on the longitudinal dynamics of an open-loop unstable aircraft [16] is considered in this section. The Forward-Swept-Wing Demonstrator's aerodynamic center moves forward its center of gravity, resulting in static instability. The given flight control system compensated for this instability by sensing flight conditions such as attitude and speed and through computer processing. The Forward-Swept-Wing gives an airplane the appearance of flying backward in Fig.



(a) An upper view.



(b) In-flight view.

Fig. 1. A view of forward-swept-wing demonstrator.

1. Its Forward-Swept-Wings were mounted well back on the fuselage, while its canards, the horizontal stabilizers to control pitch, were in front of the wings instead of on the tail. The complex geometries of the wings and canards combined to provide exceptional maneuverability, supersonic performance, and a light structure. Air moving over the Forward-Swept-Wings tended to flow inward toward the root of the wing instead of outward toward the wing tip, as it occurs on an aft wing. This reverse air flow did not allow the wing tips and their ailerons to stall at high angles of attack.

Possible uncertainties in aerodynamic and thrust effect as well as separate dynamic pressure (ρ and V) effects lead to a 12-element parameter vector.

$$p = [\rho \ V \ f_{11} \ f_{12} \ f_{13} \ f_{22} \ f_{32} \ f_{33} \ g_{11} \ g_{12} \ g_{31} \ g_{32}].$$

Velocity (V) and air-density (ρ) are modeled as uniform parameters, the remaining terms are kinematic, due to gravity, identically zero or otherwise negligible. Each parameter perturbation is distributed around the nominal value, and its correlation is assumed to be independent of each other. In terms of the element p , $A(p)$, and $B(p)$ are given by

$$A(p) = \begin{bmatrix} \frac{-2gf_{11}}{V} & \frac{\rho V^2 f_{12}}{2} & \rho V f_{13} & -g \\ \frac{-45}{V^2} & \frac{\rho V f_{22}}{2} & 1 & 0 \\ 0 & \frac{\rho V^2 f_{32}}{2} & \rho V_{33} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B(p) = \frac{\rho V^2}{2} \begin{bmatrix} g_{11} & g_{12} \\ 0 & 0 \\ g_{31} & g_{32} \\ 0 & 0 \end{bmatrix}.$$

The state components represent the forward velocity, angle of attack, pitch rate, and pitch angle. The principal control surfaces are the canard control surface and the thrust setting. The mean model and its eigenvalues for the given system are as follows:

$$A(p) = \begin{bmatrix} -0.02 & -0.3 & -0.4 & -32.2 \\ -0.001 & -1.2 & 1 & 0 \\ 0 & 18. & -0.6 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B(p) = \begin{bmatrix} -0.04 & 35. \\ 0 & 0 \\ 0.2 & -0.2 \\ 0 & 0 \end{bmatrix},$$

$$\lambda_i = \begin{bmatrix} -0.0102 \pm 0.057j & -5.1535 & 3.3539 \end{bmatrix}.$$

For illustration, ρ and V are 30% uniform distributed parameters, and the remaining elements of p are independent each other with 30% standard deviation Gaussian uncertainties. Design specifications are determined by $\mathbf{P}=0$, and the upper norm bound of the feedback matrix $\|K\|_{bound}$ is set to 100, and the desired eigenvalue and right modal matrix are given by

$$\Lambda^d = \begin{bmatrix} -32.21 & -1.09 & -5.36 & -0.0186 \end{bmatrix},$$

$$\Phi^a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Simulations have been conducted through 5000 Monte Carlo evaluations, because the system parameters are given by probabilistic uncertainty. The probabilistic stability region of a given system is shown in Fig. 2. The probability of instability with 95% confidence coefficient is 0.0788. The closed-loop condition number, the norm of the probabilistic uncertainties, and the radius of stability circle are obtained by

$$k(\Phi) = 1.2933, \quad \|E(p)\| = 5.0656,$$

$$k(\Phi)\|E(p)\| = 6.5513.$$

The condition number $k(\Phi)$ of the obtained right modal matrix of the closed-loop system is 1.2933, while the $k(\Phi_o)$ of that of the open-loop system is

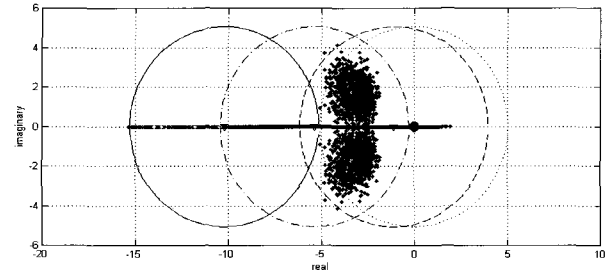


Fig. 2. The probabilistic stability region of a given system (Circles: Bounds of perturbed system, Dots: Eigenvalue distribution for probabilistic parameter).

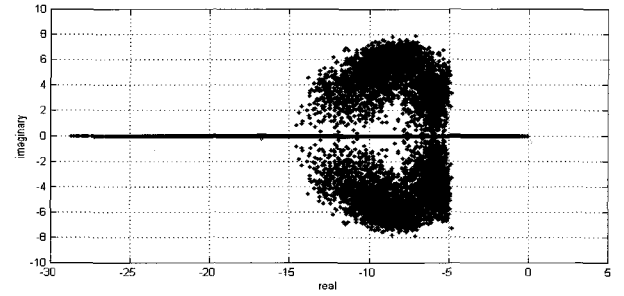


Fig. 3. The probability distribution of closed-loop eigenvalues after update of δs ($\mathbf{P} = 0$).

1069.8. The condition number of the right modal matrix of the closed-loop system considerably decreases and approaches the minimum value, 1, of the condition number. Finally, updating the eigenvalues as $\delta s = 5.047$ to $\mathbf{P} = 0$, the probability distribution of the closed-loop eigenvalues is shown in Fig. 3. Thus, it can be concluded that the proposed method can be utilized to improve the robustness of LTI systems with probabilistic parameter variations.

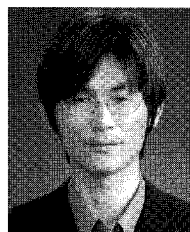
5. CONCLUSIONS

In this paper, we presented a method that had the capability of exact eigenstructure assignment with the probabilistic robustness properties for LTI systems with probabilistic parameter variations. The probabilistic parameter variations were considered as the perturbed terms in the nominal (or mean) system, and the probabilistic stability region, which is based upon the relationship between perturbed eigenvalues and nominal closed-loop eigenvalues, was presented. Monte Carlo simulations were conducted to evaluate the eigenvalues of the perturbed system. These eigenvalues can be used to develop the concept of probability of instability. The eigenstructure assignment for a system with probabilistic parameter variations was characterized in terms of the minimization of the eigenvector sensitivity by a newly proposed performance index. The usefulness of the proposed scheme was verified by a flight control

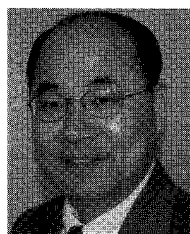
design example with probabilistic parameter variations.

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