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Estimation in a Half-Triangle Distribution Based on Multiply Type-II Censored Samples

Suk-Bok Kang[1)]

Abstract

For multiply Type-II censored samples from a half-triangle distribution, the maximum likelihood method does not admit explicit solutions. In this case, we propose some explicit estimators of the location parameter in the half-triangle distribution by the approximate maximum likelihood methods. We compare the proposed estimators in the sense of the mean squared error for various censored samples.

Keywords : Approximate Maximum Likelihood Estimator, Half-Triangle Distribution, Multiply Type-II Censored Sample,

1. Introduction

A triangle distribution was applied to a kernel function in nonparametric density estimation. Woo (2007) considered the estimation of the right-tail probability in a half-triangle distribution, and also considered the inference on the reliability, and derived the kth moment of the ratio of two independent half- triangle distributions with different supports. He introduced an example for the half-triangle distribution as follows; when a gas station derives its supply of oil-gas once per θ-day, the sales quantity X of the gas during the term of θ-days, follows a half-triangle distribution. And if X is a half-triangle random variable with support, $(0, \theta)$, $1-X/\theta$ follows a power function distribution over (0,1).

The random variable X has a half-triangle distribution if it has a probability density function (pdf) of the form

$$f(x;\theta)=\frac{2}{\theta^2}(\theta-x),\ 0<x<\theta. \tag{1.1}$$

It has been noted that in most cases, the maximum likelihood method does not

1) Professor, Department of Statistics, Yeungnam University, Gyeongsan, 712-749, Korea
E-mail : sbkang@yu.ac.kr

provide explicit estimators based on complete and censored samples. Especially, when the sample is multiply censored, the maximum likelihood method does not admit explicit solutions. Hence it is desirable to develop approximations to this maximum likelihood method which would provide us with estimators that are explicit functions of order statistics.

For multiply Type-II censoring, Upadhyay et al. (1996) considered some estimations for the exponential distribution under multiply Type-II censoring. Kong and Fei (1996) discussed the limit theorems for the maximum likelihood estimator under general multiply Type-II censoring. Kang (2005) derived the approximate maximum likelihood estimators (AMLEs) of the scale parameter and the location parameter in the extreme value distribution based on multiply Type-II censored samples. Kang and Lee (2005) derived the AMLEs of the scale and location parameters in the two-parameter exponential distribution based on multiply Type-II censored samples. They also obtained the moments of the proposed estimators. Recently, Han and Kang (2006) proposed some explicit estimators of the location and scale parameters in the Rayleigh distribution under multiply Type-II censoring.

In this paper, we derive some explicit estimators of the parameter θ in a half-triangle distribution under multiply Type-II censoring by several approximate maximum likelihood estimation methods. We also compare the proposed estimators in the sense of the mean squared error (MSE) for various censored samples.

2. Some Explicit Estimators

Suppose n items are placed on a testing experiment and that the a_1th,..., a_sth failures-times are only made available, the rest are unobserved or missing, where $1 \leq a_1 < a_2 < \cdots < a_s \leq n$ are considered to be fixed. If this censoring arises, the scheme is known as multiply Type-II censoring scheme.

Let

$$X_{a_1:n} \leq X_{a_2:n} \leq \cdots \leq X_{a_s:n} \tag{2.1}$$

be the multiply Type-II censored sample, where $X_{1:n}, \cdots, X_{n:n}$ are order statistics of $X_1, \cdots, X_n$.

Let $a_0 = 0$, $a_{s+1} = n+1$, $F(x_{a_0:n}) = 0$, $F(x_{a_{s+1}:n}) = 1$, then the likelihood function based on the multiply Type-II censored sample (2.1) is given by

$$\begin{aligned} L(\theta; z) = \frac{1}{\theta^s} & \frac{n!}{\prod_{j=1}^{s+1}(a_j - a_{j-1} - 1)!} [F(z_{a_1:n})]^{a_1 - 1} [1 - F(z_{a_s:n})]^{n - a_s} \\ & \times \prod_{j=1}^{s} f(z_{a_j:n}) \left[\prod_{j=2}^{s} [F(z_{a_j:n}) - F(z_{a_{j-1}:n})]^{a_j - a_{j-1} - 1} \right], \end{aligned} \tag{2.2}$$

where $Z_{i:n} = X_{i:n}/\theta$, $f(z) = 2(1-z)$ and $F(z) = 1-(1-z)^2$ are the pdf and the cdf of the standard half-triangle distribution, respectively.

From the equation (2.2), we obtain the likelihood equations as follows;

$$\frac{\partial \ln L}{\partial \theta} = -\frac{1}{\theta}\left[s + 2(a_1-1)\frac{1-z_{a_1:n}}{1-(1-z_{a_1:n})^2} z_{a_1:n} - 2(n-a_s)\frac{1}{1-z_{a_s:n}} z_{a_s:n} - \sum_{j=1}^{s}\frac{1}{1-z_{a_j:n}} z_{a_j:n} \right.$$
$$\left. + 2\sum_{j=2}^{s}(a_j - a_{j-1}-1)\frac{(1-z_{a_j:n})z_{a_j:n} - (1-z_{a_{j-1}:n})z_{a_{j-1}:n}}{(1-z_{a_{j-1}:n})^2 - (1-z_{a_j:n})^2} \right] \tag{2.3}$$
$$= 0$$

Since these likelihood equation is very complicated, the equation (2.3) does not admit an explicit solution for θ. So we need some approximate likelihood equations which give an explicit solution.

Let

$$\xi_i = F^{-1}(p_i) = 1 - \sqrt{1-p_i} \quad \text{where } p_i = \frac{i}{n+1},\ q_i = 1 - p_i .$$

First, by using Taylor series expansion around the points ξ_{a_1}, ξ_{a_i}, and $(\xi_{a_j}, \xi_{a_{j-1}})$, respectively, we can obtain the following results;

$$\frac{1-z_{a_1:n}}{1-(1-z_{a_1:n})^2} z_{a_1:n} \simeq \alpha_1 + \beta_1 z_{a_1:n} \tag{2.4}$$

$$\frac{1}{1-z_{a_i:n}} z_{a_i:n} \simeq -\frac{\xi_{a_i}^2}{q_{a_i}} + \frac{1}{q_{a_i}} z_{a_i:n} \tag{2.5}$$

$$\frac{(1-z_{a_j:n})z_{a_j:n} - (1-z_{a_{j-1}:n})z_{a_{j-1}:n}}{(1-z_{a_{j-1}:n})^2 - (1-z_{a_j:n})^2} \simeq \alpha_{1j} + \beta_{1j} z_{a_j:n} + \gamma_{1j} z_{a_{j-1}:n} \tag{2.6}$$

where

$$\alpha_1 = \frac{\xi_{a_1}^2}{p_{a_1}}\left(1 + \frac{2q_{a_1}}{p_{a_1}}\right)$$

$$\beta_1 = \frac{1}{p_{a_1}}\left(1 - 2\xi_{a_1} - \frac{2q_{a_1}}{p_{a_1}}\xi_{a_1}\right)$$

$$\alpha_{1j} = \frac{1}{2}K^2 + \frac{\xi_{a_j}^2 - \xi_{a_{j-1}}^2}{p_{a_j} - p_{a_{j-1}}}$$

$$\beta_{1j} = \frac{1}{p_{a_j} - p_{a_{j-1}}}\left[1 - 2\xi_{a_j} - K(1-\xi_{a_j})\right]$$

$$\gamma_{1j} = -\frac{1}{p_{a_j} - p_{a_{j-1}}}\left[1 - 2\xi_{a_{j-1}} - K(1-\xi_{a_{j-1}})\right]$$

$$K = 2\frac{(\xi_{a_j} - \xi_{a_{j-1}})(1 - \xi_{a_j} - \xi_{a_{j-1}})}{p_{a_j} - p_{a_{j-1}}}$$

From the equations (2.4), (2.5), and (2.6), we may approximate the likelihood equation of (2.3) by

$$\frac{\partial \ln L}{\partial \theta} \simeq -\frac{1}{\theta}\Bigg[s+2(a_1-1)\alpha_1+\frac{2\xi_{a_s}^2(n-a_s)}{q_{a_s}}+\sum_{j=1}^{s}\frac{\xi_{a_j}^2}{q_{a_j}}+2\sum_{j=2}^{s}(a_j-a_{j-1}-1)\alpha_{1j}$$
$$+2\beta_1(a_1-1)\,z_{a_1:n}-\frac{2(n-a_s)}{q_{a_s}}z_{a_s:n}-\sum_{j=1}^{s}\frac{1}{q_{a_j}}z_{a_j:n}$$
$$+2\sum_{j=2}^{s}(a_j-a_{j-1}-1)\beta_{1j}z_{a_j:n}+2\sum_{j=2}^{s}(a_j-a_{j-1}-1)\gamma_{1j}\,z_{a_{j-1}:n}\Bigg] \tag{2.7}$$
$$=0.$$

Upon solving the equations (2.7) for θ, the explicit estimator of θ is

$$\hat{\theta}_1 = \frac{1}{A}\Bigg[\frac{2(n-a_s)}{q_{a_s}}X_{a_s:n}-2\beta_1(a_1-1)X_{a_1:n}+\sum_{j=1}^{s}\frac{1}{q_{a_j}}X_{a_j:n}$$
$$-2\sum_{j=2}^{s}(a_j-a_{j-1}-1)\beta_{1j}X_{a_j:n}-2\sum_{j=2}^{s}(a_j-a_{j-1}-1)\gamma_{1j}X_{a_{j-1}:n}\Bigg], \tag{2.8}$$

where

$$A=s+2(a_1-1)\alpha_1+2\xi_{a_s}^2(n-a_s)/q_{a_s}+\sum_{j=1}^{s}\xi_{a_j}^2/q_{a_j}+2\sum_{j=2}^{s}(a_j-a_{j-1}-1)\alpha_{1j}$$

The estimator $\hat{\theta}_1$ is a linear function of available order statistics. So we can easily obtain the moments of $\hat{\theta}_1$ by the moments of the order statistics.

Second, by using Taylor series expansion around the points ξ_{a_1}, ξ_{a_i}, and $(\xi_{a_j}, \xi_{a_{j-1}})$, respectively, we can also obtain the following results;

$$\frac{1-z_{a_1:n}}{1-(1-z_{a_1:n})^2} \simeq \alpha_2+\beta_2 z_{a_1:n} \tag{2.9}$$

$$\frac{1}{1-z_{a_i:n}} \simeq \frac{1}{q_{a_i}}(1-2\xi_{a_i})+\frac{1}{q_{a_i}}z_{a_i:n} \tag{2.10}$$

$$\frac{1-z_{a_j:n}}{(1-z_{a_{j-1:n}})^2-(1-z_{a_j:n})^2} \simeq \alpha_{2j}+\beta_{2j}\,z_{a_j:n}+\gamma_{2j}\,z_{a_{j-1}:n} \tag{2.11}$$

$$\frac{1-z_{a_{j-1}:n}}{(1-z_{a_{j-1:n}})^2-(1-z_{a_j:n})^2} \simeq \alpha_{3j}+\beta_{3j}\,z_{a_j:n}+\gamma_{23}\,z_{a_{j-1}:n} \tag{2.12}$$

where

$$\alpha_2=\frac{1}{p_{a_1}}\left(1+\frac{2q_{a_1}}{p_{a_1}}\xi_{a_1}\right)$$

$$\beta_2=-\frac{1}{p_{a_1}}\left(1+\frac{2q_{a_1}}{p_{a_1}}\right)$$

$$\alpha_{2j}=\frac{1}{p_{a_j}-p_{a_{j-1}}}\left[1+K(1-\xi_{a_j})\right]$$

$$\beta_{2j}=-\frac{1}{p_{a_j}-p_{a_{j-1}}}\left(1+\frac{2q_{a_j}}{p_{a_j}-p_{a_{j-1}}}\right)$$

$$\gamma_{2j}=\frac{2(1-\xi_{a_j})(1-\xi_{a_{j-1}})}{(p_{a_j}-p_{a_{j-1}})^2}=-\beta_{3j}$$

$$\alpha_{3j}=\frac{1}{p_{a_j}-p_{a_{j-1}}}\left[1+K(1-\xi_{a_{j-1}})\right]$$

$$\gamma_{3j}=-\frac{1}{p_{a_j}-p_{a_{j-1}}}\left(1-\frac{2q_{a_j}}{p_{a_j}-p_{a_{j-1}}}\right)$$

From the equations (2.9), (2.10), (2.11), and (2.12), we may approximate the likelihood equation of (2.3) by

$$\begin{aligned}\frac{\partial \ln L}{\partial\theta} \simeq -\frac{1}{\theta}\Bigg[& s+2\alpha_2(a_1-1)z_{a_1:n}-\frac{2(n-a_s)(1-2\xi_{a_s})}{q_{a_s}}z_{a_s:n}+\sum_{j=1}^{s}\frac{2\xi_{a_j}-1}{q_{a_j}}z_{a_j:n}\\ &+2\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\alpha_{2j}z_{a_j:n}-\alpha_{3j}z_{a_{j-1}:n})+2\beta_2(a_1-1)z^2_{a_1:n}\\ &+2\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\beta_{2j}z^2_{a_j:n}+\gamma_{2j}z_{a_j:n}z_{a_{j-1}:n}-\beta_{3j}z_{a_j:n}z_{a_{j-1}:n}-\gamma_{3j}z^2_{a_{j-1}:n})\\ &-\frac{2(n-a_s)}{q_{a_s}}z^2_{a_s:n}-\sum_{j=1}^{s}\frac{1}{q_{a_j}}z^2_{a_j:n}\Bigg].\end{aligned} \tag{2.13}$$

Upon solving the equations (2.13) for θ, the second explicit estimator of θ is

$$\hat{\theta}_2=\frac{-B_1+\sqrt{B_1^2-4sC_1}}{2s} \tag{2.14}$$

where

$$\begin{aligned}B_1 =\ & 2\alpha_2(a_1-1)X_{a_1:n}-\frac{2}{q_{a_s}}(n-a_s)(1-2\xi_{a_s})X_{a_s:n}+\sum_{j=1}^{s}\frac{2\xi_{a_j}-1}{q_{a_j}}X_{a_j:n}\\ &+2\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\alpha_{2j}X_{a_j:n}-\alpha_{3j}X_{a_{j-1}:n})\end{aligned}$$

$$\begin{aligned}C_1 =\ & 2(a_1-1)\beta_2X^2_{a_1:n}-\frac{2}{q_{a_s}}(n-a_s)X^2_{a_s:n}-\sum_{j=1}^{s}\frac{1}{q_{a_j}}X^2_{a_j:n}\\ &+2\sum_{j=2}^{s}(a_j-a_{j-1}-1)\left(\beta_{2j}X^2_{a_j:n}+\gamma_{2j}X_{a_{j-1}:n}X_{a_j:n}-\beta_{3j}X_{a_{j-1}:n}X_{a_j:n}-\gamma_{3j}X_{a_{j-1}:n}\right).\end{aligned}$$

The second estimator $\hat{\theta}_2$ is a quadratic form and nonlinear function of available order statistics. Therefore, It is very difficult to obtain the moments of $\hat{\theta}_2$, but we can easily obtain the estimate of the parameter θ.

Third, we may use the following approximation

$$\frac{1}{1-z_{a_i:n}}\simeq 1+z_{a_i:n} \tag{2.15}$$

From the equation (2.15), we may approximate the likelihood equation of (2.3) by

$$\frac{\partial \ln L}{\partial\theta}\simeq -\frac{1}{\theta}\Bigg[s+2\alpha_2(a_1-1)z_{a_1:n}-2(n-a_s)z_{a_s:n}-\sum_{j=1}^{s}z_{a_j:n}$$

$$+2\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\alpha_{2j}z_{a_j:n}-\alpha_{3j}z_{a_{j-1}:n})+2\beta_2(a_1-1)z_{a_1:n}^2-2(n-a_s)z_{a_s:n}^2 \tag{2.16}$$
$$-\sum_{j=1}^{s}z_{a_j:n}^2+2\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\beta_{2j}z_{a_j:n}^2+\gamma_{2j}z_{a_j:n}z_{a_{j-1}:n}-\beta_{3j}z_{a_j:n}z_{a_{j-1}:n}-\gamma_{3j}z_{a_{j-1}:n}^2\Big].$$

Upon solving the equations (2.16) for θ, the third explicit estimator of θ is

$$\hat{\theta}_3=\frac{-B_2+\sqrt{B_2^2-4s\,C_2}}{2s} \tag{2.17}$$

where

$$B_2=2\alpha_2(a_1-1)X_{a_1:n}-2(n-a_s)X_{a_s:n}-\sum_{j=1}^{s}X_{a_j:n}$$
$$+2\sum_{j=2}^{s}(a_j-a_{j-1}-1)(\alpha_{2j}X_{a_j:n}-\alpha_{3j}X_{a_{j-1}:n})$$
$$C_2=2(a_1-1)\beta_2X_{a_1:n}^2-2(n-a_s)X_{a_s:n}^2-\sum_{j=1}^{s}X_{a_j:n}^2$$
$$+2\sum_{j=2}^{s}(a_j-a_{j-1}-1)\left(\beta_{2j}X_{a_j:n}^2+\gamma_{2j}X_{a_{j-i}:n}X_{a_j:n}-\beta_{3j}X_{a_{j-1}:n}X_{a_j:n}-\gamma_{3j}X_{a_{j-1}:n}\right).$$

The third estimator $\hat{\theta}_3$ is also quadratic form and nonlinear function of available order statistics. Therefore, It is very difficult to obtain the moments of $\hat{\theta}_3$, but we can easily obtain the estimate of the parameter θ.

Woo (2007) proposed two estimators of the parameter θ based on complete sample as follows;

$$\hat{\theta}_4=X_{n:n} \text{ and } \hat{\theta}_5=X_{n:n}/c(n,1) \tag{2.18}$$

where

$$c(n,k)=n\sum_{i=0}^{n-1}(-1)^i2^{n-i}\binom{n-1}{i}\Big/[(n+k+i)(n+k+i+1)]$$

Finally, for multiply Type-II censored sample (2.1), we can modify the estimators (2.18) as follows;

$$\hat{\theta}_4=X_{a_s:n} \text{ and } \hat{\theta}_5=X_{a_s:n}/c(s,1) \tag{2.19}$$

These estimators are very simple and function of the order statistic $X_{a_s:n}$, but the estimator $\hat{\theta}_4$ may be underestimate the parameter θ.

3. Simulated Results

In order to evaluate the performance of the proposed explicit estimators, we need the moments of the proposed estimators. Since it is difficult to find the moments of all proposed estimators, we simulate the MSEs of all proposed estimators through Monte Carlo simulation method. The MSEs usually depend on the parameter θ, thus we will define the relative MSE that dose not depend on the parameter by MSE/θ^2. The simulation procedure is repeated 10,000 times for the sample size $n=20(10)50$, and various choices of censoring ($m=n-s$ is the number of unobserved or missing data) under multiply Type-II censored samples. These values are given in Tables 1.

From Table 1, the estimator $\hat{\theta}_2$ is generally more efficient than the other estimators of the scale parameter θ in the sense of the MSE, and $\hat{\theta}_1$ is generally more efficient than the estimators $\hat{\theta}_3$, $\hat{\theta}_4$ and $\hat{\theta}_5$.

The estimator $\hat{\theta}_1$ is a linear function of available order statistics and the performance is good. But the estimator $\hat{\theta}_5$ is simple estimator but the performance is very bad for large sample size ($n=50$).

As expected, the MSEs of all estimators except $\hat{\theta}_5$ decrease as sample size n increases, and increase as the number of unobserved or missing data increases for fixed sample size.

<Table 1> The relative MSEs for the proposed estimators of the parameter θ

n	m	a_j	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\theta}_5$
20	0	1~20	0.0201	0.0180	0.1636	0.0476	0.0149
	1	1~19	0.0197	0.0190	0.1189	0.0950	0.0290
		2~20	0.0201	0.0174	0.1476	0.0476	0.0151
	2	1~18	0.0225	0.0222	0.0920	0.1419	0.0553
		3~20	0.0201	0.0172	0.1420	0.0476	0.0155
		2~19	0.0197	0.0190	0.1073	0.0950	0.0279
	3	1~17	0.0262	0.0260	0.0752	0.1900	0.0888
		4~20	0.0201	0.0171	0.1380	0.0476	0.0159
		2~18	0.0225	0.0224	0.0830	0.1419	0.0532
		3~19	0.0197	0.0189	0.1036	0.0950	0.0267
	4	2~17	0.0262	0.0265	0.0681	0.1900	0.0860
		4~20	0.0197	0.0189	0.1013	0.0950	0.0256
		3~18	0.0225	0.0224	0.0804	0.1419	0.0510
		2~4, 7~14, 16~20	0.0201	0.0160	0.1032	0.0476	0.0165
	5	3~17	0.0262	0.0266	0.0661	0.1900	0.0829
		4~18	0.0225	0.0223	0.0788	0.1419	0.0487
		2~6, 10~20	0.0197	0.0189	0.0888	0.0950	0.0244
	6	4~17	0.0262	0.0265	0.0650	0.1900	0.0795
		1, 2, 6~9, 12~15, 17~20	0.0201	0.0158	0.0903	0.0476	0.0185
30	0	1~30	0.0125	0.0114	0.1600	0.0326	0.0096
	1	1~29	0.0120	0.0116	0.1256	0.0645	0.0186
		2~30	0.0125	0.0110	0.1474	0.0326	0.0097
	2	1~28	0.0135	0.0133	0.1021	0.0967	0.0360
		3~30	0.0125	0.0109	0.1428	0.0326	0.0098
		2~29	0.0120	0.0115	0.1155	0.0645	0.0181
	3	1~27	0.0151	0.0150	0.0851	0.1291	0.0575
		4~30	0.0125	0.0109	0.1396	0.0326	0.0099
		2~28	0.0135	0.0133	0.0938	0.0967	0.0351
		3~29	0.0120	0.0114	0.1121	0.0645	0.0176

<Table 1> (continued)

n	m	a_j	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\theta}_5$
30	4	2~27	0.0151	0.0150	0.0779	0.1290	0.0563
		4~30	0.0120	0.0114	0.1099	0.0645	0.0171
		3~28	0.0135	0.0133	0.0911	0.0967	0.0342
		1~4, 7~14, 16~30	0.0125	0.0104	0.1193	0.0326	0.0101
	5	3~27	0.0151	0.0150	0.0757	0.1290	0.0550
		4~28	0.0135	0.0133	0.0895	0.0967	0.0333
		1~6, 10~19, 21~30	0.0125	0.0103	0.1137	0.0326	0.0103
	6	4~27	0.0151	0.0151	0.0745	0.1290	0.0537
		1,2, 6~9, 12~15, 17~30	0.0125	0.0103	0.1124	0.0326	0.0105
40	0	1~40	0.0086	0.0079	0.1582	0.0244	0.1756
	1	1~39	0.0082	0.0079	0.1302	0.0488	0.0420
		2~40	0.0086	0.0077	0.1480	0.0244	0.0197
	2	1~38	0.0090	0.0088	0.1096	0.0728	0.0125
		3~40	0.0086	0.0076	0.1442	0.0244	0.0556
		2~39	0.0082	0.0078	0.1216	0.0488	0.0239
	3	1~37	0.0098	0.0097	0.0937	0.0969	0.0385
		4~40	0.0086	0.0076	0.1416	0.0244	0.0074
		2~38	0.0090	0.0088	0.1022	0.0728	0.0234
		3~39	0.0082	0.0078	0.1186	0.0488	0.0117
	4	2~37	0.0098	0.0097	0.0872	0.0969	0.0405
		4~39	0.0082	0.0078	0.1167	0.0488	0.0125
		3~38	0.0090	0.0088	0.0998	0.0728	0.0248
		2~4, 7~14, 16~40	0.0086	0.0072	0.1252	0.0244	0.0071
	5	3~37	0.0098	0.0098	0.0852	0.0967	0.0351
		4~38	0.0090	0.0088	0.0983	0.0728	0.0208
		2~6, 10~19, 21~40	0.0086	0.0072	0.1223	0.0244	0.0082
	6	4~37	0.0098	0.0097	0.0840	0.0969	0.0393
		1,2, 6~9, 12~15, 17~40	0.0086	0.0072	0.1197	0.0244	0.0073
50	0	1~50	0.0066	0.0061	0.1576	0.0195	0.9999
	1	1~49	0.0063	0.0061	0.1339	0.0392	0.9999
		2~50	0.0066	0.0060	0.1489	0.0195	0.9999
	2	1~48	0.0068	0.0067	0.1158	0.0587	0.9999
		3~50	0.0066	0.0059	0.1453	0.0195	0.999
		2~49	0.0063	0.0060	0.1264	0.0392	0.9999

<Table 1> (continued)

n	m	a_j	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\theta}_5$
50	3	1~47	0.0075	0.0074	0.1014	0.0785	1.0003
		4~50	0.0066	0.0059	0.1429	0.0195	1.0003
		2~48	0.0068	0.0067	0.1092	0.0587	1.0003
		3~49	0.0063	0.0060	0.1234	0.0392	1.0003
	4	2~47	0.0075	0.0074	0.0955	0.0785	1.0041
		4~49	0.0063	0.0060	0.1216	0.0392	1.0046
		3~48	0.0068	0.0067	0.1067	0.0587	1.0043
		2~4, 7~14, 16~50	0.0066	0.0056	0.1297	0.0195	1.0049
	5	3~47	0.0075	0.0074	0.0933	0.0785	0.9756
		4~48	0.0068	0.0067	0.1051	0.0587	0.9743
		2~6, 10~19, 21~50	0.0066	0.0056	0.1270	0.0195	0.9707
	6	4~47	0.0075	0.0074	0.0920	0.0785	0.8308
		1, 2, 6~9, 12~15, 17~50	0.0066	0.0056	0.1248	0.0196	0.7983

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