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R-GENERALIZED FUZZY COMPACTNESS

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ABSTRACT. In this paper, we introduce the concepts of r-generalized fuzzy closed sets, r-generalized fuzzy continuous maps and several types of r-generalized compactness in fuzzy topological spaces and investigate some of their properties.

1. INTRODUCTION

R. Badard [1] introduced the concept of the fuzzy topological space which is an extension of Chang's fuzzy topological space [3]. Many mathematical structures in fuzzy topological spaces were introduced and studied. In particular, M. Demirci [5] and M. K. El Gayyar, E. E. Kerre and A. A. Ramadan [6] studied several types of compactness in fuzzy topological spaces. K. C. Chattopadhyay and S. K. Samanta [4] and S. J. Lee and E. P. Lee [7] introduced the concepts of fuzzy r-closure and fuzzy r-interior in fuzzy topological spaces and obtained some of their properties. J. Balasuramanian and P. Sundaram [2] introduced the concept of generalized fuzzy closed sets in a Chang's fuzzy topology which is an extension of generalized closed sets of N. Levine [8] in topological spaces.

In this paper, we introduce the concepts of r-generalized fuzzy closed sets, r-generalized fuzzy continuous maps and several types of r-generalized compactness in fuzzy topological spaces and investigate some of their properties.

2. PRELIMINARIES

Throughout this paper, let X be a nonempty set, $I = [0, 1]$ and $I_0 = (0, 1]$. The family of all fuzzy sets of X will be denoted by I^X. By $\tilde{0}$ and $\tilde{1}$ we denote the

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characteristic functions of ϕ and X, respectively. For any $\mu \in I^X$, μ^c denotes the complement of μ, i.e., $\mu^c = \tilde{1} - \mu$.

A *fuzzy topology* [1,9], which is also called a *smooth topology,* on X is a map $\tau : I^X \to I$ satisfying the following conditions:

(O1) $\tau(\tilde{0}) = \tau(\tilde{1}) = 1$;

(O2) $\forall \mu_1, \mu_2 \in I^X$, $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$;

(O3) for every subfamily $\{\mu_i : i \in \Gamma\} \subseteq I^X$, $\tau(\cup_{i\in\Gamma} \mu_i) \geq \wedge_{i\in\Gamma} \tau(\mu_i)$.

The pair (X, τ) is called a *fuzzy topological space* (for short, fts), which is also called a *smooth topological space.*

Definition 2.1 ([4,7]). Let (X, τ) be a fts. For $\mu \in I^X$ and $r \in I_0$, the *fuzzy r-closure* of μ is defined by

$$cl(\mu, r) = \wedge\{\rho \in I^X \mid \mu \leq \rho,\ \tau(\rho^c) \geq r\}$$

and the *fuzzy r-interior* of μ is defined by

$$int(\mu, r) = \vee\{\rho \in I^X \mid \mu \geq \rho,\ \tau(\rho) \geq r\}.$$

For $r \in I_0$, we call μ a *fuzzy r-open set* of X if $\tau(\mu) \geq r$ and μ a *fuzzy r-closed set* of X if $\tau(\mu^c) \geq r$.

Theorem 2.2 ([4]). *Let (X, τ) be a fts. Then for $\mu, \lambda \in I^X$ and $r, s \in I_0$,*

(1) $cl(\tilde{0}, r) = \tilde{0}$,

(2) $\mu \leq cl(\mu, r)$,

(3) $cl(\mu, r) \leq cl(\mu, s)$ *if* $r \leq s$,

(4) $cl(\mu \vee \lambda, r) = cl(\mu, r) \vee cl(\lambda, r)$,

(5) $cl(cl(\mu, r), r) = cl(\mu, r)$.

Theorem 2.3 ([7]). *Let (X, τ) be a fts. Then for $\mu, \lambda \in I^X$ and $r, s \in I_0$,*

(1) $int(\tilde{1}, r) = \tilde{1}$,

(2) $int(\mu, r) \leq \mu$,

(3) $int(\mu, r) \geq int(\mu, s)$ *if* $r \leq s$,

(4) $int(\mu \wedge \lambda, r) = int(\mu, r) \wedge int(\lambda, r)$,

(5) $int(int(\mu, r), r) = int(\mu, r)$.

Theorem 2.4 ([7]). *Let (X, τ) be a fts. Then for $\mu \in I^X$ and $r \in I_0$,*

(1) $int(\mu, r)^c = cl(\mu^c, r)$,

(2) $cl(\mu, r)^c = int(\mu^c, r)$.

Definition 2.5 ([7]). Let (X,τ) and (Y,σ) be fts's and $r \in I_0$. Then a map $f : (X,\tau) \to (Y,\sigma)$ is called

(1) a *fuzzy r-continuous map* if $f^{-1}(\mu)$ is a fuzzy r-open set of X for each fuzzy r-open set μ of Y, or equivalently, $f^{-1}(\mu)$ is a fuzzy r-closed set of X for each fuzzy r-closed set μ of Y.

(2) a *fuzzy r-open map* if $f(\mu)$ is a fuzzy r-open set of Y for each fuzzy r-open set μ of X.

(3) a *fuzzy r-closed map* if $f(\mu)$ is a fuzzy r-closed set of Y for each fuzzy r-closed set μ of X.

3. R-Generalized Fuzzy Closed Sets

Definition 3.1. Let (X,τ) be a fts, $\mu,\rho \in I^X$ and $r \in I_0$.

(1) A fuzzy set μ is called *r-generalized fuzzy closed* (for short, r-gfc) if $cl(\mu,r) \le \rho$ whenever $\mu \le \rho$ and $\tau(\rho) \ge r$.

(2) A fuzzy set μ is called *r-generalized fuzzy open* (for short, r-gfo) if μ^c is r-gfc.

Theorem 3.2. *Let (X,τ) be a fts and $r \in I_0$.*

(1) *If μ_1 and μ_2 are r-gfc sets, then $\mu_1 \vee \mu_2$ is a r-gfc set.*

(2) *If μ is a r-gfc set and $\mu \le \lambda \le cl(\mu,r)$, then λ is a r-gfc set.*

(3) *If μ is a fuzzy r-closed set, then μ is a r-gfc set.*

(4) *μ is a r-gfo set if and only if $\rho \le int(\mu,r)$ whenever $\rho \le \mu$ and $\tau(\rho^c) \ge r$.*

(5) *If μ_1 and μ_2 are r-gfo sets, then $\mu_1 \wedge \mu_2$ is a r-gfo set.*

(6) *If μ is a r-gfo set and $int(\mu,r) \le \lambda \le \mu$, then λ is a r-gfo set.*

(7) *If μ is a fuzzy r-open set, then μ is a r-gfo set.*

Proof. (1) Let μ_1 and μ_2 be r-gfc sets, $\mu_1 \vee \mu_2 \le \rho$ and $\tau(\rho) \ge r$. Then $\mu_1 \le \rho$ and $\mu_2 \le \rho$. Since μ_1 and μ_2 are r-gfc sets, $cl(\mu_1,r) \le \rho$ and $cl(\mu_2,r) \le \rho$. By Theorem 2.2, $cl(\mu_1 \vee \mu_2, r) = cl(\mu_1,r) \vee cl(\mu_2,r) \le \rho$. Thus $\mu_1 \vee \mu_2$ is a r-gfc set.

(2) Let $\lambda \le \rho$ and $\tau(\rho) \ge r$. Since $\mu \le \lambda$, $\mu \le \rho$. Since μ is a r-gfc set, $cl(\mu,r) \le \rho$. Since $\lambda \le cl(\mu,r)$, $cl(\lambda,r) \le cl(cl(\mu,r),r) = cl(\mu,r) \le \rho$. Hence λ is a r-gfc set.

(3) It follows directly from Definition 3.1.

(4) It follows easily from Definition 3.1 and Theorem 2.4.

(5) The proof is similar to (1).

(6) Since $int(\mu, r) \leq \lambda \leq \mu$, $\mu^c \leq \lambda^c \leq int(\mu, r)^c = cl(\mu^c, r)$. Since μ is a r-gfo set, μ^c is a r-gfc set. By (2), λ^c is a r-gfc set. Hence λ is a r-gfo set.

(7) It follows directly from Definition 3.1. □

Example 3.3. The intersection of two r-gfc sets need not be a r-gfc set and the union of two r-gfo sets need not be a r-gfo set.

Let $X = \{x_1, x_2, x_3\}$ and μ_1, μ_2 and μ_3 be fuzzy sets of X defined as

$$\mu_1(x) = \begin{cases} 0 & \text{if } x = x_2, x_3, \\ 1 & \text{if } x = x_1, \end{cases}$$

$$\mu_2(x) = \begin{cases} 0 & \text{if } x = x_3, \\ 1 & \text{if } x = x_1, x_2, \end{cases}$$

$$\mu_3(x) = \begin{cases} 0 & \text{if } x = x_2 \\ 1 & \text{if } x = x_1, x_3. \end{cases}$$

Define $\tau : I^X \to I$ by

$$\tau(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{3}{4} & \text{if } \mu = \mu_1, \\ \frac{1}{2} & \text{if } \mu = \mu_2, \\ \frac{1}{4} & \text{if } \mu = \mu_3, \\ 0 & \text{otherwise.} \end{cases}$$

Then τ is a fuzzy topology on X. It is easy to show that μ_2 and μ_3 are $\frac{3}{4}$-gfc sets. $cl(\mu_2 \wedge \mu_3, \frac{3}{4}) = cl(\mu_1, \frac{3}{4}) = \tilde{1}$, $\mu_2 \wedge \mu_3 = \mu_1$ and $\tau(\mu_1) = \frac{3}{4}$ but $cl(\mu_1 \wedge \mu_2, \frac{3}{4}) = \tilde{1} \not\leq \mu_1$. Hence $\mu_2 \wedge \mu_3$ is not a $\frac{3}{4}$-gfc set.

By taking complement in the above example, we know that the union of two r-gfo sets need not be a r-gfo set.

Example 3.4. Every r-gfc set need not be a fuzzy r-closed set and every r-gfo set need not be a fuzzy r-open set.

Let $X = \{x_1, x_2, x_3\}$ and μ_1 and μ_2 be fuzzy sets of X defined as

$$\mu_1(x) = \begin{cases} 0 & \text{if } x = x_2, x_3, \\ 1 & \text{if } x = x_1, \end{cases}$$

$$\mu_2(x) = \begin{cases} 0 & \text{if } x = x_3, \\ 1 & \text{if } x = x_1, x_2. \end{cases}$$

Define $\tau : I^X \to I$ by

$$\tau(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{2}{3} & \text{if } \mu = \mu_1, \\ \frac{1}{3} & \text{if } \mu = \mu_2, \\ 0 & \text{otherwise.} \end{cases}$$

Then τ is a fuzzy topology on X. It is easy to show that μ_2 is a $\frac{2}{3}$-gfc set. Since $\tau(\mu_2^c) = 0 \not\geq \frac{2}{3}$, μ_2 is not a fuzzy $\frac{2}{3}$-closed set. Since μ_2 is a $\frac{2}{3}$-gfc set, μ_2^c is a $\frac{2}{3}$-gfo set. Since $\tau(\mu_2^c) = 0 \not\geq \frac{2}{3}$, μ_2^c is not a fuzzy $\frac{2}{3}$-open set.

Definition 3.5. Let (X, τ) be a fts. For $\mu \in I^X$ and $r \in I_0$, the *r-generalized fuzzy closure of* μ is defined by

$$gcl(\mu, r) = \wedge\{\rho \in I^X \mid \mu \leq \rho,\ \rho \text{ is r-gfc}\}.$$

and the *r-generalized fuzzy interior of* μ is defined by

$$gint(\mu, r) = \vee\{\rho \in I^X \mid \mu \geq \rho,\ \rho \text{ is r-gfo}\}.$$

Theorem 3.6. *Let (X, τ) be a fts. Then for $\mu, \lambda \in I^X$ and $r, s \in I_0$,*

(1) $gcl(\tilde{0}, r) = \tilde{0}$,

(2) $\mu \leq gcl(\mu, r)$,

(3) $gcl(\mu, r) \leq gcl(\mu, s)$ *if* $r \leq s$,

(4) $gcl(\mu, r) \leq gcl(\lambda, r)$ *if* $\mu \leq \lambda$,

(5) $gcl(\mu \vee \lambda, r) = gcl(\mu, r) \vee gcl(\lambda, r)$,

(6) $gcl(gcl(\mu, r), r) = gcl(\mu, r)$,

(7) $gcl(\mu, r) \leq cl(\mu, r)$.

Proof. (1), (2),(3) and (4) are easily obtained from Definition 3.5.

(5) Since $\mu \leq \mu \vee \lambda$ and $\lambda \leq \mu \vee \lambda$, $gcl(\mu, r) \leq gcl(\mu \vee \lambda, r)$ and $gcl(\lambda, r) \leq gcl(\mu \vee \lambda, r)$ by (4). Hence $gcl(\mu, r) \vee gcl(\lambda, r) \leq gcl(\mu \vee \lambda, r)$.

Conversely, suppose that $gcl(\mu, r) \vee gcl(\lambda, r) \not\geq gcl(\mu \vee \lambda, r)$. Then there exist $x \in X$ and $t \in (0, 1)$ such that $gcl(\mu, r)(x) \vee gcl(\lambda, r)(x) < t < gcl(\mu \vee \lambda, r)(x)$. Since $gcl(\mu, r)(x) < t$ and $gcl(\lambda, r)(x) < t$, there exist r-gfc sets μ_1 and λ_1 with $\mu \leq \mu_1$ and $\lambda \leq \lambda_1$ such that $\mu_1(x) < t$ and $\lambda_1(x) < t$. Since $\mu \vee \lambda \leq \mu_1 \vee \lambda_1$ and $\mu_1 \vee \lambda_1$ is a r-gfc set by Theorem 3.2, $gcl(\mu \vee \lambda, r)(x) \leq (\mu_1 \vee \lambda_1)(x) < t$. This is a contradiction. Hence $gcl(\mu, r) \vee gcl(\lambda, r) \geq gcl(\mu \vee \lambda, r)$. Thus $gcl(\mu, r) \vee gcl(\lambda, r) = gcl(\mu \vee \lambda, r)$.

(6) $gcl(\mu, r) \leq gcl(gcl(\mu, r), r)$ by (2) and (4).

Suppose that $gcl(\mu, r) \not\geq gcl(gcl(\mu, r), r)$. Then there exist $x \in X$ and $t \in (0, 1)$ such that $gcl(\mu, r)(x) < t < gcl(gcl(\mu, r), r)(x)$. Since $gcl(\mu, r)(x) < t$, there exists a r-gfc set μ_1 with $\mu \leq \mu_1$ such that $gcl(\mu, r)(x) \leq \mu_1(x) < t$. Since $\mu \leq \mu_1$, $gcl(\mu, r) \leq \mu_1$ and $gcl(gcl(\mu, r), r) \leq \mu_1$ by (4). Hence

$$gcl(gcl(\mu, r), r)(x) \leq \mu_1(x) < t.$$

This is a contraction. Hence $gcl(\mu, r) \geq gcl(gcl(\mu, r), r)$. Thus $gcl(gcl(\mu, r), r) = gcl(\mu, r)$.

(7) Since $cl(\mu, r)$ is a fuzzy r-closed set, $cl(\mu, r)$ is a r-gfc set by Theorem 3.2. Hence $gcl(\mu, r) \leq cl(\mu, r)$ by Definition 3.5. □

Theorem 3.7. *Let (X, τ) be a fts. Then for $\mu, \lambda \in I^X$ and $r, s \in I_0$,*

(1) $gint(\tilde{1}, r) = \tilde{1}$,
(2) $gint(\mu, r) \leq \mu$,
(3) $gint(\mu, r) \geq gint(\mu, s)$ *if* $r \leq s$,
(4) $gint(\mu, r) \leq gint(\lambda, r)$ *if* $\mu \leq \lambda$,
(5) $gint(\mu \wedge \lambda, r) = gint(\mu, r) \wedge gint(\lambda, r)$,
(6) $gint(gint(\mu, r), r) = gint(\mu, r)$,
(7) $int(\mu, r) \leq gint(\mu, r)$.

Proof. The proof is similar to Theorem 3.6. □

Theorem 3.8. *Let (X, τ) be a fts. Then for $\mu \in I^X$ and $r \in I_0$,*

(1) $gcl(\mu, r)^c = gint(\mu^c, r)$,
(2) $gint(\mu, r)^c = gcl(\mu^c, r)$.

Proof. (1) From Definition 3.5, we have

$$\begin{aligned} gcl(\mu, r)^c &= (\wedge\{\rho \in I^X \mid \mu \leq \rho,\ \rho \text{ is r-gfc}\})^c \\ &= \vee\{\rho^c \in I^X \mid \mu^c \geq \rho^c,\ \rho^c \text{ is r-gfo}\} \\ &= \vee\{\lambda \in I^X \mid \mu^c \geq \lambda,\ \lambda \text{ is r-gfo}\} \\ &= gint(\mu^c, r). \end{aligned}$$

(2) The proof is similar to (1). □

Theorem 3.9. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$. If $f : (X, \tau) \to (Y, \sigma)$ is a fuzzy r-continuous and fuzzy r-closed map, then $f(\mu)$ is a r-gfc set of Y for each r-gfc set μ of X.*

Proof. Let $f(\mu) \leq \rho$ and $\sigma(\rho) \geq r$. Then $\mu \leq f^{-1}(\rho)$. Since f is a fuzzy r-continuous map, $\tau(f^{-1}(\rho)) \geq r$. Since μ is a r-gfc set, $cl(\mu, r) \leq f^{-1}(\rho)$, i.e., $f(cl(\mu, r)) \leq \rho$. Since f is a fuzzy r-closed map and $cl(\mu, r)$ is a fuzzy r-closed set, $f(cl(\mu, r))$ is a fuzzy r-closed set. Hence $cl(f(\mu), r) \leq cl(f(cl(\mu, r)), r) = f(cl(\mu, r)) \leq \rho$. Thus $f(\mu)$ is a r-gfc set of Y. □

Theorem 3.10. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a bijective, fuzzy r-continuous and fuzzy r-open map, then $f(\mu)$ is a r-gfo set of Y for each r-gfo set μ of X.*

Proof. Let $\rho \leq f(\mu)$ and $\sigma(\rho^c) \geq r$. Since f is injective, $f^{-1}(\rho) \leq f^{-1}(f(\mu)) = \mu$. Since f is a fuzzy r-continuous map and ρ is a fuzzy r-closed set, $f^{-1}(\rho)$ is a fuzzy r-closed set, i.e., $\tau((f^{-1}(\rho))^c) \geq r$. Since μ is a r-gfo set of X, $f^{-1}(\rho) \leq int(\mu, r)$. Since f is a fuzzy r-open map and $int(\mu, r)$ is a fuzzy r-open set, $f(int(\mu, r))$ is a fuzzy r-open set and so $f(int(\mu, r)) = int(f(int(\mu, r)), r)$. Since f is surjective, $\rho = f(f^{-1}(\rho)) \leq f(int(\mu, r)) = int(f(int(\mu, r)), r) \leq int(f(\mu), r)$. Hence $f(\mu)$ is a r-gfo set of Y. □

4. R-Generalized Fuzzy Continuous Maps

Definition 4.1. Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map.

(1) f is called *r-generalized fuzzy continuous* (for short, r-gf-continuous) if $f^{-1}(\mu)$ is a r-gfc set of X for each fuzzy r-closed set μ of Y.
(2) f is called *strongly r-generalized fuzzy continuous* (for short, strongly r-gf-continuous) if $f^{-1}(\mu)$ is a fuzzy r-closed set of X for each r-gfc set μ of Y.
(3) f is called *r-generalized fuzzy irresolute* (for short, r-gf-irresolute) if $f^{-1}(\mu)$ is a r-gfc set of X for each r-gfc set μ of Y.
(4) f is called *r-generalized fuzzy open* (for short, r-gf-open) if $f(\mu)$ is a r-gfo set of Y for each fuzzy r-open set μ of X.
(5) f is called *strongly r-generalized fuzzy open* (for short, strongly r-gf-open) if $f(\mu)$ is a r-gfo set of Y for each r-gfo set μ of X.

Remark 4.2. Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map.

(1) If f is fuzzy r-continuous, then f is r-gf-continuous.
(2) If f is r-gf-irresolute, then f is r-gf-continuous.
(3) If f is strongly r-gf-continuous, then f is r-gf-irresolute.
(4) If f is fuzzy r-open, then f is r-gf-open.
(5) If f is strongly r-gf-open, then f is r-gf-open.
(6) If f is bijective, fuzzy r-continuous and fuzzy r-open, then f is strongly r-gf-open.

Example 4.3. The converse of Remark 4.2(1) is not true, i.e., every r-gf-continuous map need not be a fuzzy r-continuous map.

Let $X = \{x_1, x_2\}$ and μ_1 and μ_2 be fuzzy sets of X defined as

$$\mu_1(x) = \begin{cases} \frac{1}{4} & \text{if } x = x_1, \\ \frac{1}{2} & \text{if } x = x_2, \end{cases}$$

$$\mu_2(x) = \begin{cases} \frac{1}{3} & \text{if } x = x_1, \\ \frac{2}{3} & \text{if } x = x_2. \end{cases}$$

Define $\tau : I^X \to I$ and $\sigma : I^X \to I$ by

$$\tau(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0} \text{ or } \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_2, \\ 0 & \text{otherwise.} \end{cases}$$

$$\sigma(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0} \text{ or } \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_1, \\ 0 & \text{otherwise,} \end{cases}$$

then τ and σ are fuzzy topologies on X. It is easy to show that an identity map $id_X : (X, \tau) \to (X, \sigma)$ is $\frac{1}{2}$-gf-continuous. But $id_X : (X, \tau) \to (X, \sigma)$ is not fuzzy $\frac{1}{2}$-continuous because μ_1 is a fuzzy $\frac{1}{2}$-open set in (X, σ) but $id_X^{-1}(\mu_1) = \mu_1$ is not a fuzzy $\frac{1}{2}$-open set in (X, τ).

Example 4.4. The converse of Remark 4.2(2) is not true, i.e., every r-gf-continuous map need not be a r-gf-irresolute map.

Let $X = \{x_1, x_2\}$ and μ_1, μ_2 and μ_3 be fuzzy sets of X defined as

$$\mu_1(x) = \begin{cases} \frac{1}{5} & \text{if } x = x_1, \\ \frac{2}{5} & \text{if } x = x_2, \end{cases}$$

$$\mu_2(x) = \begin{cases} \frac{1}{3} & \text{if } x = x_1, \\ \frac{2}{3} & \text{if } x = x_2, \end{cases}$$

$$\mu_3(x) = \begin{cases} \frac{1}{4} & \text{if } x = x_1, \\ \frac{2}{3} & \text{if } x = x_2. \end{cases}$$

Define $\tau : I^X \to I$ and $\sigma : I^X \to I$ by

$$\tau(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0} \text{ or } \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_2, \\ 0 & \text{otherwise}, \end{cases}$$

$$\sigma(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0} \text{ or } \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_1, \\ 0 & \text{otherwise}, \end{cases}$$

then τ and σ are fuzzy topologies on X. It is easy to show that an identity map $id_X : (X, \tau) \to (X, \sigma)$ is $\frac{1}{2}$-gf-continuous. Clearly, μ_3 is a $\frac{1}{2}$-gfc set in (X, σ). Since $\mu_3 \leq \mu_2$ and $\tau(\mu_2) = \frac{1}{2}$ but $cl(\mu_3, \frac{1}{2}) = \tilde{1} \not\leq \mu_2$, $id_X^{-1}(\mu_3) = \mu_3$ is not a $\frac{1}{2}$-gfc set in (X, τ). Hence $id_X : (X, \tau) \to (X, \sigma)$ is not $\frac{1}{2}$-gf-irresolute.

Theorem 4.5. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a map. Then the following are equivalent:*

(1) *f is r-gf-continuous.*
(2) *$f^{-1}(\mu)$ is a r-gfo set of X for each fuzzy r-open set μ of Y.*

Proof. It is straightforward. □

Theorem 4.6. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a map. Then the following are equivalent:*

(1) *f is strongly r-gf-continuous.*
(2) *$f^{-1}(\mu)$ is a fuzzy r-open set of X for each r-gfo set μ of Y.*

Proof. It is straightforward. □

Theorem 4.7. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$. If $f : (X, \tau) \to (Y, \sigma)$ is a r-gf-continuous map, then $f(gcl(\mu, r)) \leq cl(f(\mu), r)$ for each $\mu \in I^X$.*

Proof. For each $\mu \in I^X$, $cl(f(\mu), r)$ is a fuzzy r-closed set of Y. Since f is r-gf-continuous, $f^{-1}(cl(f(\mu), r))$ is a r-gfc set of X. $\mu \leq f^{-1}(cl(f(\mu), r))$ and so $gcl(\mu, r) \leq f^{-1}(cl(f(\mu), r))$ by Definition 3.5. Hence $f(gcl(\mu, r)) \leq cl(f(\mu), r)$. □

Theorem 4.8. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$. Then $f : (X, \tau) \to (Y, \sigma)$ is a r-gf-irresolute map if and only if $f^{-1}(\mu)$ is a r-gfo set of X for each r-gfo set μ of Y.*

Proof. It is straightforward. □

Theorem 4.9. *Let (X,τ), (Y,σ) and (Z,ν) be fts's and $r \in I_0$. If $f : (X,\tau) \to (Y,\sigma)$ is a r-gf-irresolute map and $g : (Y,\sigma) \to (Z,\nu)$ is a r-gf-continuous map, then $g \circ f : (X,\tau) \to (Z,\nu)$ is a r-gf-continuous map.*

Proof. It is straightforward. □

Theorem 4.10. *Let (X,τ) and (Y,σ) be fts's and $r \in I_0$. If $f : (X,\tau) \to (Y,\sigma)$ is a r-gf-irresolute map, then*

(1) $f(gcl(\mu,r)) \leq gcl(f(\mu),r)$ *for each* $\mu \in I^X$,
(2) $gcl(f^{-1}(\mu),r) \leq f^{-1}(gcl(\mu,r))$ *for each* $\mu \in I^Y$,
(3) $f^{-1}(gint(\mu,r)) \leq gint(f^{-1}(\mu),r)$ *for each* $\mu \in I^Y$.

Proof. (1) For each $\mu \in I^X$, we have

$$\begin{aligned} f^{-1}(gcl(f(\mu),r)) &= f^{-1}(\wedge\{\rho \in I^Y \mid f(\mu) \leq \rho,\ \rho \text{ is r-gfc}\}) \\ &\geq f^{-1}(\wedge\{\rho \in I^Y \mid \mu \leq f^{-1}(\rho),\ \rho \text{ is r-gfc}\}) \\ &\geq \wedge\{f^{-1}(\rho) \in I^X \mid \mu \leq f^{-1}(\rho),\ f^{-1}(\rho) \text{ is r-gfc}\}) \\ &\geq \wedge\{\lambda \in I^X \mid \mu \leq \lambda,\ \lambda \text{ is r-gfc}\} \\ &= gcl(\mu,r). \end{aligned}$$

Hence $f(gcl(\mu,r)) \leq gcl(f(\mu),r)$.

(2) For each $\mu \in I^Y$, we have

$$\begin{aligned} f^{-1}(gcl(\mu,r)) &= f^{-1}(\wedge\{\rho \in I^Y \mid \mu \leq \rho,\ \rho \text{ is r-gfc}\}) \\ &\geq f^{-1}(\wedge\{\rho \in I^Y \mid f^{-1}(\mu) \leq f^{-1}(\rho),\ \rho \text{ is r-gfc}\}) \\ &\geq \wedge\{f^{-1}(\rho) \in I^X \mid f^{-1}(\mu) \leq f^{-1}(\rho),\ f^{-1}(\rho) \text{ is r-gfc}\}) \\ &\geq \wedge\{\lambda \in I^X \mid f^{-1}(\mu) \leq \lambda,\ \lambda \text{ is r-gfc}\} \\ &= gcl(f^{-1}(\mu),r). \end{aligned}$$

Hence $gcl(f^{-1}(\mu),r) \leq f^{-1}(gcl(\mu,r))$.

(3) For each $\mu \in I^Y$, we have

$$\begin{aligned} f^{-1}(gint(\mu,r)) &= f^{-1}(\vee\{\rho \in I^Y \mid \rho \leq \mu,\ \rho \text{ is r-gfo}\}) \\ &\leq f^{-1}(\vee\{\rho \in I^Y \mid f^{-1}(\rho) \leq f^{-1}(\mu),\ \rho \text{ is r-gfo}\}) \\ &\leq \vee\{f^{-1}(\rho) \in I^X \mid f^{-1}(\rho) \leq f^{-1}(\mu),\ f^{-1}(\rho) \text{ is r-gfo}\}) \end{aligned}$$

$$\leq \vee\{\lambda \in I^X \mid \lambda \leq f^{-1}(\mu),\ \lambda \text{ is r-gfo}\}$$
$$= gint(f^{-1}(\mu), r).$$

Hence $f^{-1}(gint(\mu, r)) \leq gint(f^{-1}(\mu), r)$. □

Theorem 4.11. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$. If $f : (X, \tau) \to (Y, \sigma)$ is a strongly r-gf-open map, then $f(gint(\mu, r)) \leq gint(f(\mu), r)$ for each $\mu \in I^X$.*

Proof. For each $\mu \in I^X$, we have

$$\begin{aligned} f(gint(\mu, r)) &= f(\vee\{\rho \in I^X \mid \rho \leq \mu,\ \rho \text{ is r-gfo}\}) \\ &\leq f(\vee\{\rho \in I^X \mid f(\rho) \leq f(\mu),\ \rho \text{ is r-gfo}\}) \\ &\leq \vee\{f(\rho) \in I^Y \mid f(\rho) \leq f(\mu),\ f(\rho) \text{ is r-gfo}\}) \\ &\leq \vee\{\lambda \in I^Y \mid \lambda \leq f(\mu),\ \lambda \text{ is r-gfo}\} \\ &= gint(f(\mu), r). \end{aligned}$$

Hence $f(gint(\mu, r)) \leq gint(f(\mu), r)$. □

5. Several Types of R-Generalized Fuzzy Compactness

A collection $\{\mu_i \mid i \in \Gamma\}$ of fuzzy r-open sets of X is called a fuzzy r-open cover of X if $\vee_{i\in\Gamma}\mu_i = \tilde{1}$.

A collection $\{\mu_i \mid i \in \Gamma\}$ of r-gfo sets of X is called a r-gfo cover of X if $\vee_{i\in\Gamma}\mu_i = \tilde{1}$.

Definition 5.1. Let (X, τ) be a fts and $r \in I_0$.

(1) (X, τ) is called *fuzzy r-compact* if for every fuzzy r-open cover $\{\mu_i \mid i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0}\mu_i = \tilde{1}$.

(2) (X, τ) is called *nearly fuzzy r-compact* if for every fuzzy r-open cover $\{\mu_i \mid i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0} int(cl(\mu_i, r), r) = \tilde{1}$.

(3) (X, τ) is called *almost fuzzy r-compact* if for every fuzzy r-open cover $\{\mu_i \mid i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0} cl(\mu_i, r) = \tilde{1}$.

(4) (X, τ) is called *strongly nearly fuzzy r-compact* if for every fuzzy r-open cover $\{\mu_i \mid i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0} gint(gcl(\mu_i, r), r) = \tilde{1}$.

(5) (X, τ) is called *strongly almost fuzzy r-compact* if for every fuzzy r-open cover $\{\mu_i \mid i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0} gcl(\mu_i, r) = \tilde{1}$.

(6) (X, τ) is called *fuzzy r-regular* if each fuzzy r-open set μ of X can be written as $\mu = \vee\{\rho \in I^X \mid \tau(\rho) \geq \tau(\mu),\ cl(\rho, r) \leq \mu\}$.

Definition 5.2. Let (X, τ) be a fts and $r \in I_0$.

(1) (X, τ) is called *r-generalized fuzzy compact* (for short, r-gf-compact) if for every r-gfo cover $\{\mu_i \mid i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i \in \Gamma_0} \mu_i = \tilde{1}$.

(2) (X, τ) is called *nearly r-generalized fuzzy compact* (for short, nearly r-gf-compact) if for every r-gfo cover $\{\mu_i \mid i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i \in \Gamma_0} gint(gcl(\mu_i, r), r) = \tilde{1}$.

(3) (X, τ) is called *almost r-generalized fuzzy compact* (for short, almost r-gf-compact) if for every r-gfo cover $\{\mu_i \mid i \in \Gamma\}$ of X, there exists a finite subset Γ_0 of Γ such that $\vee_{i \in \Gamma_0} gcl(\mu_i, r) = \tilde{1}$.

(4) (X, τ) is called *r-generalized fuzzy regular* (for short, r-gf-regular) if each r-gfo set μ of X can be written as $\mu = \vee\{\rho \in I^X \mid \rho \text{ is r-gfo}, gcl(\rho, r) \leq \mu\}$.

Theorem 5.3. *Let (X, τ) be a fts and $r \in I_0$. Then*

(1) *If (X, τ) is r-gf-compact, then (X, τ) is fuzzy r-compact.*

(2) *If (X, τ) is almost r-gf-compact, then (X, τ) is strongly almost fuzzy r-compact.*

(3) *If (X, τ) is strongly almost fuzzy r-compact, then (X, τ) is almost fuzzy r-compact.*

(4) *If (X, τ) is nearly r-gf-compact, then (X, τ) is strongly nearly fuzzyr-compact.*

Proof. It is straightforward. □

Theorem 5.4. *Let (X, τ) be a fts and $r \in I_0$. Then (X, τ) is r-gf-compact $\Rightarrow$ (X, τ) is nearly r-gf-compact $\Rightarrow$ (X, τ) is almost r-gf-compact.*

Proof. It is straightforward. □

Theorem 5.5. *Let (X, τ) be a fts and $r \in I_0$. Then (X, τ) is fuzzy r-compact $\Rightarrow$ (X, τ) is nearly fuzzy r-compact $\Rightarrow$ (X, τ) is almost fuzzy r-compact.*

Proof. It is straightforward. □

Theorem 5.6. *Let (X, τ) be a fts and $r \in I_0$. Then (X, τ) is fuzzy r-compact $\Rightarrow$ (X, τ) is strongly nearly fuzzy r-compact $\Rightarrow$ (X, τ) is strongly almost fuzzy r-compact.*

Proof. It is straightforward. □

Theorem 5.7. *Let (X,τ) be a fts and $r \in I_0$. If (X,τ) is almost r-gf-compact and r-gf-regular, then (X,τ) is r-gf-compact.*

Proof. Let $\{\mu_i |\ i \in \Gamma\}$ be a r-gfo cover of X. Since (X,τ) is r-gf-regular, $\mu_i = \vee_{j_i \in J_i}\{\rho_{j_i} \in I^X |\ \rho_{j_i}$ is r-gfo, $gcl(\rho_{j_i}, r) \leq \mu_i\}$ for each $i \in \Gamma$. Since $\vee_{i\in\Gamma}\mu_i = \vee_{i\in\Gamma}(\vee_{j_i\in J_i}\rho_{j_i}) = \tilde{1}$ and (X,τ) is almost r-gf-compact, there exists a finite subfamily $\{\rho_j \in I^X |\ \rho_j$ is r-gfo, $j \in J\}$ such that $\vee_{j\in J} gcl(\rho_j, r) = \tilde{1}$. Since for each $j \in J$ there exists $i \in \Gamma$ such that $gcl(\rho_j, r) \leq \mu_i$, we have $\vee_{i\in\Gamma_0}\mu_i = \tilde{1}$, where Γ_0 is a finite subset of Γ. Hence (X,τ) is r-gf-compact. □

Theorem 5.8. *Let (X,τ) be a fts and $r \in I_0$. If (X,τ) is almost fuzzy r-compact and fuzzy r-regular, then (X,τ) is fuzzy r-compact.*

Proof. The proof is similar to Theorem 5.7. □

Theorem 5.9. *Let (X,τ) and (Y,σ) be fts's and $r \in I_0$ and let $f : (X,\tau) \to (Y,\sigma)$ be a surjective r-gf-continuous map. If (X,τ) is r-gf-compact, then (Y,σ) is fuzzy r-compact.*

Proof. It is straightforward. □

Theorem 5.10. *Let (X,τ) and (Y,σ) be fts's and $r \in I_0$ and let $f : (X,\tau) \to (Y,\sigma)$ be a surjective strongly r-gf-continuous map. If (X,τ) is fuzzy r-compact, then (Y,σ) is r-gf-compact.*

Proof. Let $\{\mu_i |\ i \in \Gamma\}$ be a r-gfo cover of Y. Since f is strongly r-gf-continuous, by Theorem 4.6 $\{f^{-1}(\mu_i)|\ i \in \Gamma\}$ is a fuzzy r-open cover of X. Since (X,τ) is fuzzy r-compact, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0} f^{-1}\ (\mu_i) = \tilde{1}_X$. Since f is surjective, $\tilde{1}_Y = f(\tilde{1}_X) = f(\vee_{i\in\Gamma_0} f^{-1}(\mu_i)) = \vee_{i\in\Gamma_0} f(f^{-1}(\mu_i)) = \vee_{i\in\Gamma_0}\mu_i$, i.e., $\vee_{i\in\Gamma_0}\mu_i = \tilde{1}_Y$. Hence (Y,σ) is r-gf-compact. □

Theorem 5.11. *Let (X,τ) and (Y,σ) be fts's and $r \in I_0$ and let $f : (X,\tau) \to (Y,\sigma)$ be a surjective r-gf-continuous map. If (X,τ) is almost r-gf-compact, then (Y,σ) is almost fuzzy r-compact.*

Proof. Let $\{\mu_i | \ i \in \Gamma\}$ be a fuzzy r-open cover of Y. Since f is r-gf-continuous, by Theorem 4.5 $\{f^{-1}(\mu_i) | \ i \in \Gamma\}$ is a r-gfo cover of X. Since (X, τ) is almost r-gf-compact, there exists a finite subset Γ_0 of Γ such that $\vee_{i \in \Gamma_0} gcl(f^{-1}(\mu_i), r) = \tilde{1}_X$. Since f is surjective,

$$\tilde{1}_Y = f(\tilde{1}_X) = f(\vee_{i \in \Gamma_0} gcl(f^{-1}(\mu_i), r)) = \vee_{i \in \Gamma_0} f(gcl(f^{-1}(\mu_i), r)).$$

Since f is r-gf-continuous, by Theorem 4.7 $f(gcl(f^{-1}(\mu_i), r)) \leq cl(f(f^{-1}(\mu_i)), r)$.

Hence $\tilde{1}_Y = \vee_{i \in \Gamma_0} \ f(gcl(f^{-1}(\mu_i), r)) \leq \vee_{i \in \Gamma_0} cl(f(f^{-1}(\mu_i)), r) = \vee_{i \in \Gamma_0} cl(\mu_i, r)$.

Thus $\vee_{i \in \Gamma_0} \ cl(\mu_i, r) = \tilde{1}_Y$.

Hence (Y, σ) is almost fuzzy r-compact. □

Theorem 5.12. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a surjective fuzzy r-continuous map. If (X, τ) is strongly almost fuzzy r-compact, then (Y, σ) is almost fuzzy r-compact.*

Proof. The proof is similar to Theorem 5.11. □

Theorem 5.13. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a surjective r-gf-irresolute map. Then*

(1) *If (X, τ) is r-gf-compact, then (Y, σ) is r-gf-compact.*
(2) *If (X, τ) is almost r-gf-compact, then (Y, σ) is almost r-gf-compact.*

Proof. (1) It is straightforward.

(2) Let $\{\mu_i | \ i \in \Gamma\}$ be a r-gfo cover of Y. Since f is r-gf-irresolute, by Theorem 4.8 $\{f^{-1}(\mu_i) | \ i \in \Gamma\}$ is a r-gfo cover of X. Since (X, τ) is almost r-gf-compact, there exists a finite subset Γ_0 of Γ such that $\vee_{i \in \Gamma_0} gcl(f^{-1}(\mu_i), r) = \tilde{1}_X$. Since f is surjective, $\tilde{1}_Y = f(\tilde{1}_X) = f(\vee_{i \in \Gamma_0} gcl(f^{-1}(\mu_i), r)) = \vee_{i \in \Gamma_0} \ f(gcl(f^{-1}(\mu_i), r))$. Since f is r-gf-irresolute, by Theorem 4.10 $f(gcl(f^{-1}(\mu_i), \ r)) \leq gcl(f(f^{-1}(\mu_i)), r)$. Hence $\tilde{1}_Y = \vee_{i \in \Gamma_0} f(gcl(f^{-1}(\mu_i), r)) \leq \vee_{i \in \Gamma_0} gcl \ (f(f^{-1}(\mu_i)), r) = \vee_{i \in \Gamma_0} gcl(\mu_i, r)$. Thus $\vee_{i \in \Gamma_0} gcl(\mu_i, r) = \tilde{1}_Y$. Hence (Y, σ) is almost r-gf-compact. □

Theorem 5.14. Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a surjective, fuzzy r-continuous and r-gf-irresolute map. If (X, τ) is strongly almost fuzzy r-compact, then (Y, σ) is strongly almost fuzzy r-compact.

Proof. The proof is similar to Theorem 5.13. □

Theorem 5.15. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a surjective, r-gf-irresolute and strongly r-gf-open map. If (X, τ) is nearly r-gf-compact, then (Y, σ) is nearly r-gf-compact.*

Proof. Let $\{\mu_i | \ i \in \Gamma\}$ be a r-gfo cover of Y. Since f is r-gf-irresolute, by Theorem 4.8 $\{f^{-1}(\mu_i) | \ i \in \Gamma\}$ is a r-gfo cover of X. Since (X, τ) is nearly r-gf-compact, there exists a finite subset Γ_0 of Γ such that $\vee_{i\in\Gamma_0} gint(gcl(f^{-1}(\mu_i), r), r) = \tilde{1}_X$. Since f is surjective, $\tilde{1}_Y = f(\tilde{1}_X) = f(\vee_{i\in\Gamma_0} gint(gcl(f^{-1}(\mu_i), r), r)) = \vee_{i\in\Gamma_0} f(gint(gcl(f^{-1}(\mu_i), r), r))$. Since f is strongly r-gf-open, by Theorem 4.11 $f(gint(gcl\ (f^{-1}(\mu_i), r), r)) \leq gint(f(gcl(f^{-1}(\mu_i), r)), r)$ for each $i \in \Gamma$. Since f is r-gf-irresolute, by Theorem 4.10 $f(gcl(f^{-1}(\mu_i), r)) \leq gcl(f(f^{-1}(\mu_i)), r)$. Hence we have

$$\begin{aligned}
\tilde{1}_Y &= \vee_{i\in\Gamma_0} f(gint(gcl(f^{-1}(\mu_i), r), r)) \\
&\leq \vee_{i\in\Gamma_0} gint(f(gcl(f^{-1}(\mu_i), r)), r) \\
&\leq \vee_{i\in\Gamma_0} gint(gcl(f(f^{-1}(\mu_i)), r), r) \\
&= \vee_{i\in\Gamma_0} gint(gcl(\mu_i, r), r).
\end{aligned}$$

Thus $\vee_{i\in\Gamma_0} gint(gcl(\mu_i, r), r) = \tilde{1}_Y$. Hence (Y, σ) is nearly r-gf-compact. □

We obtain the following corollary from Theorem 5.15 and Remark 4.2.

Corollary 5.16. *Let (X, τ) and (Y, σ) be fts's and $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a bijective, fuzzy r-continuous, fuzzy r-open and r-gf-irresolute map. If (X, τ) is nearly r-gf-compact, then (Y, σ) is nearly r-gf-compact.*

Theorem 5.17. *Let (X, τ) and (Y, σ) be fts's, $r \in I_0$ and let $f : (X, \tau) \to (Y, \sigma)$ be a surjective, fuzzy r-continuous, r-gf-irresolute and strongly r-gf-open map. If (X, τ) is strongly nearly fuzzy r-compact, then (Y, σ) is strongly nearly fuzzy r-compact.*

Proof. The proof is similar to Theorem 5.15. □

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