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Testing the domestic financial data for the normality of the innovation based on the GARCH(1,1) model

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Abstract

Since Bollerslev (1986), the GARCH model has been popular in analysing the volatility of the financial time series. In real data analysis, practitioners conventionally put the normal assumption on the innovation random variables of the GARCH model, which is often violated. In this paper, we analyse the domestic financial data based on the GARCH(1,1) model and among existing normality tests, perform the Jarque-Bera test based on the residuals. It is shown that the innovation based on the GARCH(1,1) model dose not follow the normality assumption.

Keywords : GARCH Model, Jarque-Bera Test, Kurtosis, Normality Test, Skewness

1. Introduction

The volatility of the asset returns is crucial to perform the task of risk management, derivative pricing, and risk hedging. For instance, the volatility is the key factor to decide the price of options and used as the most important input in Black-Scholes model. To estimate volatility, EWMA(exponentially weighted moving average) model has been used. However, it has a drawback not to show the attribute of mean reversion, which is usually shown by a variance rate. Among the existing volatility models, the following GARCH(p,q) (generalized autoregressive conditional heteroscedastic) model (Bollerslev (1986)) has been popular in analysing the volatility of the financial time series:

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$$X_t = \epsilon_t \sqrt{h_t}\,,$$
$$h_t = \omega + \sum_{i=1}^{q} \alpha_i X_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}.$$

where $\omega > 0, \alpha_i \geq 0, 1 \leq i \leq q, \beta_j \geq 0, 1 \leq j \leq p$ and the innovations ϵ_t are i.i.d. random variables with $E(\epsilon_t) = 0$ and $E(\epsilon_t^2) = 1$.

In actual practice, it is conventional to put the normal assumption on the innovation random variables of the GARCH model. However, it is often revealed that real data analysis exhibit evidence against the normal assumption (Lazar and Alexander (2004)). Therefore, many researchers and practitioners have focused on the normality test for the innovations of GARCH models. In fact, there exist several test procedures such as the Kolmogorov-Smirnov test (Berkes and Horvath (2001), and Lee and Taniguchi (2005)), Bickel-Rosenblatt test (Horvath and Zitikis (2006)) and the Jarque-Bera (JB) test (Park et al. (2007)). Among these existing normality tests, we adopt the JB test which is well known to be efficient and easy to implement. Further, it is well known that the Kolmogorov-Smirnov test tends to produce low powers, and the Bickel-Rosenblatt test has a demanding problem like optimal bandwidth selection problem.

In this paper, we performed the JB test for the daily log returns of the domestic financial data based on the GARCH(1,1) model, which is widely used by the practitioners in financial circles (cf. Park and Lee (2007) and Hansen and Lunde (2005) and Tsay (2005)). Financial data consist of KOSPI200 (Korea Stock Price Index 200), KOSDAQ(Korea Securities Dealers Automated Quotation) index, Samsung electronics and Posco individual stock price.

2. Normality test of the domestic financial data

For the daily data $\{x_t\}$ of the domestic financial time series, we assume that the log return

$$y_t = \log \frac{x_t}{x_{t-1}}, \quad t = 1, \ldots, T$$

satisfies the GARCH(1,1) model:

$$\begin{aligned} y_t &= \epsilon_t \cdot \sqrt{h_t} \\ h_t &= \omega + \alpha y_{t-1}^2 + \beta h_{t-1} \end{aligned} \qquad (2.1)$$

Figure 1 shows that KOSDAQ index and the Posco index is more volatile than the KOSPI200 index and the Samsung index, respectively. Moreover, KOSDAQ index seems to be skewed to the left. Further, we can notice that the volatilities are reduced recently, especially for KOSPI200 and Samsung electronics index.

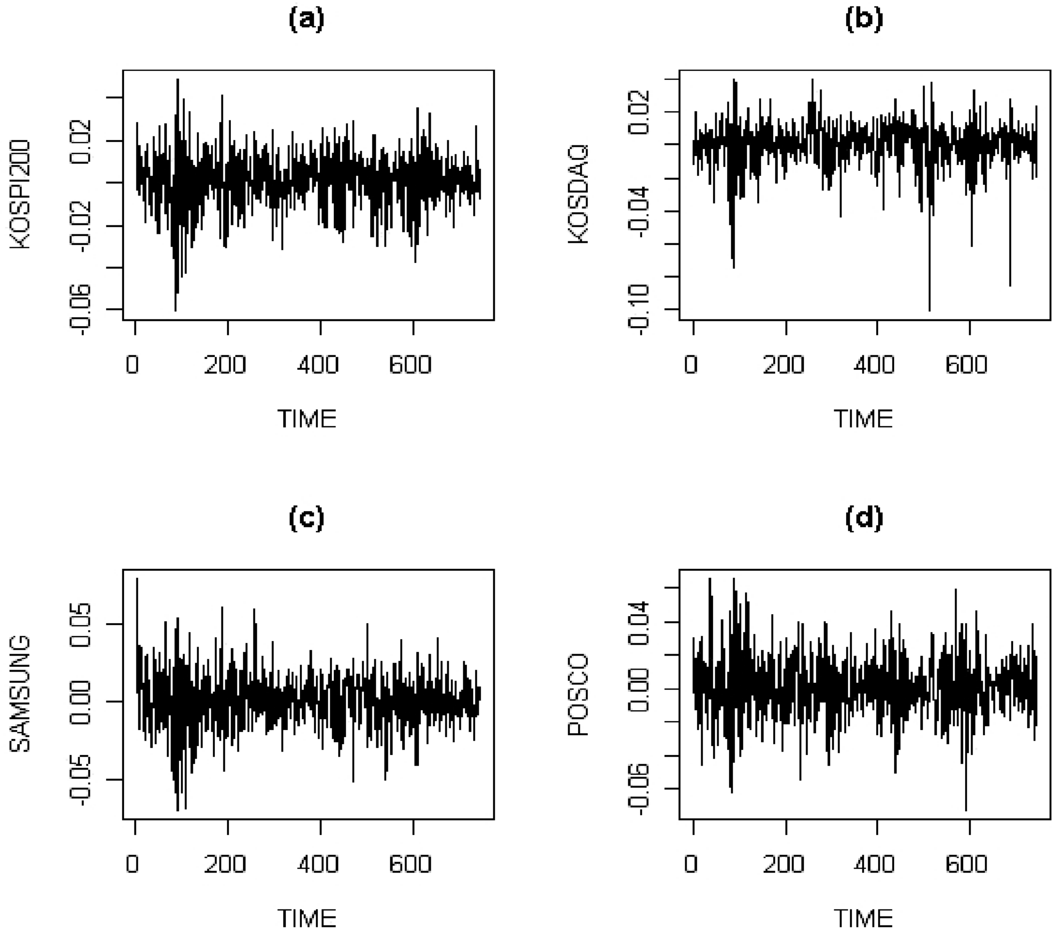

Figure 1. Time series plot of the daily log returns of (a) KOSPI200 Index, (b) KOSDAQ index, (c) Samsung stock and (d) Posco stock from January 2004 to December 2006.

In this section. we consider the problem of testing the following hypotheses:

H_0 : ϵ_t's are normally distributed. vs.
H_1 : Not H_0.

In order to perform a test, we adopt the Jarque-Bera (JB) test proposed by

Park et al (2007), which is previously developed in IID case by Jarque and Bera (1980) and Bera and Jarque (1981). To construct the JB test, we employ the quasi maximum likelihood estimators $\hat{\theta}_T = (\hat{\omega}, \hat{\alpha}, \hat{\beta})'$ (cf. Francq and Zakoian (2004)) and obtain the residuals

$$\tilde{\epsilon}_t = \frac{y_t}{\sqrt{\tilde{h}_t}}, \quad t = 1, \cdots, T$$

where $\tilde{h}_t$'s are defined recursively by using $\tilde{h}_t = \hat{\omega} + \hat{\alpha} y_{t-1}^2 + \hat{\beta}\tilde{h}_{t-1}$, and the initial r.v.'s are chosen as y_1^2. Based on these residuals, the JB test statistic $\tilde{S}_T$ is constructed as follows:

$$\tilde{S}_T = T(\tilde{\tau}^2/6 + (\tilde{\kappa} - 3)^2/24),$$

where

$$\tilde{\tau} = \frac{\frac{1}{T}\sum_{t=1}^{T}(\tilde{\epsilon}_t - \bar{\tilde{\epsilon}})^3}{(\frac{1}{T}\sum_{t=1}^{T}(\tilde{\epsilon}_t - \bar{\tilde{\epsilon}})^2)^{3/2}}, \quad \tilde{\kappa} = \frac{\frac{1}{T}\sum_{t=1}^{T}(\tilde{\epsilon}_t - \bar{\tilde{\epsilon}})^4}{(\frac{1}{T}\sum_{t=1}^{T}(\tilde{\epsilon}_t - \bar{\tilde{\epsilon}})^2)^2}, \quad \bar{\tilde{\epsilon}} = \frac{1}{T}\sum_{t=1}^{T}\tilde{\epsilon}_t.$$

Under some conditions and H_0, Park et al. (2007) showed that the limiting distribution of $\tilde{S}_T$ is the χ_2^2, which is identical to that of the test statistic based on true errors. We reject H_0 if $\tilde{S}_T > C_\alpha$, where the critical value C_α is the $(1-\alpha)$-quantile point of χ_2^2.

Here, by using the daily log return data of the domestic financial time series from January 2, 2004 to December 28, 2006 (cf. Figure 1), we have the fitted GARCH(1,1) models in (2.1) as follows:

$$\begin{aligned}
&\text{KOSPI200} &&: h_t = 0.0000046 + 0.0738 y_{t-1}^2 + 0.8971 h_{t-1} \\
&\text{KOSDAQ} &&: h_t = 0.0000288 + 0.2628 y_{t-1}^2 + 0.6153 h_{t-1} \\
&\text{SAMSUNG} &&: h_t = 0.0000188 + 0.1462 y_{t-1}^2 + 0.8173 h_{t-1} \\
&\text{POSCO} &&: h_t = 0.0000073 + 0.0861 y_{t-1}^2 + 0.8945 h_{t-1}
\end{aligned}$$

Since $\alpha + \beta \simeq 1$, we can see that all the indices except KOSDAQ index are modeled as almost IGARCH(1,1) model which is typical in asset return data. And the conditional volatility of KOSDAQ index are more dependent on the previous return.

Figure 2, which is the Quantile-Quantile plot based on the residuals, shows that the true error is heavier-tailed for all cases and distinguishably, more skewed for the KOSDAQ data than the normal distribution.

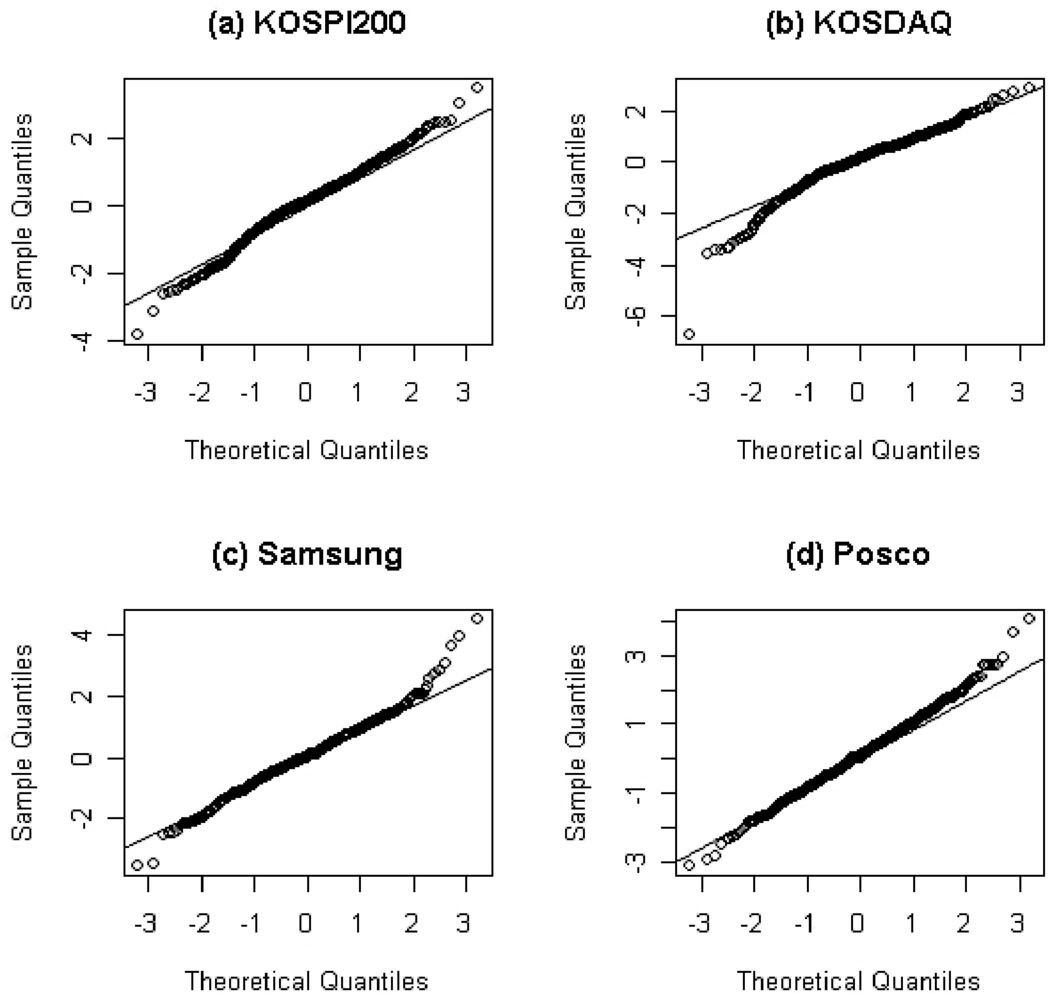

Figure 2. Quantile-Quantile Plot for the residuals of (a) KOSPI200 Index, (b) KOSDAQ index, (c) Samsung stock and (d) Posco stock.

The figures in Tables 1 indicate the skewness, kurtosis and the JB test statistics for the residuals of the fitted GARCH (1,1) models of the daily log returns. According to the JB test statistics, it is obtained that the null hypothesis is rejected at the significant level $\alpha = 0.05$ in all cases, which is mainly due to the heavy-tailedness rather than that skewness. Further, it can be noted that the innovations of log return of KOSDAQ data is most heavy tailed and skewed among others.

	Skewness	Kurtosis	JB test (P-value)
KOSPI200	-0.31	3.65	24.69 (0.0000)
KOSDAQ	-1.05	6.70	561.36 (0.0000)
SAMSUNG	0.14	4.49	71.53 (0.0000)
POSCO	0.13	3.46	8.64 (0.0133)

Table 1. The skewness, kurtosis and the JB test statistics for the daily log returns of the domestic financial data.

So far, it is revealed that the innovations of the domestic financial time series data is heavier-tailed and more skewed than the normal innovations. In fact, it is known that the statistical inference of GARCH models based on the normal innovations is badly affected, when the true innovation is heavier-tailed and more skwed (cf. Hall, P. and Yao, Q. (2003)). Although our results are based on the GARCH(1,1) model, the non-normality of innovations are widely discovered in many applications of GARCH(p,q) models. Therefore, practitioners should be aware of the impact of non-normality of GARCH innovations. Recently, many researchers have developed the new methods to overcome the non-normality of innovations. We leave the practical applications of those methods as a future task.

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