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Reliability P(Y<X) in Two Independent Uniform and Weibull-Gamma

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Abstract

We consider estimation of reliability P(Y<X), when X and Y are two independent Weibull random variable and uniform random variable, respectively, and also consider the estimation problem when X and Y are two different independent Gamma random variable and uniform random variable, respectively.

Keywords : Hyper Geometric Function, Reliability, Weibull Distribution.

1. Introduction

Many authors have considered properties of Weibull and uniform distributions in Johnson, et al(1994). McCool(1991) considered inference on reliability P(X<Y) when X and Y have the same Weibull distributions. Ali & Woo(2005a,b) studied inference on reliability P(Y<X) when X and Y are the same power function distributions and the same Levy distributions. And in recent, Kim(2006), and Lee & Won(2006) and Woo(2007) studied inference on reliability in two independent exponentiated uniform distributions, exponential distributions, and triangle distributions.

For application of this case P(Y<X), X, representing time to sustain temperature, is a uniform random variable and Y, representing time to sustain life of a vacuum tube, is a Weibull random variable, and X, representing time to sustain temperature, is a uniform random variable and Y, representing to time to sustain a velocity of particles in a uniform vapor, is a gamma random variable.

In this paper, we derive the density of quotient X/Y, and consider point and interval estimations of reliability P(Y<X) when X and Y are independent Weibull

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random variable and uniform random variable, respectively, and also derive the density of quotient X/Y, and consider the estimation problem when X and Y are two different independent gamma random variable and uniform random variable, respectively.

2. Weibull and Uniform

In this section, we consider the case for the estimation of P(Y<X) when (X, Y) is a pair of Weibull and uniform random variables, respectively. As an application of this case, X, representing time to sustain temperature, is a uniform random variable and Y, representing time to sustain life of a vacuum tube, is Weibull random variable.

2.1 Quotient and reliability

Let X and Y be independent Weibull and uniform random variables with the densities, respectively:

$$f_X(x)=\frac{\alpha}{\beta^{\alpha}}x^{\alpha-1}e^{-(x/\beta)^{\alpha}}, \quad \text{if} \quad 0<x<\infty$$

$$f_Y(y)=1/\theta, \quad \text{if} \quad 0<y<\theta \text{ , where } \alpha, \beta, \text{ and } \theta \text{ are positive.} \tag{2.1}$$

Let W=X/Y. Then, from the quotient distribution in Rohatgi(1976, p.141) and the formula 3.381(1) in Gradshteyn and Ryzhik(1965. p.317), the density of W=X/Y is given by:

$$f_W(w)=w^{-2}\cdot\gamma(1+\frac{1}{\alpha},\ \rho^{\alpha}\cdot w^{\alpha})\,/\,\rho, \quad \text{if } 0<w<\infty \tag{2.2}$$

where $\gamma(\alpha,x)$ is the incomplete gamma function. and $\rho\equiv\theta/\beta$.

By transformation of variable, $x\equiv\rho^{\alpha}\cdot w^{\alpha}$ in the density (2.2), and the formula 13.42 in Oberhettinger(1974, p.144) and the formula 15.1.20 in Abramowitz and Stegun (1970, p.556), integration of the function in (2.2) over$(0,\infty)$ is 1.

From the density (2.2) and the formula 13.42 in Oberhettinger(1974, p.144), we can obtain the k-th moment of W=X/Y when X and Y are independent Weibull random variable and uniform random variable, respectively.

$$E(W^k)=\Gamma(\frac{k}{\alpha}+1)\cdot {}_2F_1(1,1+\frac{k}{\alpha};2+\frac{1}{\alpha};1)/((1+\alpha)\rho^k), \quad \text{if } \alpha+1>k$$

where ${}_2F_1(a,b;c;x)$ is the generalized hypergeometric function, and $\Gamma(a)$ is the gamma function.

Especially if k=1 and 2, then we derive the expectation and 2nd moment of W:

$$E(W) = \Gamma(\frac{1}{\alpha}+1) \cdot {}_2F_1(1, \frac{1}{\alpha}+1 ; \frac{1}{\alpha}+2 ; 1)/((\alpha+1)\rho)$$

and $$E(W^2) = \Gamma(\frac{2}{\alpha}+1) \cdot {}_2F_1(1, \frac{2}{\alpha}+1 ; \frac{1}{\alpha}+2 ; 1)/((1+\alpha)\rho), \quad \text{if } \alpha > 1 .$$

Remark. The numerical values of F(a,b;c;x) can be evaluated by an integral representation in Abramowitz and Stegun (1970, p.558).

From the density (2.2), and the formulas 3.381(1) in Gradshteyn and Ryzhik(1965, p.317), we obtain the reliability P(Y<X):

Fact 1. When X and Y are independent Weibull random variable and uniform random variable with the densities (2.1) with known α, respectively, then, for $\rho \equiv \theta/\beta$, $R \equiv P(Y < X) = e^{-\rho^{\alpha}} + \frac{1}{\rho} \cdot \gamma(\frac{1}{\alpha}+1, \rho^{\alpha})$ is a function of ρ.

Since $\frac{d}{dx}\gamma(\alpha,x) = x^{\alpha-1} \cdot e^{-x}$, so $\frac{d}{d\rho}P(Y < X) = -\gamma(\frac{1}{\alpha}+1, \rho^{\alpha}) < 0$,
and hence we obtain the following:
Fact 2. If X and Y are independent Weibull random variable and uniform random variable and the shape parameter α in the density (2.1) is known, then reliability R=P(Y<X) is a monotone decreasing function of $\rho \equiv \theta/\beta$.

2.2. Estimating reliability P(Y<X)

From Facr 2, because R=P(Y<X) is a monotone function of ρ when the shape parameter α in the density (2.1) is known, an inference on the reliability is equivalent to an inference on ρ (see McCool(1991)).

So we consider estimation on $\rho \equiv \theta/\beta$ when β and θ are parameters in the densities (2.1), instead of estimating R=P(Y<X), when the shape parameter α in the density (2.1) is known.

Assume $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be two independent samples from X and Y with the densities (2.1), respectively and the shape parameter α in the density (2.1) is known. Then, from the formula 3.381(4) in Gradshteyn and Ryzhik(1965, p.317), we derive the following results:
Fact 3. (a) If $X_1, X_2, \ldots, X_m$ are a sample drawn from a Weibulll distribution with density(2.1), then $Z \equiv \sum_{i=1}^{m} X_i^{\alpha}$ follows a gamma random variable with shape parameter m and scale parameter β^{α}, $E(1/Z^{1/\alpha}) = \Gamma(m-1/\alpha)/(\Gamma(m) \cdot \beta)$, and $E(1/X^{2/\alpha}) = \Gamma(m-2/\alpha)/(\Gamma(m) \cdot \beta^2)$, if $m\alpha > 2$.

(b) If X follows a gamma distribution with mean a and variance ab^2, then $E(1/X) = 1/((a-1)b)$ and $E(1/X^2) = 1/((a-1)(a-2)b^2)$, if $a > 2$.

When the shape parameter α in the density (2.1) is known, the MLE $\hat{\beta}$ of β and the MLE $\hat{\theta}$ of θ are given by:

$$\hat{\beta} = m^{-1/\alpha} \cdot (\sum_{i=1}^{m} X_i^{\alpha})^{1/\alpha} \quad \text{and} \quad \hat{\theta} = Y_{(n)},$$

respectively, where $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$.

Hence the MLE $\hat{\rho}$ of ρ is $\hat{\rho} = \hat{\theta}/\hat{\beta} = m^{1/\alpha} \cdot Y_{(n)} / (\sum_{i=1}^{m} X_i^{\alpha})^{1/\alpha}$.

From the results in Fact 3 and density of the greatest order statistics $Y_{(n)}$:

$$f_{Y_{(n)}}(y) = \frac{n}{\theta^n} y^{n-1}, 0 < y < \theta \quad , \tag{2.3}$$

we obtain the expectation and variance of $\hat{\rho}$:

$$E(\hat{\rho}) = m^{1/\alpha} \cdot \frac{n \cdot \Gamma(m - \frac{1}{\alpha})}{(n+1) \cdot \Gamma(m)} \cdot \rho, \quad \text{if } m\alpha > 1 \tag{2.4}$$

$$Var(\hat{\rho}) = \frac{m^{2/\alpha} \cdot n}{\Gamma(m)} [\frac{\Gamma(m - \frac{2}{\alpha})}{n+2} - \frac{n \cdot \Gamma^2(m - \frac{1}{\alpha})}{(n+1)^2 \Gamma(m)}] \rho^2 \quad \text{if } \alpha m > 2 \ . \tag{2.5}$$

From the expectation (2.4), an unbiased estimator $\tilde{\rho}$ of ρ is defined as:

$$\tilde{\rho} = \frac{(n+1)\Gamma(m)}{n \cdot \Gamma(m - \frac{1}{\alpha})} \cdot \frac{Y_{(n)}}{(\sum_{i=1}^{m} X_i^{\alpha})^{1/\alpha}} \ .$$

Hence, from the results in Fact 3, we also obtain the variance of $\tilde{\rho}$:

$$Var(\tilde{\rho}) = \frac{(n+1)^2 \Gamma(m)}{m \cdot \Gamma^2(m - \frac{1}{\alpha})} [\frac{\Gamma(m - \frac{2}{\alpha})}{n+2} - \frac{n \cdot \Gamma^2(m - \frac{1}{\alpha})}{(n+1)^2 \cdot \Gamma(m)}] \rho^2 \ , \text{ if } \alpha m > 2 \ . \tag{2.6}$$

From the results (2.4), (2.5) and (2,6), Table 1 shows mean squares errors(MSE) of the MLE $\hat{\rho}$ and the unbiased estimator $\tilde{\rho}$ of $\rho \equiv \theta/\beta$:

<Table 1> Mean square errors of MLE $\hat{\rho}$ and unbiased estimator $\tilde{\rho}$ (unit: ρ^2)

m	n	$\alpha=2$		$\alpha=3$		$\alpha=4$		$\alpha=6$	
		$\hat{\rho}$	$\tilde{\rho}$	$\hat{\rho}$	$\tilde{\rho}$	$\hat{\rho}$	$\tilde{\rho}$	$\hat{\rho}$	$\tilde{\rho}$
10	10	0.03582	0.03672	0.02272	0.02062	0.01875	0.01516	0.01625	0.01134
10	20	0.02998	0.03049	0.01441	0.04486	0.09526	0.01564	0.00632	0.00526
10	30	0.02961	0.02922	0.01308	0.01324	0.00784	0.00782	0.00435	0.00402
20	10	0.02401	0.02169	0.01849	0.01419	0.01675	0.01161	0.01564	0.00978
20	20	0.01151	0.01555	0.00887	0.00810	0.00670	0.00553	0.00524	0.00371
20	30	0.01410	0.01429	0.00703	0.00686	0.00469	0.00429	0.00310	0.00248
30	10	0.02076	0.01706	0.01729	0.01218	0.01618	0.01049	0.01546	0.00929
30	20	0.01145	0.01095	0.00725	0.00609	0.00586	0.00442	0.00492	0.00322
30	30	0.00972	0.00970	0.00525	0.00486	0.00375	0.00318	0.00272	0.00199

From Table 1, we observe the followings:

Fact 4. The unbiased estimator $\tilde{\rho}$ performs better than the MLE $\hat{\rho}$ in a sense of MSE, when (i) $\alpha \geqq 4$, m and n=10, 20, 30, (ii) $\alpha = 3$, m =20, 30 and n=10, 20, 30, and (iii) $\alpha = 2$, m=30 and n=10, 20, 30.

Now when the shape parameter α in the density (2.1) is known, we consider an interval estimator of ρ . From the quotient density in Rohatgi(1975, p.141) and the formula 3.381(1) in Gradshteyn and Ryzhik(1965, p.317), $Q \equiv \rho \cdot (\sum_{i=1}^{m} X_i^{\alpha})^{1/\alpha} / Y_{(n)}$ is a pivot quantity having the following density:

$$f_Q(x) = n \cdot x^{-n-1} \cdot \gamma(m + \frac{n}{\alpha}, x^{\alpha}) / \Gamma(m), \quad \text{if } x > 0 . \tag{2.7}$$

From the formula 13.42 in Oberhettinger(1974, p.144) and the formula 15.1.20 in Abramowitz and Stegun (1970, p.556), integration of $f_Q(x)$ in (2.7) over$(0, \infty)$ is unity. For given $0 < p_i < 1,\ i = 1, 2,$ there exist $l(p_1)$ and $u(p_2)$ such that

$$\int_0^{l(p_1)} f_Q(x)dx = p_1 \ \text{ and } \int_{u(p_2)}^{\infty} f_Q(x)dx = p_2 . \tag{2.8}$$

Based on the density (2.7) of the pivotal quantity $Q \equiv \rho \cdot \sum_{i=1}^{m} X_i / Y_{(n)}$, when α is known, a $(1 - p_1 - p_2)100\%$ confidence interval of ρ is:

$$(l(p_1) \cdot Y_{(n)} / (\sum_{i=1}^{m} X_i^{\alpha})^{1/\alpha},\ u(p_2) \cdot Y_{(n)} / (\sum_{i=1}^{m} X_i^{\alpha})^{1/\alpha}) .$$

As applying an asymptotic confidence interval, since the MLE $\hat{\rho}$ is consistent estimator of ρ from the results (2.4) & (2.5), an asymptotic confidence interval of

ρ is given by: For a given $0<\gamma<1$,

$$\left(\hat{\rho}-z_{\gamma/2}\cdot\hat{\rho}\sqrt{\frac{m^{2/\alpha}\cdot n}{\Gamma(m)}\left[\frac{\Gamma(m-\frac{2}{\alpha})}{n+2}-\frac{n\cdot\Gamma^2(m-\frac{1}{\alpha})}{(n+1)^2\Gamma(m)}\right]},\right.$$

$$\left.\hat{\rho}+z_{\gamma/2}\cdot\hat{\rho}\sqrt{\frac{m^{2/\alpha}\cdot n}{\Gamma(m)}\left[\frac{\Gamma(m-\frac{2}{\alpha})}{n+2}-\frac{n\cdot\Gamma^2(m-\frac{1}{\alpha})}{(n+1)^2\Gamma(m)}\right]}\right),$$

is an $(1-\gamma)100\%$ asymptotic confidence interval of ρ,

where $\hat{\rho}=m^{1/\alpha}Y_{(n)}/(\sum_{i=1}^{m}X_i^{\alpha})^{1/\alpha}$ and $\int_u^{\infty}\varphi(t)dt=\gamma/2$, $u\equiv z_{\gamma/2}$, $\varphi(t)$ is the standard normal density.

3. Gamma and uniform

In this section, we consider the case for the estimation of P(Y<X) when (X, Y) is a pair of gamma and uniform random variables, respectively. As an application of this case, X, representing time to sustain temperature, is a uniform random variable and Y, representing to time to sustain a velocity of particles in a uniform vapor, is gamma random variable.

3.1. Quotient and reliability

Let X and Y be independent gamma and uniform random variables with the densities: $f_X(x)=\frac{1}{\Gamma(\alpha)\sigma^{\alpha}}x^{\alpha-1}e^{-x/\sigma}$, if $0<x<\infty$

$f_Y(y)=1/\theta$, if $0<y<\theta$, where $\alpha, \sigma,$ and θ are positive. (3.1)

Let W=X/Y. Then, from the quotient distribution in Rohatgi(1976, p.141) and the formula 3.381(1) in Gradshteyn and Ryzhik(1965, p.317), the density of W=X/Y is given by:

$$f_W(w)=\frac{1}{\eta\cdot\Gamma(\alpha)}w^{-2}\cdot\gamma(\alpha+1,\eta\cdot w),\quad \text{if } 0<w<\infty \tag{3.2}$$

where $\gamma(\alpha,x)$ is the incomplete gamma function. and $\eta\equiv\theta/\sigma$.

By transformation of variable, $x\equiv\eta\cdot w$ in the density (3.2), and the formula 13.42 in Oberhettinger(1974, p.144) and the formula 15.1.20 in Abramowitz and Stegun (1970, p.556), integration of $f_W(w)$ in (3.2) over $(0,\infty)$ is obvious 1:

From the density (3.2) and the formula 13.42 in Oberhettinger(1974, p.144), we can obtain the mgf of W=X/Y when X and Y are independent gamma random variable and uniform random variable, respectively, each having the density (3.1)

$$m_W(t) = \frac{1}{1+\alpha} \cdot (1-\frac{t}{\eta})^{-\alpha} \cdot {}_2F_1(1,\alpha\,;\alpha+2\,;(1-\frac{t}{\eta})^{-1}). \quad \text{if } t<0$$

where ${}_2F_1(a,b\,;c\,;x)$ is the generalized hypergeometric function.

From the density (3.2) and the formulas 3.381(1) in Gradshteyn and Ryzhik(1965, p.141), we obtain the reliability P(Y<X):

Fact 5. When X and Y are independent gamma random variable and uniform random variable, respectively, each having densities (3.1), then, for $\eta \equiv \theta/\sigma$,

$$R \equiv P(Y<X) = 1 + \frac{1}{\Gamma(\alpha)}[\frac{1}{\eta} \cdot \gamma(\alpha+1,\eta) - \gamma(\alpha,\eta)] \ .$$

Since $\frac{d}{dx}\gamma(\alpha,x) = x^{\alpha-1} \cdot e^{-x}$, and hence $\frac{d}{d\eta}[\frac{1}{\eta} \cdot \gamma(\alpha+1,\eta) - \gamma(\alpha,\eta)] < 0$, and hence we can obtain the following:

Fact 6. When X and Y are independent gamma random variable and uniform random variable, respectively, each having densities (3.1), the reliability R=P(Y<X) is a monotone decreasing function of $\eta \equiv \theta/\sigma$, when α is known.

3.2. Estimating reliability P(Y<X)

From Fact 6, because R=P(Y<X) is a monotone function of η when the shape parameter α in the density (3.1) is known, an inference on the reliability is equivalent to an inference on η (see McCool(1991)).

And hence, we only consider estimation on $\eta \equiv \theta/\sigma$ when σ and θ are parameters in the densities (3.1), instead of estimating R=P(Y<X).

Assume $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be two independent samples from X and Y with the densities (3.1), respectively. The MLE $\hat{\sigma}$ of σ and the MLE $\hat{\theta}$ of θ are: $\hat{\sigma} = \sum_{i=1}^{m} X_i/(m\alpha)$, $\hat{\theta} = Y_{(n)}$. if α is known positive.

Therefore, the MLE $\hat{\eta}$ of η is $\hat{\eta} = \hat{\theta}/\hat{\sigma} = m\alpha Y_{(n)}/\sum_{i=1}^{m} X_i$. Based on that $\sum_{i=1}^{m} X_i$ follows a gamma distribution with $\alpha \cdot m$ and σ in Fact 3 and the greatest order statistics $Y_{(n)}$ has the density (2.3), we can obtain the expectation and variance of $\hat{\eta}$:

$$E(\hat{\eta}) = \frac{nm\alpha}{(n+1)(m\alpha-1)} \cdot \eta \tag{3.3}$$

$$Var(\hat{\eta}) = \frac{m^2 n \alpha^2 (m\alpha + n^2 + 2n - 1)}{(m\alpha-1)^2 (m\alpha-2)(n+1)^2 (n+2)} \cdot \eta^2, \quad \text{if } \alpha m > 2. \tag{3.4}$$

From the expectation (3.3), an unbiased estimator $\tilde{\eta}$ of η is defined as:

$$\tilde{\eta} = (n+1)(m\alpha-1)\, Y_{(n)} \,/\, (n\sum_{i=1}^{m} X_i) \ .$$

Hence, from the density (2.3) and Fact 3(b) we also obtain the variance of $\tilde{\eta}$:

$$Var(\tilde{\eta}) = \frac{m\alpha + n^2 + 2n - 1}{n(n+2)(m\alpha - 2)} \cdot \eta^2, \quad \text{if } m\alpha > 2 \ . \qquad (3.5)$$

From the results (3.3), (3.4) and (3.5), Table 2 shows mean squares errors(MSE) of the MLE $\hat{\eta}$ and the unbiased estimator $\tilde{\eta}$:

<Table 2> Mean square errors of MLE $\hat{\eta}$ and unbiased estimator $\tilde{\eta}$ (unit:η^2)

m	n	$\alpha=0.25$		$\alpha=0.5$		$\alpha=1$	
		$\hat{\eta}$	$\tilde{\eta}$	$\hat{\eta}$	$\tilde{\eta}$	$\hat{\eta}$	$\tilde{\eta}$
10	10	4.91414	2.025	0.46338	0.3444	0.13721	0.13435
10	20	5.40115	2.00682	0,51299	0.33636	0.14622	0.12756
10	30	5.58669	2.00313	0.53377	0.33472	0.15155	0.12617
20	10	0.46337	0.34444	0.13721	0.13438	0.06078	0.06435
20	20	0.51299	0.33636	0.14822	0.12756	0.06825	0.05792
20	30	0.53377	0.33472	0.15155	0.12617	0.05914	0.05665
30	10	0.21329	0.19167	0.08217	0.08589	0.04277	0.04435
30	20	0.23259	0.18450	0.08306	0.07937	0.03717	0.03807
30	30	0,24184	0.18305	0.08527	0.07805	0.03688	0.03679

(to continue)

m	n	$\alpha=2$		$\alpha=4$	
		$\hat{\eta}$	$\tilde{\eta}$	$\hat{\eta}$	$\tilde{\eta}$
10	10	0.06078	0.06435	0.03488	0.03487
10	20	0.05825	0.05796	0.02787	0.02865
10	30	0.05914	0.05666	0.02703	0.02738
20	10	0.03488	0.03486	0.02432	0.02126
20	20	0.02787	0.02865	0.01533	0.01512
20	30	0.02703	0.02738	0.01373	0.01388
30	10	0.02768	0.02572	0.02112	0.01688
30	20	0.01933	0.01955	0.01150	0.01077
30	30	0.01798	0.01830	0.00965	0.00953

From Table 2, we observe the followings:

Fact 7. (a) The unbiased estimator $\tilde{\eta}$ performs better than the MLE $\hat{\eta}$ in a sense of MSE, when (i) $\alpha=0.25$, m and n=10, 20, 30, (ii) $\alpha=4$, m=30 and n=10, 20, 30, (iii) $\alpha=0.5$, m=10, 20 and n=10, 20, 30. and (iv) $\alpha=1$, m=10 and n=10, 20, 30. (b) In other cases, the unbiased estimator $\tilde{\eta}$ and the MLE $\hat{\eta}$ don't dominate each other.

Now when the shape parameter α in the density (3.1) is known, we consider an interval estimator of η , from the formula 3.381(1) in Gradshteyn and Ryzhik(1965, p.141), $Q \equiv \eta \cdot \sum_{i=1}^{m} X_i / Y_{(n)}$ is a pivot quantity having the following density:

$$f_Q(x) = \frac{n}{\Gamma(m\alpha)} \cdot x^{-n-1} \cdot \gamma(m\alpha + n, x), \quad \text{if} \quad x > 0 \ . \tag{3.6}$$

From the formula 13.42 in Oberhettinger(1974, p.144) and the formula 15.1.20 in Abramowitz and Stegun (1970, p.558), integration of $f_Q(x)$ in (3.6) over $(0, \infty)$ is one. For given $0 < p_i < 1$, $i = 1, 2$, there exist $l(p_1)$ and $u(p_2)$ such that

$$\int_0^{l(p_1)} f_Q(x)dx = p_1, \quad \int_{u(p_2)}^{\infty} f_Q(x)dx = p_2 \ . \tag{3.7}$$

Based on the density(2.7) of the pivotal quantity $Q \equiv \eta \sum_{i=1}^{m} X_i / Y_{(n)}$,
a $(1 - p_1 - p_2)100\%$ confidence interval of η is given as:

$$\left(l(p_1) \cdot Y_{(n)} / \sum_{i=1}^{m} X_i, \quad u(p_2) \cdot Y_{(n)} / \sum_{i=1}^{m} X_i \right) \ .$$

As applying an asymptotic confidence interval, since the MLE $\hat{\eta}$ is consistent estimator of η from the results (3.3) & (3.4), an asymptotic confidence interval of η is given by: For a given $0 < \gamma < 1$,

$$\left(\hat{\eta} - z_{\gamma/2} \cdot \hat{\eta} \cdot \sqrt{\frac{m^2 n \alpha^2 (m\alpha + n^2 + 2n - 1)}{(m\alpha - 1)^2 (m\alpha - 2)(n+1)^2 (n+2)}}, \right.$$
$$\left. \widehat{\eta} + z_{\gamma/2} \cdot \hat{\eta} \cdot \sqrt{\frac{m^2 n \alpha^2 (m\alpha + n^2 + 2n - 1)}{(m\alpha - 1)^2 (m\alpha - 2)(n+1)^2 (n+2)}} \right)$$

is an $(1 - \gamma)100\%$ asymptotic confidence interval of η, where $\hat{\eta} = m\alpha Y_{(n)} / \sum_{i=1}^{m} X_i$ and $\int_u^{\infty} \varphi(t)dt = \gamma/2$, $u \equiv z_{\gamma/2}$, $\varphi(t)$ is the standard normal density.

References

1. Abramowitz, M. and Stegun, I. A. (1970), *Handbook of Mathematical Functions*, Dover Publications Inc., New York.
2. Ali, M.M. and Woo, J. (2005a), Inferences on reliability P(Y<X) in a power function distribution, *J. of Statistics & Management Systems*, 8-3, 681-686.
3. Ali, M.M. and Woo, J. (2005b), Inference on Reliability P(Y<X) in the

Levy Distribution, *Math. and Computer Modelling* 41, 965-971.

4. Gradshteyn, I. S. and Ryzhik, I. M.(1965), *Tables of Integrals, Series, and Products*, Academic Press, New York.
5. Johnson, N.L., Kotz, S. and Balakrishnan, N. (1995), *Continue Univariate Distribution*, Houghton Mifflin Com., Boston.
6. Kim, J.(2006), Truncated Point and Reliability in a Right Truncated Ray- leigh Distribution, *J. of the Korean Data & Information Science Society* 17-4, 1343-1348.
7. Lee, C. and Won, H. (2006), Inference on Reliability in Exponentiated Uniform Distribution, *J. of the Korean Data & Information Science Society* 17-2, 507- 514.
8. McCool, J. I,(1991), Inference on P(X<Y) in the Weibull case, *Commun. Statist.-Simula.*, 20(1), 129-148.
9. Oberhettinger, F.(1974), *Tables of Mellin Transform*, Springer-Verlag New York.
10. Rohatgi, V. K.(1976), *An Introduction To Probability Theory and Mathematical Statistics*, John Wiley & Sons, New York.
11. Woo. J. (2007), Reliability in a Half-Triangle Distribution and a Skew-Symmetric Distribution, *J. of the Korean Data & Information Science Society* 18-2, 543-552.]

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