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A Case Study of Developing Students' Ability to Design Algorithm in LOGO Environment

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The algorithmic idea has been a kind of necessary mathematics quality for modern people in this information society. In China the algorithm was represented fully as one of the new mathematics contents in the secondary level for the first time when The *Standards of Mathematics Curriculum for the Senior High School* was promulgated in 2003, so the research about the teaching algorithm undoubtedly has its practical implications for mathematics education. In this paper, with the conceptual framework of The Mathematics Task Framework as the research tool, an algorithmic teaching case based on LOGO software was introduced in detail, and data by ways of observations, interviews and worksheets were collected, then the case was analyzed. The results showed that the teaching of algorithm is feasible and effective in the LOGO environment. Some beneficial implications about the instructional design of algorithm were also discussed.

Keywords: algorithm, LOGO, case study
ZDM Classification: U50
MSC2000 Classification: 97C30, 97U50

INTRODUCTION

An algorithm refers the step-by-step systematic procedure used to accomplish an operation, which is characterized as finiteness, definiteness, input, output, effectiveness and named by the ninth-century Arabian mathematician Mohammed Al-Khowarizmi. Due to the ways of mechanical operation, the algorithm hasn't been emphasized much in the long history of mathematics education (Peng 2004).

With the rapid development in modern information technology, the algorithm begins to play a fundamental role in the development of science, technology and society, and even penetrates into every aspects of life. Being considered as a kind of necessary mathematics quality for modern people, the algorithmic idea is gradually emphasized in educational circles, which arouses the interests of the related research from mathematics education. The initial work can trace back to 1978, in which Engel outlined the comprehensive topics of the mathematics curriculum at school from an algorithmic standpoint (pp. 255–274). Similarly, Ziegenbalg argued that the concept of algorithm belongs to one of those fundamental concepts of mathematics (pp. 239–241).

At present, as the process of problem solving and mathematical application in the authentic life are highlighted in modern mathematics teaching reform, the learning and understanding of the algorithmic process are especially emphasized. It's a trend in mathematics teaching reform that the traditional way of teaching algorithm should be changed in such a way that students design their own algorithms and solve realistic problems through the algorithmic ideas. Furthermore, students should be able to decide their own approaches and steps (Xu 2001, pp. 179–200).

Although algorithm is a distinctive feature of mathematics in ancient China (Ma *et al.* 1991), the term algorithm hasn't been represented in the mathematical textbooks for schools until 2003, when the *Standards of Mathematics Curriculum for the Senior High School* was promulgated in China and the algorithm was represented fully as one of the new contents for the first time. This implies that the research about algorithm is a new issue and not much work has been done in this field in China, except the research from the angles of cognitive psychology (Xu 2003), curricular value (J. Li 2004; Liu 2003a; 2003b, pp. 12–13) and the significance of learning algorithm (Y. Li 2004). In particular, the problems about which undoubtedly should have the practical implications for mathematics education, such as how to design the mathematics teaching according to students' real level, especially, how to integrate the information technology into the algorithmic teaching, are topics worthy of research while still are scarce.

In the following, a case of algorithmic teaching where the main classroom task is to draw a pentagram based on LOGO software (*cf.* Fu 2002, pp. 45–48) will be introduced in detail, and data including observations, interviews and worksheets will also be collected, then in-depth analysis will be shown with the conceptual framework of The Mathematics Task Framework as the research tool. In the last, some beneficial implications about the instructional design of algorithm will be discussed.

CONCEPTUAL FRAMEWORK

The framework that guided much of this study is based on Mathematical Tasks Framework developed by Stein and her colleagues which focus on the cognitive demand of mathematical tasks and the various phases tasks pass through in their instructional use (Stein and Smith 1998). This framework is depicted in Figure 1.

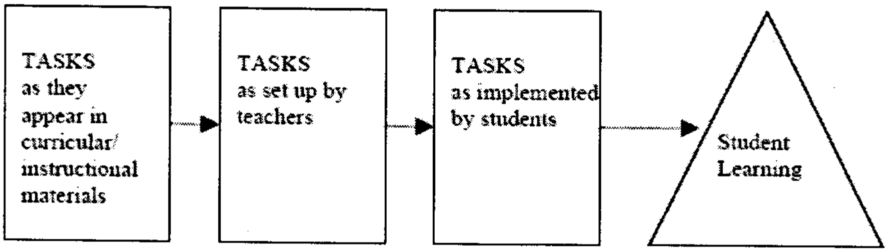

Figure 1. Mathematical Tasks Framework

In describing the framework, Stein, Grover & Henningsen (1996) write:

> "Instructional tasks are seen as passing through three phases: first, as curricular materials; second, as set up by the teacher in the classroom; and third, as implemented by students during the lesson."

Certain cognitive demands are inherent in the way that a mathematical task is written. For example, tasks that ask students to memorize a fact or to perform an algorithm rarely encourage a certain type of mathematical thinking. Tasks that ask students to look for patterns, generalize, make connections, or think conceptually encourage a different kind of thinking (Stein & Smith 1998).

In my research, the Mathematical Tasks Framework guided my data collection, analysis, and reporting. To use this framework as a research tool because I was particularly interested in understanding how the students perform their classroom tasks pertaining to the ideas found in the Mathematical Tasks Framework. While organizing the teaching case, I was guided by my use of the Mathematical Tasks Framework in making decisions about what data to collect. These data could have been analyzed in a way that I was guided by my use of the Mathematical Tasks Framework in choosing a coherent way of thinking about how to organize and interpret the data.

THE CASE OF ALGORITHMIC TEACHING

During the implementation of the algorithmic teaching, several computer languages such as Basic, C, LOGO, Mat lab and so on, are practiced in different schools in China. Contrary to other languages, LOGO is the most controversial one because LOGO is thought as the lowest level which is just suitable for little children. The present situation is that LOGO is not popular in China although it had wide influence in mathematics teaching in 1980's. But there is still a persistent small study group who works on LOGO including not only developing Chinese version but also creating new LOGO orders just like building blocks, to adapt to mathematics teaching in middle and high school, like developing teaching design for the preparation for the entrance examination to university. With the new curriculum reform, algorithm has been listed as an important learning content for students, LOGO is considered how to play its role in the new challenging by this study group, based on the data from the LOGO experiment, one of the aims in this paper is to test whether it is feasible to implement algorithmic teaching in LOGO environment. The case embodied in this study is a lesson about the teaching of algorithm, and it is highly typical of the lessons in LOGO environment and of my previous experience attending in this study group.

Task: drawing a pentagram

The content of drawing a pentagram is chosen from *the colorful LOGO-figures world*, a subsection of a learning textbook, *LOGO experiment*, as a help of new textbooks compiled according to the *Standards of Mathematics Curriculum during Compulsory Education* (*cf.* Ministry of Education of Peoples' Republic of China 2001). Although the case is taken from junior high school, the results showed that it can realize the notion of the *Standards of Mathematics Curriculum for the Senior High School,* so it is helpful not only for the instructional design of algorithm in senior high school, but also for how to develop algorithmic idea in junior high school.

Teaching condition

The students have learned some basic geometry, and they can use some basic and simple LOGO orders to operate. Each one has an individual computer in a well-furnished computer laboratory, which is linked by local area network, through which students and teacher can communicate freely.

Teaching process

Phrase 1 Review (about 3 minutes):

The teacher guides students to review the LOGO orders, FD (FORWORD), BK (BACKWARD), RT (RIGHT), LT (LEFT), which will be used in this lesson, through the strategy of asking-answer way.

Phrase 2 Learning the new LOGO order (about 10 minutes):

The students begin to learn the new order REPEAT, with the help of the teacher, through drawing the triangle, quadrangle and pentagon. In this stage, students can describe the complex recycling process and use the format:

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REPEAT 3 [FD 60 RT 120]
REPEAT 4 [FD 60 RT 90]
REPEAT 5 [FD 60 RT 72]
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Phrase 3 Exploration of the experiment (about 27 minutes):

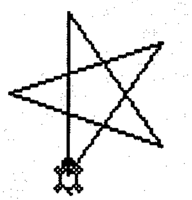
The teacher shows a model of a pentagram and encourages students to draw it through computer. To many people, pentagram is a very popular geometry figure, it can be seen many places such as the Chinese National Flag.

Many students are very excited when the teacher asks them to draw a pentagram, because they are familiar with it, and most of them have the experience in drawing a pentagram with a paper-pencil. But in the face of computer based on LOGO, they feel out of place, because it's difficult for them without considerable mathematics knowledge and the thinking way of precise expression.

The angle is the core of solving the algorithmic problem. Based on the knowledge of measure of angle in the seventh grade, the teacher guides students to review the conception of supplementary angles and adjacent angles, then tells them the characteristics of the pentagram (like "every angle is 36°.").

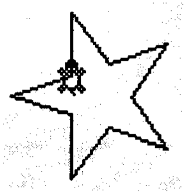
Students devote themselves to draw quickly, and some who are good at computer can color the pentagrams.

The students find the following approaches to draw pentagrams:



Regular pentagram (two algorithms)
REPEAT 5 [FD 60 RT 144]
REPEAT 5 [FD 60 LT 144]

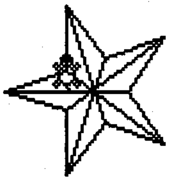
Figure 2. Regular pentagrm



Hollow pentagram (four algorithms)
REPEAT 5 [FD 25 RT 144 FD 25 LT 72]
REPEAT 5 [FD 25 LT 144 FD 25 RT 72]
REPEAT 5 [FD 25 RT 72 FD 25 LT 144]
REPEAT 5 [FD 25 LT 72 FD 25 RT 144]

Figure 3. Hollow pentagram

After discussion with students about the meaning of the data 5, 60, 144, 25, 72, the teacher gives another approach of drawing.



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Solid pentagram ( added by the teacher )
REPEAT 5 [FD 25 LT 18 BK 25 * 1.9 FD 25 * 1.9
                RT 162 FD 25 LT 72 ]
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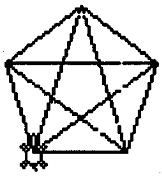
Figure 4. Solid pentagram

Phrase 4 Exploration and innovation (about 5 minutes):



Fill in the blanks：

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REPEAT 5 [RT      FD 100 RT ]
      RT 90
REPEAT 5 [FD 100*        LT 72]
      LT 90
END
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Figure 5. Pentagon

This practice provides an approach through connecting the diagonal of the pentagon. Especially and meaningfully, this is a bridge of the pentagram and pentagon. It motivates students to further understand the algorithm of drawing a pentagram, and do prepare for the continuing learning of the pentagon in the future.

DISSCUSSION AND CONCLUSION

This lesson is designed from the perspectives of how to explore algorithm and how to design various algorithm, aiming at developing and deepening students' algorithmic idea through the experience of the algorithmic diversity. We'd like to analyze the obtained lessons from the exploration of algorithm, algorithmic diversity and mistakes during the study of algorithm.

Students' exploration of algorithm

In this case, taking advantage of the characteristics of LOGO language, such as intuitive and easy-operate, the teacher guided students to explore the way of solving problem during the operation, to develop their own algorithmic ideas step by step and foster their ability to solve realistic problems (drawing a pentagram from many aspects) by using the algorithmic ideas.

Students can experience the algorithmic characteristics of finiteness, definiteness, input, output and effectiveness during drawing the pentagram, and deeply impressed with the input language of FD … and RT …, recycling language of REPEAT. …

The following are some samples of how students felt and thoughts about exploration of the algorithm, which we got from the observation and interview in the classroom.

> It's easier to understand the approaches of drawing a pentagram through computer, when we can think carefully as well as looking at the turtle moving (the order is operated by a turtle).

> It's interesting to draw different colors and sizes of pentagrams (by using the order of color, students color red, yellow and blue for the pentagrams).

Programming is not a difficult task (students can summarize the step of drawing a pentagram as a program, and then put different numerical data into the parameter, thus making the pentagram variable in size. In fact, it has realized the transition from mathematics language to procedure language).

> The turtle can finish my demands well, for which I'm highly required that no mistakes would happen during the operation.

What are mentioned above show that it's effective to arouse students' enthusiasm and to let them develop their own algorithmic ideas with the teacher' guiding.

Students' various algorithms

There are six algorithms of drawing a pentagram developed by the students, which can be divided into two types of algorithms. According to the moving way of the turtle, the first type is based on the drawing through connecting the diagonal of the pentagram (there are two approaches, see figure 2), and the second type is based on the drawing through moving along the sides of the pentagram (there are four approaches, see figure 3).

Other two students used this approach: [FD 60 RT 144 FD 60 RT 144 FD 60 RT 144 FD 60 RT 144 FD 60 RT 144], which we call regular algorithm, for it don't need the order of REPEAT, only but FD and RT. It seems like the ordinary paper-pencil way, which can be finished step by step.

From 55 students' procedure records, we got 49. There were 18 students who used the first or second type of algorithm, and there are 4 students who used both (see Table1).

Table 1. The types of the algorithms and the corresponding number of students

Types of algorithm	First	Second	First & Second	Regular	Wrong
Numbers of student	18	18	4	2	7

According to our interview, the students who used the first or the second were affected by the order of REPEAT taught by the teacher, because it can simple the repeated process.

There are only 2 students who used the regular algorithm and we know that they didn't catch the meaning of REPEAT, so they chose the way of step by step.

- These results indicate that:
- Teacher' action has highly active or negative effects on students' learning of algorithm;
- The algorithmic structure of pentagram in students' mind and LOGO language can help students to express the algorithmic model;

Students' previous mathematics knowledge and skills have effects on the subsequent learning of algorithm. The first type of algorithm is simpler in expression than the second and more difficult to get. While the results show the number in any of these two types are the same. It is recorded by the students that because of the transition of the ordinary paper-pencil drawing, they form the habit of connecting the vertex of pentagram to draw. As for the second, it is intuitional, although relatively complex in expression and easier to find for them.

Students' mistakes

There are 7 students whose algorithms are wrong, of course, they didn't get the pentagram on computer. There are some ordinary mistakes in their algorithms, for example, procedure of REPEAT 5 [FD 25 LT 144 FD 25 RT 72] is replaced by the procedure of REPEAT 5 [FD 25 RT 144 FD 25 LT 36]. The mistakes result from the failing to understand the mathematics basic knowledge.

Some students know that every angle of the pentagram is 36° with the help of the teacher, so 144° of the parameter of LT, rather than 36°, which can be learned only though computing according to the theorem of the total of internal angles of a triangle. It shows that students haven't deeply understood the relationship between the angles. Further more, the angle of LF or RT is also important, which easy to be mistaken without a logic and precise thinking process.

The analysis above suggests the students have obtained the expected goals and the teaching algorithm is feasible and effective in the LOGO environment.

IMPLICATIONS FOR THE INSTRUCTIONAL DESIGN

In this section, we'll look back some characteristics of the instructional design of algorithm in this case.

Roles of the teacher and students

The learning of algorithm is a kind of uncreative learning in the psychology. In this case, the teacher didn't indoctrinate the existing algorithm, but let students try themselves to draw a pentagram to experience the initiative of the algorithm through their constructive learning in the LOGO environment, thus transiting the learning of algorithm into a kind of creative learning. It's helpful to eliminating their fear and hate to mathematics and algorithm, also promote students to understand the constructive process of algorithm (Dowling & Noss 1990, pp. 451– 457). If students can create an algorithm, it is an indication that he or she has not only understood the algorithm, but also has known how to apply the algorithm to ordinary life.

So the role of the teacher is to create actively environment for students and to help them take part in exploring and constructing their own algorithm, rather than to teach the existing algorithm. Also, from the time of students' participation, total to 32 minutes, we can see that students are the dominant of the learning.

Task-directed to arouse the motivation

The motivation can arouse students' enthusiasm for learning; provide the direction and goal of learning. In this case, an interesting and familiar problem (drawing a pentagram) was given, catching students right now, and then resulted in requirement of recognition in the algorithm learning during the operation, thus the mathematics knowledge need to be taught and learned naturally. In the LOGO environment, the teacher can choose much more realistic problems, by using the strategies of task-directed to guide the students.

Taking advantage of the LOGO network

The previous research shows that LOGO can help students toward more intuitive mathematical strategies rather than analytic strategies, and it's something of scaffold for the learning (Hoyles & Noss 1992, pp. 6–9). Its good characteristics of friendly face, convenient language and simple operation, are easy to be learned and well liked by students. The most important thing is that it is an open system that can allow students to create new orders, namely creating new algorithms. When teaching algorithm, we should make use of it.

Working on a computer individually allows them to devote to the exploration of algorithm freely. Furthermore, through the monitoring system, the teacher can monitor everyone, and every student can ask for help from the teacher. Students can communicate their algorithms and cooperate with one another to share resources and make progress together.

REFERENCES

Dowling, P. & Noss, R. (1990): *Mathematics versus the National Curriculum.* London: Falmer Press. MATHDI **1992c**.03540

Engel, A. (1978): The role of algorithms and computers in teaching mathematics at school. In: H. Kunle & H. Athen (Eds.), *Proceedings of the third International Congress on Mathematical Education* (held at Karlsruhe, Germany, Aug. 16–21, 1976) (pp. 265–275). MATHDI **1979a**.00120

Fu, M. (2002): *LOGO Laboratory.* Beijing: Electron and Industry Press.

Hoyles, C. & Noss, R. (1992): *Learning Mathematics and Logo.* London: MTT Press.

Li, J. (2004): Concept and Educational Value of Algorithm (In Chinese). *Journal of Mathematics Education* **11(3)**, 46–47. MATHDI **2005c**.01294

Li, Y. (2004): Concept and Learning Value of Algorithm. *Mathematics Bulletin* **40(2)**, 7–8.

Liu, Z. (2003a): Understand and Thought about the Algorithm Based on the Standard of the New Curriculum. *Mathematics Message* **21(2)**, 6–7.

_____ (2003b): *The Standards of Mathematics Curriculum for the Senior High School.* Beijing: People's Education Press.

Ministry of Education of Peoples' Republic of China (2001): *Standards of Mathematics Curriculum during Compulsory Education,* Experiment Draft (in Chinese). Beijing: Beijing Normal University Press.

Ma, Z.; Wang, H.; Sun, H. & Wang, Y. (1991): *A concise history of mathematics education.* Nanning: Guangxi Educational Press.

Peng, A. (2004): *A study of Algorithm Curriculum and its Instructional Design in the Mathematics Course of Senior High School.* Unpublished thesis (M. Ed). Guiyang, Guizhou: Guizhou Normal University.

Stein, M. S.; Grover, B. W. & Henningsen, M. (1996): Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal* **33(2)**, 455–488. MATHDI **1998a**.00423

Stein, M. K. & Smith, M. S. (1998): Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School* **3(4)**, 268–275.

Xu, B. (2001): *Perspective of Mathematics Education.* Shanghai: East Normal University Press.

Xu, B. (2003): *A Study of Cognitive Structure in the Algorithmic Concept.* Shanghai: East Normal University Press.

Ziegenbalg, J. (2002): Algorithm — Fundamental for Mathematics and Mathematics Education. In: J.Wang, & B. Xu, (Eds.), *Trends and Challenges in Mathematics* (pp. 239–241). Shanghai: East China Normal University Press. MATHDI **2004e**.04092