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Estimation for the Generalized Extreme Value Distribution Based on Multiply Type-II Censored Samples

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Abstract

In this paper, we derive the approximate maximum likelihood estimators of the scale parameter and the location parameter in a generalized extreme value distribution under multiply Type-II censoring by the approximate maximum likelihood estimation method. We compare the proposed estimators in the sense of the mean squared error for various censored samples.

Keywords : Approximate Maximum Likelihood Estimator, Generalized Extreme Value Distribution, Multiply Type-II Censored Sample

1. Introduction

The generalized extreme value distribution has the following cumulative distribution function (cdf)

$$F(x)=\begin{cases}\exp\left[-\left\{1-\lambda\left(\dfrac{x-\theta}{\sigma}\right)\right\}^{\frac{1}{\lambda}}\right], & -\infty<x<\theta+\dfrac{\sigma}{\lambda} \quad ,\lambda>0\\ & \theta+\dfrac{\sigma}{\lambda}<x<\infty \quad ,\lambda<0\\ \exp\left\{-\exp\left(-\dfrac{x-\theta}{\sigma}\right)\right\}, & -\infty<x<\infty \quad ,\lambda=0\end{cases} \tag{1.1}$$

where θ, σ and λ are the location, scale and shape parameters, respectively.

In the special case of $\lambda=0$, the distribution is the Gumbel-type distribution or

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the extreme value distribution. Extreme value distributions have been used in the analysis of data concerning floods, extreme sea levels and air pollution problems.

Prescott and Walden (1980) provided for the information matrix of the three-parameter generalized extreme-value distribution. Hosking (1984) considered tests of the hypothesis $H_0 : \lambda = 0$ and compared their small-sample properties when testing against the two-sided alternative $H_1 : \lambda \neq 0$ and the one-sided alternatives $H_1^- : \lambda < 0$ and $H_1^+ : \lambda > 0$. Johnson et al. (1994), and Kotz and Nadarajah (2000) studied the properties of the generalized extreme value distribution. The parameter λ may be used to model a wide range of tail behavior. The case $\lambda > 0$ is that of a polynomially decreasing tail function and therefore corresponds to a long-tailed parent distribution. The case $\lambda = 0$ is that of an exponentially decreasing tail, while $\lambda < 0$ is the case of a finite upper endpoint and is therefore short-tailed.

In many life test studies, it is common that the lifetimes of test units may not be able to record exactly. For example, multiply Type-II censored sampling arises in a life-testing experiment whenever the experimenter does not observe the failure times of some units placed on a life-test.

Balakrishnan et al. (1995) derived the estimators for the location and scale parameters of the extreme value distribution under multiply Type-II censoring. Balakrishnan et al. (2004) discussed point and interval estimation for the extreme value distribution under progressively Type-II censoring. Kang (2005) proposed the approximate maximum likelihood estimators (AMLEs) of the location and the scale parameters of the extreme value distribution. Recently, Han and Kang (2006) derived AMLEs of the scale parameter and the location parameter in the two-parameter Rayleigh distribution under multiply Type-II censoring by the approximate maximum likelihood estimation method when two parameters are unknown. Lin et al. (2006) discussed the maximum likelihood estimates of the parameters of the log-gamma distribution based on progressively Type-II censored samples.

In this paper, we derive the AMLEs of the scale parameter σ and the location parameter θ when shape parameter $\lambda \neq 0$ under multiply Type-II censored sample. We also compare the proposed estimators in the sense of the mean squared error (MSE) for various censored samples.

2. Approximate Maximum Likelihood Estimators

Let us assume that the following multiply Type-II censored sample from a sample of size n is

$$X_{a_1:n} < X_{a_2:n} < \cdots < X_{a_s:n} \tag{2.1}$$

where $1 \leq a_1 < a_2 < \cdots < a_s \leq n$.

$$a_0 = 0, \quad a_{s+1} = n+1, \quad F(x_{a_0:n}) = 0, \quad F(x_{a_{s+1}:n}) = 1. \tag{2.2}$$

The likelihood function based on the multiply Type-II censored sample (2.1) can be written as

$$L = n!\prod_{j=1}^{s} f(x_{a_j:n})\prod_{j=1}^{s+1}\frac{\left[F(x_{a_j:n}) - F(x_{a_{j-1}:n})\right]^{a_j - a_{j-1}-1}}{(a_j - a_{j-1}-1)!}. \tag{2.3}$$

The random variable $Z_{i:n} = (X_{i:n}-\theta)/\sigma$ then has a standard generalized extreme value distribution with the probability density function (pdf) and the cdf;

$$f(z) = \exp[-\{1-\lambda z\}^{1/\lambda}]\{1-\lambda z\}^{(1-\lambda)/\lambda},$$

$$F(z) = \exp[-\{1-\lambda z\}^{1/\lambda}], \quad \begin{array}{ll} -\infty < z < 1/\lambda, & \lambda > 0 \\ 1/\lambda < z < \infty, & \lambda < 0. \end{array}$$

On differentiating the log-likelihood function with respect to σ in turn and equation to zero, we obtain the estimating equations as

$$\begin{aligned}\frac{\partial \ln L}{\partial \sigma} = -\frac{1}{\sigma}\Bigg[&(a_1 - 1)\frac{f(Z_{a_1:n})}{F(Z_{a_1:n})}Z_{a_1:n} - (n-a_s)\frac{f(Z_{a_s:n})}{1-F(Z_{a_s:n})}Z_{a_s:n} + \sum_{j=1}^{s}\frac{f'(Z_{a_j:n})}{f(Z_{a_j:n})}Z_{a_j:n} \\ &-(1-\lambda)\sum_{j=1}^{s}\frac{Z_{a_j:n}}{1-\lambda Z_{a_j:n}} + \sum_{j=2}^{s}(a_j - a_{j-1}-1)\frac{f(Z_{a_j:n})Z_{a_j:n} - f(Z_{a_{j-1}:n})Z_{a_{j-1}:n}}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})}\Bigg] \\ = 0.&\end{aligned} \tag{2.4}$$

Since the likelihood equations is very complicated, the equation (2.4) does not admit an explicit solution for σ.

Let

$$\xi_i = F^{-1}(p_i) = \frac{1}{\lambda}\left[1 - \{\ln(1/p_i)\}^{\lambda}\right]$$

where $p_i = \dfrac{i}{n+1}$, $q_i = 1 - p_i$, λ is known.

First, we can approximate these functions by

$$\frac{f(Z_{a_j:n})}{F(Z_{a_j:n})}Z_{a_j:n} \approx \kappa_{1j} + \delta_{1j}Z_{a_j:n} \tag{2.5}$$

$$\frac{f(Z_{a_s:n})}{1-F(Z_{a_s:n})}Z_{a_s:n} \approx \kappa_1 + \delta_1 Z_{a_s:n} \tag{2.6}$$

$$\frac{Z_{a_j:n}}{1-\lambda Z_{a_j:n}} \approx \kappa_{2j} + \delta_{2j}Z_{a_j:n} \tag{2.7}$$

and

$$\frac{f(Z_{a_j:n})Z_{a_j:n} - f(Z_{a_{j-1}:n})Z_{a_{j-1}:n}}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})} \approx a_{1j} + \beta_{1j}Z_{a_j:n} + \gamma_{1j}Z_{a_{j-1};n} \tag{2.8}$$

where

$$\kappa_{1j} = -\frac{\xi_{a_j}^2}{p_{a_j}}\left[f'(\xi_{a_j}) - \frac{f^2(\xi_{a_j})}{p_{a_j}}\right], \qquad \delta_{1j} = \frac{1}{p_{a_j}}\left[f(\xi_{a_j}) + f'(\xi_{a_j})\xi_{a_j} - \frac{f^2(\xi_{a_j})}{p_{a_j}}\xi_{a_j}\right]$$

$$\kappa_1 = -\frac{\xi_{a_s}^2}{q_{a_s}}\left[f'(\xi_{a_s}) + \frac{f^2(\xi_{a_s})}{q_{a_s}}\right], \qquad \delta_1 = \frac{1}{q_{a_s}}\left[f(\xi_{a_s}) + f'(\xi_{a_s})\xi_{a_s} + \frac{f^2(\xi_{a_s})}{q_{a_s}}\xi_{a_s}\right]$$

$$\kappa_{2j} = -\frac{\lambda\xi_{a_j}^2}{\left[1-\lambda\xi_{a_j}\right]^2}, \qquad \delta_{2j} = \frac{1}{\left[1-\lambda\xi_{a_j}\right]^2}$$

$$\alpha_{1j} = K^2 - \frac{f'(\xi_{a_j})\xi_{a_j}^2 - f'(\xi_{a_{j-1}})\xi_{a_{j-1}}^2}{p_{a_j} - p_{a_{j-1}}}$$

$$\beta_{1j} = \frac{1}{p_{a_j} - p_{a_{j-1}}}\left[(1-K)f(\xi_{a_j}) + f'(\xi_j)\xi_{a_j}\right],$$

$$\gamma_{1j} = -\frac{1}{p_{a_j} - p_{a_{j-1}}}\left[(1-K)f(\xi_{a_{j-1}}) + f'(\xi_{j-1})\xi_{a_{j-1}}\right]$$

$$K = \frac{f(\xi_{a_j})\xi_{a_j} - f(\xi_{a_{j-1}})\xi_{a_{j-1}}}{p_{a_j} - p_{a_{j-1}}}$$

By substituting the equations (2.5), (2.6), (2.7) and (2.8) into the equation (2.4), we can derive an estimator of σ as follows;

$$\hat{\sigma}_{1i} = \frac{B_1 + C_1\hat{\theta}_i}{A_1}, \ i = 0,1 \tag{2.9}$$

where

$$A_1 = s + (a_1 - 1)\kappa_{11} - (n - a_s)\kappa_1 + \sum_{j=1}^{s}\kappa_{1j} - (1-\lambda)\sum_{j=1}^{s}\kappa_{2j} + \sum_{j=2}^{s}(a_j - a_{j-1} - 1)\alpha_{1j}$$

$$B_1 = (a_1 - 1)\delta_{11}X_{a_1:n} - (n - a_s)\delta_1 X_{a_s:n} + \sum_{j=1}^{s}\delta_{1j}X_{a_j:n} - (1-\lambda)\sum_{j=1}^{s}\delta_{2j}X_{a_j:n}$$
$$+ \sum_{j=2}^{s}(a_j - a_{j-1} - 1)(\beta_{1j}X_{a_j:n} + \gamma_{1j}X_{a_{j-1}:n})$$

$$C_1 = (a_1 - 1)\delta_{11} - (n - a_s)\delta_1 + \sum_{j=1}^{s}\delta_{1j} - (1-\lambda)\sum_{j=1}^{s}\delta_{2j} + \sum_{j=2}^{s}(a_j - a_{j-1} - 1)(\beta_{1j} + \gamma_{1j}).$$

Second, we can also approximate these functions by

$$\frac{f(Z_{a_j:n})}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})} \approx a_{2j} + \beta_{2j}Z_{a_j:n} + \gamma_{2j}Z_{a_{j-1}:n} \tag{2.10}$$

and

$$\frac{f(Z_{a_{j-1}:n})}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})} \approx a_{3j} + \beta_{3j}Z_{a_j:n} + \gamma_{3j}Z_{a_{j-1}:n} \tag{2.11}$$

where

$$\alpha_{2j} = \frac{1}{p_{a_j} - p_{a_{j-1}}}\left[(1+K)f(\xi_{a_j}) - f'(\xi_j)\xi_{a_j}\right]$$

$$\beta_{2j} = \frac{1}{p_{a_j} - p_{a_{j-1}}}\left[f'(\xi_{a_j}) - \frac{f^2(\xi_{a_j})}{p_{a_j} - p_{a_{j-1}}}\right], \qquad \gamma_{2j} = \frac{f(\xi_{a_j})f(\xi_{a_{j-1}})}{\left[p_{a_j} - p_{a_{j-1}}\right]^2}$$

$$\alpha_{3j} = \frac{1}{p_{a_j} - p_{a_{j-1}}}\left[(1+K)f(\xi_{a_{j-1}}) - f'(\xi_{j-1})\xi_{a_{j-1}}\right], \quad \beta_{3j} = -\frac{f(\xi_{a_j})f(\xi_{a_{j-1}})}{\left[p_{a_j} - p_{a_{j-1}}\right]^2}$$

$$\gamma_{3j} = \frac{1}{p_{a_j} - p_{a_{j-1}}}\left[f'(\xi_{a_{j-1}}) + \frac{f^2(\xi_{a_{j-1}})}{p_{a_j} - p_{a_{j-1}}}\right].$$

By substituting the equations (2.5), (2.6), (2.7), (2.10) and (2.11) into the equation (2.4), we can derive another estimator of σ as follows;

$$\hat{\sigma}_{2i} = \frac{-B_{2i} + \sqrt{B_{2i}^2 - 4A_2C_{2i}}}{2A_2}, \; i = 0,1 \tag{2.12}$$

where

$$A_2 = s + (a_1 - 1)\kappa_{11} - (n - a_s)\kappa_1 + \sum_{j=1}^{s}\kappa_{1j} - (1-\lambda)\sum_{j=1}^{s}\kappa_{2j}$$

$$\begin{aligned} B_{2i} = {} & (a_1 - 1)\delta_{11}X_{a_1:n} - (n - a_s)\delta_1 X_{a_s:n} + \sum_{j=1}^{s}\delta_{1j}X_{a_j:n} - (1-\lambda)\sum_{j=1}^{s}\delta_{2j}X_{a_j:n} \\ & + \sum_{j=2}^{s}(a_j - a_{j-1} - 1)(\alpha_{2j}X_{a_j:n} - \alpha_{3j}X_{a_{j-1}:n}) \\ & - \left\{(a_1 - 1)\delta_{11} - (n - a_s)\delta_1 + \sum_{j=1}^{s}\delta_{1j} - (1-\lambda)\sum_{j=1}^{s}\delta_{2j} + \sum_{j=2}^{s}(a_j - a_{j-1} - 1)(\alpha_{2j} - \alpha_{3j})\right\}\hat{\theta}_i \end{aligned}$$

$$\begin{aligned} C_{2i} = {} & \sum_{j=2}^{s}(a_j - a_{j-1} - 1)\left\{\beta_{2j}(X_{a_j:n} - \hat{\theta}_i)^2 + 2\gamma_{2j}(X_{a_j:n} - \hat{\theta}_i)(X_{a_{j-1}:n} - \hat{\theta}_i)\right. \\ & \left. - \gamma_{3j}(X_{a_{j-1}:n} - \hat{\theta}_i)^2\right\}. \end{aligned}$$

Let $\hat{\theta}_0 = \theta_0$ be known location parameter. Now we obtain the estimator of the location parameter θ. From the equation (2.3), the likelihood equation for θ is obtained as

$$\begin{aligned} \frac{\partial \ln L}{\partial \theta} = {} & -\frac{1}{\sigma}\left[(a_1 - 1)\frac{f(Z_{a_1:n})}{F(Z_{a_1:n})} - (n - a_s)\frac{f(Z_{a_s:n})}{1 - F(Z_{a_s:n})} + \sum_{j=1}^{s}\frac{f'(Z_{a_j:n})}{f(Z_{a_j:n})}\right. \\ & \left. -(1-\lambda)\sum_{j=1}^{s}\frac{1}{1 - \lambda Z_{a_j:n}} + \sum_{j=2}^{s}(a_j - a_{j-1} - 1)\frac{f(Z_{a_j:n}) - f(Z_{a_{j-1}:n})}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})}\right] \\ = {} & 0. \end{aligned} \tag{2.13}$$

Equation (2.13) does not admit an explicit solution for θ. But we can approximate these functions by

$$\frac{f(Z_{a_j:n})}{F(Z_{a_j:n})} \approx \kappa_{3j} - \frac{\kappa_{1j}}{\xi_{a_j}^2} Z_{a_j:n} \tag{2.14}$$

$$\frac{f(Z_{a_s:n})}{1 - F(Z_{a_s:n})} \approx \kappa_2 - \frac{\kappa_1}{\xi_{a_s}^2} Z_{a_s:n} \tag{2.15}$$

$$\frac{1}{1 - \lambda Z_{a_j:n}} \approx \kappa_{4j} + \delta_{2j} Z_{a_j:n} \tag{2.16}$$

$$\frac{f(Z_{a_j:n}) - f(Z_{a_{j-1}:n})}{F(Z_{a_j:n}) - F(Z_{a_{j-1}:n})} \approx \alpha_{4j} + \beta_{4j}Z_{a_j:n} + \gamma_{4j}Z_{a_{j-1}:n} \tag{2.17}$$

where

$$\kappa_{3j} = \frac{1}{p_{a_j}}\left[f(\xi_{a_j}) - f'(\xi_{a_j})\xi_{a_j} + \frac{f^2(\xi_{a_j})}{p_{a_j}}\xi_{a_j}\right]$$

$$\kappa_2 = \frac{1}{q_{a_s}}\left[f(\xi_{a_s}) - f'(\xi_{a_s})\xi_{a_s} - \frac{f^2(\xi_{a_s})}{q_{a_s}}\xi_{a_s}\right], \quad \kappa_{4j} = -\frac{1 - 2\lambda\xi_{a_j}}{\left[1 - \lambda\xi_{a_j}\right]^2}$$

$\alpha_{4j} = \alpha_{2j} - \alpha_{3j}$, $\beta_{4j} = \beta_{2j} - \beta_{3j}$, and $\gamma_{4j} = \gamma_{2j} - \gamma_{3j}$.

By substituting the equations (2.14), (2.15), (2.16), and (2.17) into the equation

(2.13), we can derive an estimator of θ as follows;

$$\hat{\theta}_1 = \frac{A_0 B_1 - A_1 B_0}{A_0 C_1 - A_1 C_0} \tag{2.18}$$

where

$$A_0 = (a_1 - 1)\kappa_{31} - (n - a_s)\kappa_2 + \sum_{j=1}^{s} \kappa_{3j} - (1-\lambda)\sum_{j=1}^{s} \kappa_{4j} + \sum_{j=2}^{s} (a_j - a_{j-1} - 1)\alpha_{4j}$$

$$B_0 = (a_1 - 1)\delta_{31} X_{a_1 : n} - (n - a_s)\delta_2 X_{a_s : n} + \sum_{j=1}^{s} \delta_{3j} X_{a_j : n} - (1-\lambda)\sum_{j=1}^{s} \delta_{4j} X_{a_j : n} + \sum_{j=2}^{s} (a_j - a_{j-1} - 1)(\beta_{4j} X_{a_j : n} + \gamma_{4j} X_{a_{j-1} : n})$$

$$C_0 = (a_1 - 1)\delta_{31} - (n - a_s)\delta_2 + \sum_{j=1}^{s} \delta_{3j} - (1-\lambda)\sum_{j=1}^{s} \delta_{4j} + \sum_{j=2}^{s} (a_j - a_{j-1} - 1)(\beta_{4j} + \gamma_{4j}).$$

3. The Simulated Results

The mean squared errors of the proposed estimators are simulated by Monte Carlo simulation method. The simulation procedure is repeated 10,000 times for the sample size $n = 20, 50$, $\lambda = -0.5, 0.5, 0.0$ and various choices of censoring ($m = n - s$ is the number of unobserved or missing data) under multiply Type-II censored samples. These values are given in Table 1, Table 2, and Table 3.

From Table 1, 2, and 3, we have the following results;

i) $\hat{\sigma}_{20}$ and $\hat{\sigma}_{21}$ are more efficient than $\hat{\sigma}_{10}$ and $\hat{\sigma}_{11}$ in the sense of the MSE. The MSEs of the proposed estimators when $\lambda > 0$ are smaller than these MSEs when $\lambda < 0$.

ii)As expected, the MSE of all estimators decreases as sample size n increases. For fixed sample size, the MSE increases generally as m increases.

iii) The MSEs of the estimators for the left censored cases are larger than the right censored cases when the number of censoring m is fixed.

<Table 1> The relative mean squared errors for the estimators of the location parameter θ and the scale parameter σ when $\lambda=-0.5$.

n	m	a_j	$\hat{\sigma}_{10}$	$\hat{\sigma}_{20}$	$\hat{\theta}$	$\hat{\sigma}_{11}$	$\hat{\sigma}_{21}$
20	0	1~20	0.041904	0.041904	0.077451	0.085822	0.085822
	1	1~19	0.035268	0.035268	0.070976	0.069568	0.069568
		2~20	0.055485	0.055485	0.079201	0.103008	0.103008
	2	1~18	0.035046	0.035046	0.069574	0.067929	0.067929
		3~20	0.074829	0.074829	0.080035	0.121141	0.121141
		2~19	0.044326	0.044326	0.072956	0.080487	0.080487
	3	1~17	0.035599	0.035599	0.068929	0.068261	0.068261
		4~20	0.100342	0.100342	0.080942	0.141106	0.141106
		2~18	0.043555	0.043555	0.071536	0.077895	0.077895
		3~19	0.056959	0.056959	0.074495	0.091475	0.091475
	4	2~17	0.044009	0.044009	0.070858	0.077922	0.077922
		4~19	0.072987	0.072987	0.076513	0.103000	0.103000
		3~18	0.055440	0.055440	0.073160	0.087956	0.087956
		2~4 7~14 16~20	0.055408	0.064707	0.079582	0.103628	0.118826
	5	3~17	0.055929	0.055929	0.072481	0.087908	0.087908
		4~18	0.070617	0.070617	0.075322	0.098667	0.098667
		2~6 10~19	0.044249	0.045174	0.073269	0.081034	0.085174
	6	4~17	0.071113	0.071113	0.074679	0.098555	0.098555
		1 2 6~9 12~15 17~20	0.041223	0.055410	0.078594	0.086882	0.108593
50	0	1~50	0.011350	0.011350	0.026210	0.026603	0.026603
	1	1~49	0.010973	0.010973	0.025702	0.025346	0.025346
		2~50	0.012777	0.012777	0.026497	0.028720	0.028720
	2	1~48	0.011008	0.011008	0.025618	0.025301	0.025301
		3~50	0.014483	0.014483	0.026645	0.030718	0.030718
		2~49	0.012240	0.012240	0.025974	0.027158	0.027158
	3	1~47	0.011034	0.011034	0.025550	0.025265	0.025265
		4~50	0.016364	0.016364	0.026726	0.032811	0.032811
		2~48	0.012256	0.012256	0.025884	0.027064	0.027064
		3~49	0.013796	0.013796	0.026122	0.028905	0.028905
	4	2~47	0.012269	0.012269	0.025811	0.026994	0.026994
		4~49	0.015474	0.015474	0.026213	0.030707	0.030707
		3~48	0.013797	0.013797	0.026029	0.028775	0.028775
		2~4 7~14 16~50	0.012777	0.012768	0.026510	0.028736	0.029470
	5	3~47	0.013803	0.013803	0.025954	0.028683	0.028683
		4~48	0.015456	0.015456	0.026120	0.030541	0.030541
		2~6 10~19 21~50	0.012768	0.012882	0.026530	0.028741	0.029729
	6	4~47	0.015449	0.015449	0.026045	0.030424	0.030424
		1 2 6~9 12~15 17~50	0.011324	0.011187	0.026274	0.026672	0.027541

<Table 2> The relative mean squared errors for the estimators of the location parameter θ and the scale parameter σ when $\lambda = 0.5$.

n	m	a_j	$\hat{\sigma}_{10}$	$\hat{\sigma}_{20}$	$\hat{\theta}$	$\hat{\sigma}_{11}$	$\hat{\sigma}_{21}$
20	0	1~20	0.023042	0.023042	0.049890	0.031080	0.031080
	1	1~19	0.021952	0.021952	0.049999	0.030011	0.030011
		2~20	0.023682	0.023682	0.052137	0.033192	0.033192
	2	1~18	0.024575	0.024575	0.050239	0.032052	0.032052
		3~20	0.024301	0.024301	0.054856	0.035539	0.035539
		2~19	0.022630	0.022630	0.052215	0.032175	0.032175
	3	1~17	0.028368	0.028368	0.050738	0.034879	0.034879
		4~20	0.025108	0.025108	0.057818	0.038497	0.038497
		2~18	0.025585	0.025585	0.052347	0.034646	0.034646
		3~19	0.023273	0.023273	0.054906	0.034574	0.034574
	4	2~17	0.029818	0.029818	0.052696	0.037953	0.037953
		4~19	0.024078	0.024078	0.057837	0.037526	0.037526
		3~18	0.026543	0.026543	0.054958	0.037515	0.037515
		2~4 7~14 16~20	0.023550	0.019867	0.052328	0.033075	0.030109
	5	3~17	0.031250	0.031250	0.055168	0.041405	0.041405
		4~18	0.027628	0.027628	0.057847	0.040925	0.040925
		2~6 10~19	0.022546	0.020839	0.052559	0.032158	0.031022
	6	4~17	0.032829	0.032829	0.057945	0.045484	0.045484
		1 2 6~9 12~15 17~20	0.022646	0.017593	0.050828	0.030852	0.026896
50	0	1~50	0.006984	0.006984	0.019929	0.010691	0.010691
	1	1~49	0.006508	0.006508	0.019922	0.010285	0.010285
		2~50	0.007031	0.007031	0.020339	0.010960	0.010960
	2	1~48	0.006975	0.006975	0.019959	0.010650	0.010650
		3~50	0.007111	0.007111	0.020733	0.011283	0.011283
		2~49	0.006560	0.006560	0.020332	0.010561	0.010561
	3	1~47	0.007573	0.007573	0.019993	0.011133	0.011133
		4~50	0.007174	0.007174	0.021054	0.011555	0.011555
		2~48	0.007044	0.007044	0.020360	0.010952	0.010952
		3~49	0.006640	0.006640	0.020726	0.010885	0.010885
	4	2~47	0.007661	0.007661	0.020386	0.011465	0.011465
		4~49	0.006700	0.006700	0.021047	0.011151	0.011151
		3~48	0.007143	0.007143	0.020747	0.011305	0.011305
		2~4 7~14 16~50	0.007019	0.006426	0.020389	0.010954	0.010451
	5	3~47	0.007779	0.007779	0.020768	0.011844	0.011844
		4~48	0.007217	0.007217	0.021064	0.011593	0.011593
		2~6 10~19 21~50	0.007025	0.006409	0.020385	0.010965	0.010440
	6	4~47	0.007874	0.007874	0.021079	0.012164	0.012164
		1 2 6~9 12~15 17~50	0.006925	0.006005	0.020121	0.010653	0.009874

<Table 3> The relative mean squared errors for the estimators of the location parameter θ and the scale parameter σ when $\lambda=0.0$.

n	m	a_j	$\hat{\sigma}_{10}$	$\hat{\sigma}_{20}$	$\hat{\theta}$	$\hat{\sigma}_{11}$	$\hat{\sigma}_{21}$
20	0	1～20	0.031451	0.031451	0.057008	0.033012	0.033012
	1	1～19	0.032803	0.032803	0.057039	0.034512	0.034512
		2～20	0.034956	0.034956	0.058293	0.035988	0.035988
	2	1～18	0.033997	0.033997	0.057090	0.035876	0.035876
		3～20	0.038794	0.038794	0.060265	0.039243	0.039243
		2～19	0.036592	0.036592	0.058285	0.037733	0.037733
	3	1～17	0.035799	0.035799	0.057410	0.038000	0.038000
		4～20	0.042935	0.042935	0.062962	0.042938	0.042938
		2～18	0.038148	0.038148	0.058279	0.039418	0.039418
		3～19	0.040760	0.040760	0.060217	0.041266	0.041266
	4	2～17	0.040396	0.040396	0.058517	0.041925	0.041925
		4～19	0.045317	0.045317	0.062873	0.045331	0.045331
		3～18	0.042739	0.042739	0.060153	0.043320	0.043320
		2～4 7～14 16～20	0.034889	0.034470	0.058451	0.035942	0.036337
	5	3～17	0.045622	0.045622	0.060282	0.046362	0.046362
		4～18	0.047806	0.047806	0.062759	0.047843	0.047843
		2～6 10～19	0.036489	0.035373	0.058533	0.037660	0.037555
	6	4～17	0.051438	0.051438	0.062781	0.051523	0.051523
		1 2 6～9 12～15 17～20	0.031254	0.031521	0.057525	0.032840	0.034091
50	0	1～50	0.011909	0.011909	0.022186	0.012822	0.012822
	1	1～49	0.012095	0.012095	0.022192	0.013031	0.013031
		2～50	0.012549	0.012549	0.022345	0.013362	0.013362
	2	1～48	0.012263	0.012263	0.022219	0.013231	0.013231
		3～50	0.013261	0.013261	0.022529	0.013958	0.013958
		2～49	0.012740	0.012740	0.022351	0.013575	0.013575
	3	1～47	0.012429	0.012429	0.022242	0.013428	0.013428
		4～50	0.013944	0.013944	0.022676	0.014541	0.014541
		2～48	0.012923	0.012923	0.022376	0.013787	0.013787
		3～49	0.013469	0.013469	0.022533	0.014185	0.014185
	4	2～47	0.013107	0.013107	0.022395	0.014000	0.014000
		4～49	0.014167	0.014167	0.022679	0.014781	0.014781
		3～48	0.013668	0.013668	0.022556	0.014411	0.014411
		2～4 7～14 16～50	0.012558	0.012454	0.022363	0.013367	0.013398
	5	3～47	0.013871	0.013871	0.022572	0.014641	0.014641
		4～48	0.014384	0.014384	0.022699	0.015023	0.015023
		2～6 10～19 21～50	0.012555	0.012538	0.022370	0.013365	0.013509
	6	4～47	0.014610	0.014610	0.022713	0.015272	0.015272
		1 2 6～9 12～15 17～50	0.011944	0.011836	0.022241	0.012843	0.012936

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