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Characteristics of Problem on the Area of Probability and Statistics for the Korean College Scholastic Aptitude Test[1]

Lee, Kang Sup
Department of Mathematics Education, Dankook University, Yongin, Gyeonggi-do 448-160, Korea;
E-mail: leeks@dankook.ac.kr

Kim, Jong Gyu
Dongducheon Foreign Language High School, Dongducheon, Gyeonggi-do 483-020, Korea;
E-mail: kim7704@hanmail.net

Hwang, Dong Jou
Graduate School of Education, Ajou University, Suweon, Gyeonggi-do 443-749, Korea;
E-mail: djhwang@kedi.re.kr

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In this study, we gave 132 high school students fifteen probabilities and nine statistics problems of the Korean College Scholastic Aptitude Test and then analyzed their answer using the classical test theory and the item response theory. Using the classical test theory (the Testian 1.0) we get the item reliability (0.730 ~ 0.765), and using the item response theory (the Bayesian 1.0) we get the item difficulty (–2.32 ~ 0.83) and discrimination (0.55 ~ 2.71). From results, we find out what and why students could not understand well.

Keywords: KCSAT, reliability, difficulty, discrimination
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I. INTRODUCTION

The national wide college entrance examination in Korea initiated at 1969, and then its name and structure are changed several times. Current examination system named "Korean College Scholastic Aptitude Test (KCSAT)" was established at 1993 and

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composed of six categories of subject such as Korean, Mathematics, English, Natural Science, Social Science, and Second Foreign Language (KICE, 2007).

We have analyzed the problems of probability and statistics which is one of the hardest parts for students using the items given in KCSAT from 1994 to 2004 (Lee & Kim, 2004). And we find out what and why students could not understand well.

Im (1993) has analyzed the mathematics problems of the second to seventh experimental tests for the Korean College Scholastic Aptitude Test. National Assessment Governing Board, U. S. Departmanrt of Education (2002) has analyzed the problems (including problems of probability and statistics) of the National Assessment of Educational Progress (NAEP) mathematics assessments. The level of mathematical ability, including conceptual understanding, procedural knowledge, and problem-solving, is regarded as playing a central role in defining item descriptors and achieving balance across the tasks for each grade level in the NAEP mathematics assessment. The framework reflects an integrated view of school mathematics. Percentage of items allotted to each of the five strands, families of tasks/items to measure the depth of student knowledge in mathematics, items requiring students to construct a response, manipulative materials used to measure student knowledge and problem-solving abilities, and review for potential item bias are also discussed (NAEP, 2002).

II. METHOD

1. Participants

132 students of Uijeongbu Girls' High Sschool in Kyunggi-do were participated in this study. They had to have an entrance examination for high school after middle school, and their academic achievements are in high levels.

2. Projects

Project 1 is getting the item reliability and the degree of preferable on incorrect answers using the classical test theory. Project 2 is getting the item difficulty and discrimination using the item response theory, finding the hard parts for students and providing the basic data for further math education.

3. Instruments

This study analyzed the problems in KCSAT from 1994 to 2004. The items of probability were four subjective questions and eleven multiple choices, and the items of statistics were nine multiple choices, as shown in appendix.

4. Data analysis:

This study got the item difficulty and discrimination using Bayesian 1.0 based on two parameter item response model, and the item reliability and degree of preferable on incorrect answers using the classical test theory, Testian 1.0 (Seong, 2002).

III. ANALYSIS OF RESULTS

1. The item difficulty and discrimination

The item difficulty shows how difficult the items are and the discrimination shows how well the items discriminate students according to their abilities.

1) In the difficulty, item 1995-07 and 1996-05 were very easy, item 1994-18, 1997-08, 1998-24, 1999-12, 1996-12, 1997-12, 2002-08 were easy, item 1995-12, 1995-16, 1996-22, 1997-11, 1998-11, 1998-28, 1999-22, 2000-13, 2000-29, 2001-28, 2001-10 were average, and item 1998-14, 2002-30, 2003-11, 2004-14 were difficult as shown in table1.
2) In the discrimination, item 1998-14, 1999-12, 2003-11 were low, item 1994-18, 1995-07, 1995-12, 1995-16, 1996-05, 1996-22, 1997-11, 1997-12, 1998-24, 2000-13, 2002-08, 2004-14 were average, item 1997-08, 1998-28, 2000-29, 2001-28, 1996-12, 1998-11, 1999-22 were high, and item 2002-30, 2001-10 were very high as shown in table1.

2. The item reliability and the degree of preferable on incorrect answers

1) The reliability means how consistently and accurately it can measure something. When we measure something by a method and there is much discrepancy, the method is not reliable. Cronbach α is employed as a measure of reliability in this study, and we have the value 0.765 for probability part and 0.730 for statistics part.
2) Making answer sheets for multiple choices decides the item's quality and also influences for measuring the item difficulty. It helps make answer sheets with more preferable and better incorrect answers to analyze an answer of each item and investigate the efficiency of incorrect answers. The item 2003-11 is concerned an independent trial, and many students answered 0.2×0.8^9 while the correct answer was 0.8^8. The incorrect answer was more preferable than the correct one. It was because students misrecognized the case number that could happen.

Table 1. Difficulty and discrimination

Item	Difficulty	Discrimination
1994-18	−1.34	1.22
1995-07	−2.32	1.15
1995-16	0.22	0.89
1996-05	−2.21	1.32
1996-22	−0.01	0.89
1997-08	−1.30	1.52
1998-14	0.50	0.60
1998-24	−0.69	1.34
1998-28	−0.23	1.42
1999-12	−0.97	0.55
2000-29	0.01	1.60
2001-28	−0.37	1.46
2002-30	0.75	1.90
2003-11	0.83	0.55
2004-14	0.52	1.33
1995-12	0.29	0.91
1996-12	−1.85	1.52
1997-11	0.17	0.92
1997-12	−0.68	1.04
1998-11	−0.23	1.63
1999-22	−0.39	1.49
2000-13	0.01	1.11
2001-10	−0.38	2.71
2002-08	−0.91	0.81

IV. CONCLUSION

First, the items 1995-07 and 1996-05 were very easy because they could be solved with only basic knowledge. Item 1998-14, 2002-30, 2003-11 and 2004-14 were concerned on probability, and had high difficulty. Three of them were independent trials, so we could see that students had difficulty in independent trials. Especially, they seemed to solve the item 1996-22, the proof, only by some means not by proving perfectly. Generally speaking, students had more difficulty in probability than in statistics. There was not much difference between subjective questions and multiple choices in item difficulty.

Second, three items were not very discriminative, twelve were right and seven were very discriminative. So the items were generally discriminative. Also the reliability, 0.765

for probability and 0.730 for statistics by Cronbach α were comparatively good.

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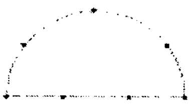
APPENDIX: THE PROBLEMS OF PROBABILITY AND STATISTICS IN KCSAT

1. 1994-18. Suppose the probability of a doctor diagnosing cancer on someone who actually has cancer is 98%, and the probability of diagnosing cancer on someone without cancer is 92%. The doctor diagnosed 400 people with cancer and 600 people without cancer. If we choose one out of these 1000 people at random, what is the probability that the person is actually diagnosed with cancer? [1 point]

① 39.2% ② 40.0% ③ 40.8% ④ 44.0% ⑤ 44.8%

2. 1995-7. There are 7 points on the half circle as in the following picture.
How many triangles can you make with the three points of the seven? [1 point]



① 34 ② 33 ③ 32 ④ 31 ⑤ 30

3. 1996-5. How many ways are there to arrange the letters, P, A, S, S? [1 point]

① 6 ② 8 ③ 12 ④ 18 ⑤ 24

4. 1996-22. There is a box, in which it has 10 balls with a natural number written on each one, arranging from 1 to 10. After mixing up the balls, when we take out two, one at a time, the probability of the number on the second ball being bigger than the number on the first ball is 1/2. The following is the proof of this. (Assuming without replacement.)

> (proof) Let's suppose the number on the first one is X_1, the number on the second one is X_2, and the probability is p. Suppose A_n is the case for $X_1 = n$ and B_n is the case for $X_2 \geq n+1$ about the natural number n, from 1 to 10. Then,
>
> $$p = \sum_{n=1}^{10} \boxed{(A)} \; P(A_n) = \sum_{n=1}^{9} \frac{10-n}{9} \cdot \boxed{(B)} = \frac{1}{2}$$

In the proof, what are the appropriate answers for (A) and (B)? [1.5 points]

① $P(B_n \mid A_n)$, $\frac{1}{10}$ ② $P(B_n \mid A_n)$, $\frac{9}{10}$ ③ $P(A_n \cap B_n)$, $\frac{1}{10}$

④ $P(B_n)$, $\frac{1}{9}$ ⑤ $P(B_n)$, $\frac{1}{10}$

5. 1997-8. The sales number of soft drinks for a certain company depends highly on the average temperature of that summer. According to the past, the probability of meeting their yearly target is 0.8 when the average temperature of that summer is higher than normal year, 0.6 when it is similar and 0.3 when it is lower. According to the weather forecast for next summer, the probability of higher temperature is 0.4, similar is 0.5, and lower is 0.1. What is the probability that the company can achieve their target figure next year? [2 points]

① 0.55 ② 0.60 ③ 0.65 ④ 0.70 ⑤ 0.75

6. 1998-14. Choose all the right answers out of the following. (Assuming the probabilities of the appearance on head and tail of the coin are same.) [2 points]

a. When you toss a coin 10 times, the probabilities that you will see head 4 times and tail 6 times is same. b. The probability that you see head 5 times when you toss the coin 10 times, and the probability that you see head10 times when you toss it 20 times are same. c. When you toss the coin 10 times, the probability that the times you see head are less than 5 times is higher than 0.5.

① a ② c ③ a, b ④ a, c ⑤ a, b, c

7. 1999-12. When you take out a ball from a box containing two white balls and two black ones and toss a coin three times if the ball is white, and 4 times if it is black. What is the probability of seeing head three times? (Assuming the probabilities of the appearance on head and tail of the coin are same.) [3 points]

① $\frac{3}{16}$ ② $\frac{5}{16}$ ③ $\frac{7}{16}$ ④ $\frac{9}{16}$ ⑤ $\frac{11}{16}$

8. 2002-30. The price of a product gets either 10 % higher or 10 % lower by the probability of 0.5 every month. The price is ₩500 now. After two months, if the price is less than ₩500 you will get the money subtraction the price after 2 months from ₩500, and if the price is higher than ₩500, you won't get any money. Find the expected value of the money you'll get after two months up to two decimal places. (Assuming the price changes in the first month and the second month are independent variables.) [3 points]

9. 2003-11. In a soccer game, even after over time teams A and B are still tied and decided to settle it with penalty kicks. When five players from each team take turns one at a time, starting with team A, what is the probability that team B will win 5:4? (Assuming each player's shot is an independent trial and the probability of success is 0.8.)

① 0.2×0.8^8 ② 0.8^8 ③ 0.2×0.8^9 ④ 0.8^9 ⑤ 0.8^{10}

10. 2004-14. When you make 4 digit number using 1, 2, 3 repeatedly, how many numbers have both 1 and 2? [3 points]

① 58 ② 56 ③ 54 ④ 52 ⑤ 50

11. 1996-12. Choose the biggest standard deviation of the followings.

① 1,5,1,5,1,5,1,5,1,5 ② 1,5,1,5,1,5,3,3,3,3 ③ 2,4,2,4,2,4,2,4,2,4
④ 2,4,2,4,2,4,3,3,3,3 ⑤ 4,4,4,4,4,4,4,4,4,4

12. 1997-12. Suppose the salaries for newly employed college graduates in Korea, U.S. and Japan at a certain year were distributed normal with mean ₩800,000, $2,000 and ￥180,000 each, and the standard deviation ₩100,000, $300, and ￥25,000 each. When the salaries of the new employee, A, B, C chosen at random from three countries are ₩940,000, $2,250, and ￥210,000 each, choose the right order of getting higher salary relatively than in his own country out of the followings. [2 points]

① A, B, C ② A, C, B ③ B, A, C ④ C, A, B ⑤ C, B, A

13. 2001-10. When it is provided that the probability density function of continuous random valuable X defined in [0, 1] is $f(x)=ax+a$, what is the value of a? [3 points]

① $\frac{1}{3}$ ② $\frac{2}{3}$ ③ 1 ④ $\frac{3}{2}$ ⑤ 2

14. 2000-13. Suppose the decimal point numbers, after dividing a rolled number by 4 on a die is defined as random variable X. What is the average of X? (Assuming the probability of getting each number of a die is same.) [3 points]

① 2 ② $\frac{5}{3}$ ③ $\frac{3}{2}$ ④ $\frac{4}{3}$ ⑤ 1

15. 2002-8. When we suppose the standard deviations of the three things, A: natural numbers from 1 to 50, B: natural numbers from 51 to 100, and C: even numbers between 1 and 100, are a, b, c in order, which is the correct relationship between the three? [3 points]

① $a = b = c$ ② $a = b \pi c$ ③ $a \pi b = c$ ④ $a \pi b \pi c$
⑤ $a \pi c \pi b$