Time-Varying Multipath Channel Estimation with Superimposed Training in CP-OFDM Systems

Qinghai Yang and Kyung Sup Kwak

ABSTRACT—Based on superimposed training methods, a novel time-varying multipath channel estimation scheme is proposed for orthogonal frequency division multiplexing systems. We first develop a linear least square channel estimator, and meanwhile find the optimal superimposed sequences with respect to the channel estimates' mean square error. Next, a low-rank approximated channel estimator is obtained by using the singular value decomposition. As demonstrated in simulations, the proposed scheme achieves not only better performance but also higher bandwidth efficiency than the conventional pilot-aided approach.

Keywords—Superimposed training, SVD, OFDM.

I. Introduction

In orthogonal frequency division multiplexing (OFDM) systems, the time-varying multipath channels give rise to a loss of subcarrier orthogonality, resulting in intercarrier interference (ICI). Estimating such channels is a big challenge and efforts to meet this challenge are usually based on training methods. Among them, a common approach is to frequency-multiplex training pilots with data symbols [1]. Since pilots will be ICI corrupted, too many pilots have to be inserted to attain an accurate channel estimate, leading to poor bandwidth efficiency.

Channel estimation with superimposed training has been proposed recently for its potential bandwidth efficiency. For time-varying fast flat-fading scenarios, a Kalman channel tracking approach with superimposed training was developed in [2], and an algorithm based on selective superimposed sequences was proposed in [3]. With the block-fading channel assumption, a superimposed training scheme was presented in [4] using the first order statistics of the data. A data-dependent superimposed sequence was derived in [5] for single-carrier transmissions. The above methods are no longer feasible for the channel assumed in this letter, wherein the multipath channel is time-varying during one OFDM symbol period. To find a feasible alternative is the motivation of our investigation.

In this letter, a novel time-domain channel estimation scheme for time-varying multipath channels is developed by using superimposed sequences. The idea behind this scheme is to split the one-OFDM-symbol-period time-domain channel into equi-spaced time-slotted subchannels, so that the time-variation for each subchannel can be assumed to be negligible; then, each subchannel is estimated by a linear least square (LS) estimator. For the cyclic-prefix (CP) OFDM system, optimal training sequences are proposed with respect to minimizing the trace of the channel estimates' mean square error (MSE) matrix. In addition, if the Doppler spread and channel power are already known, an approximated channel estimation scheme is derived by reduced rank technique.

II. Signal Model

OFDM converts serial data streams into parallel blocks of size K and modulates these blocks using inverse fast Fourier transform (IFFT). Time domain samples can be obtained as

$$s(n) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} d(k) e^{j2\pi kn/K}, \quad n \in [0,\ K-1], \qquad (1)$$

where $d(k)$ is the data symbol transmitted over the k-th subcarrier. A known training symbol $c(n)$ is added to $s(n)$, that is,

Manuscript received May 2, 2006; revised July 17, 2006.
This research was supported by the University IT Research Center Project (INHA UWB-ITRC), Korea
Qinghai Yang (phone: +82 32 864 8935, email: yangqing_hai@hotmail.com) and Kyung Sup Kwak (email: kskwak@inha.ac.kr) are with Graduate School of Information Technology and Telecommunications, Inha University, Incheon, Korea.

$$x(n)=s(n)+c(n). \tag{2}$$

In this letter, we assume that $d(k)$ is zero mean, while $E\{|d(k)|^2\}=\rho_d$. (The expectation function is denoted by $E\{\cdot\}$.) With the relationship in (1), $s(n)$ is random and has the same distribution as $d(k)$. We define the average powers of $c(n)$ and $x(n)$ as ρ_c and $\bar{\rho}$, respectively, which have the following relations:

$$\rho_c=\alpha\bar{\rho}, \tag{3}$$

$$\rho_d=(1-\alpha)\bar{\rho},\ \ (0<\alpha<1), \tag{4}$$

where α is the power allocation fraction.

Let us form a vector $\mathbf{x}=(x(0),\cdots,x(K-1))^T$. (In this letter, superscripts T, $*$, H denote transpose, conjugate, and Hermitian, respectively). Before transmission of $\mathbf{x}$, a CP (a replica of the last L elements of $\mathbf{x}$) is inserted to eliminate inter-symbol interference. Here, L denotes the number of discrete paths in a multipath channel. The received signal can be observed at the receiver as

$$y(n)=\sum_{l=0}^{L}h(l,n)[s(n-l)+c(n-l)]+\eta(n), \tag{5}$$

where $h(l, n)$ represents the channel impulse response for the l-th path at time n with the channel power σ_l^2 and $\eta(n)$ denotes complex-valued additive white Gaussian noise with zero mean and variance σ_n^2. In this letter, we assume that the multipath channel is modeled as wide sense stationary uncorrelated scattering (WSSUS) and Rayleigh fading. Let us define $\mathbf{h}(n)=(h(0,n),\cdots,h(L,n))^T$. Following from a Jakes Doppler profile [9], the cross-correlation matrix of $\mathbf{h}(n)$ can be expressed as

$$E\{\mathbf{h}(n)\mathbf{h}(n')^H\}=J_0(2\pi f_d T_s|n-n'|)\Delta, \tag{6}$$

where Δ is the diagonal process denoted by $\Delta=\mathrm{diag}(\sigma_0^2,\cdots,\sigma_L^2)$, $J_0(\cdot)$ represents the zero order Bessel function of the first kind, f_d denotes the maximum Doppler spread, and T_s is the sampling period (appropriately chosen and equal to the symbol period). If we define $\phi(n)=(c(n-0),\cdots,c(n-L))$, (5) can be further rewritten as

$$y(n)=\phi(n)\mathbf{h}(n)+\upsilon(n), \tag{7}$$

where $\upsilon(n)=\sum_{l=0}^{L}h(l,n)s(n-l)+\eta(n)$. We can obtain

$$E\{\upsilon(n)\upsilon(n')^*\}=\varpi\delta(n-n'), \tag{8}$$

where $\varpi=\rho_d\sum_{l=0}^{L}\sigma_l^2+\sigma_\eta^2$. We note that thanks to the CP, it yields $x(-l)=x(K-l)$ for $l\in[1,L]$.

III. Channel Estimation

1. LS Channel Estimator

Although the channel cannot be assumed to be constant for one OFDM symbol period, it may allow us to assume that the time-variation is negligible for a smaller time period. In this letter, we split K time intervals into G equi-spaced slots of M periods ($K=MG$) so that the time-variation of $\mathbf{h}(n)$ is negligible within one slot. The time index can be expressed as $n=gM+m$ for $g\in[0,G-1]$ and $m\in[0,M-1]$; thus, we may define a subchannel for the g-th time slot as

$$\mathbf{h}_g=\mathbf{h}(gM+m),\quad \forall m. \tag{9}$$

Collecting the received signals within one slot to form a vector $\mathbf{y}_g=(y(gM+0),\cdots,y(gM+M-1))^T$ yields

$$\mathbf{y}_g=\Phi_g\mathbf{h}_g+\boldsymbol{\upsilon}_g, \tag{10}$$

where Φ_g and $\boldsymbol{\upsilon}_g$ are obtained from the corresponding stack of $\phi(n)$ and $\upsilon(n)$, respectively. A linear LS channel estimate of $\mathbf{h}_g$ can be obtained as

$$\hat{\mathbf{h}}_g=(\Phi_g^H\Phi_g)^{-1}\Phi_g^H\mathbf{y}_g. \tag{11}$$

The MSE matrix is defined as the auto-correlation matrix of the channel estimate error $\tilde{\mathbf{h}}_g=\hat{\mathbf{h}}_g-\mathbf{h}_g$, that is,

$$\tilde{\mathbf{R}}_{\mathbf{h}_g}=E\{\tilde{\mathbf{h}}_g\tilde{\mathbf{h}}_g^H\}=\varpi(\Phi_g^H\Phi_g)^{-1}. \tag{12}$$

2. Proposed Training Sequences

Optimal training sequences are designed with respect to minimizing the trace of $\tilde{\mathbf{R}}_{\mathbf{h}_g}$, denoted by $\mathrm{Tr}(\tilde{\mathbf{R}}_{\mathbf{h}_g})$. Following from the majorization theory, the minimization of $\mathrm{Tr}(\tilde{\mathbf{R}}_{\mathbf{h}_g})$ requires matrix $\tilde{\mathbf{R}}_{\mathbf{h}_g}$ to be diagonal and thus for matrix $\Phi_g^H\Phi_g$ to be diagonal.

Let $M=PQ$ with index $m=qP+p$ ($m\in[0,M-1], q\in[0,Q-1], p\in[0,P-1]$). The index n can be further expressed by indices g, p, and q as $n=gM+qP+p$. The training sequences are proposed in this letter as

$$c(gM+qP+p)=\sqrt{\rho_c e^{-j2\pi qp/Q}}\quad (P\geq L+1). \tag{13}$$

It follows that $\Phi_g^H\Phi_g=\rho_c MI_{L+1}$ and matrix $\tilde{\mathbf{R}}_{\mathbf{h}_g}$ becomes diagonal. Accordingly, $\hat{\mathbf{h}}_g$ can be rewritten as

$$\hat{\mathbf{h}}_g=\frac{1}{\rho_c M}\Phi_g^H\mathbf{y}_g. \tag{14}$$

Remark: It is noted that G, P, and Q must be chosen properly to be integers, that is, with power of 2 (K is power of 2). Keeping in mind that $P \geq L+1$, we generally select P as $P = 2^{\lceil \log_2(L+1) \rceil}$, where $\lceil \cdot \rceil$ denotes the integer ceiling operation. The training sequences are periodic (M period) so that $\Phi_0 = \Phi_g, \forall g$. In addition, our optimal design is necessarily based on a CP transmission (see section II).

3. Low-Rank Approximations

Low-rank approximated channel estimation is obtained by using the singular value decomposition (SVD). With the advantage of low complexity, the low rank approximated technique has been employed for channel frequency response estimation [6] and interference cancellation [7].

Let us define $\mathbf{H} = (\mathbf{h}_0^T, \cdots, \mathbf{h}_{G-1}^T)^T$ and the corresponding LS channel estimate $\hat{\mathbf{H}} = (\hat{\mathbf{h}}_0^T, \cdots, \mathbf{h}_{G-1}^T)^T$. A linear minimum mean square error (MMSE) channel estimate of $\mathbf{H}$ can be obtained as

$$\overline{\mathbf{H}} = \mathbf{R_H}(\mathbf{R_H} + \frac{\varpi}{\rho_c M} I_{G(L+1)})^{-1} \hat{\mathbf{H}}, \quad (15)$$

where $\mathbf{R_H} = E\{\mathbf{H}\mathbf{H}^H\}$ can be deduced from (6) as

$$\mathbf{R_H} = \mathbf{J} \otimes \Delta, \quad (16)$$

where $\mathbf{J}$ is a $G \times G$ matrix with the (g, g')th elements $[\mathbf{J}]_{g,g'} = J_0(2\pi f_d T_s |g-g'| M)$, and $\otimes$ means the Kronecker product. With SVD, it yields that

$$\mathbf{J} = U \Lambda U^H, \quad (17)$$

where U is the unitary matrix containing the singular vectors and Λ is a diagonal matrix containing the singular values $\lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_{G-1}$ on its diagonal. As a result, we can write

$$\mathbf{R_H} = (U \otimes I_{L+1})(\Lambda \otimes \Delta)(U^H \otimes I_{L+1}). \quad (18)$$

Accordingly,

$$\overline{\mathbf{H}} = (U \otimes I_{L+1}) \overline{\Theta} (U^H \otimes I_{L+1}) \hat{\mathbf{H}}, \quad (19)$$

where $\overline{\Theta}$ is a diagonal matrix with the ($g(L+1)+l$)th diagonal element

$$\overline{\gamma}_{l,g} = \frac{\sigma_l^2 \lambda_g}{\sigma_l^2 \lambda_g + \dfrac{\varpi}{\rho_c M}}, \quad \text{for } g \in [0, G-1] \text{ and } l \in [0, L]. \quad (20)$$

Some trivial eigenvalues λ_g (for $g \in [\kappa+1, G-1]$) may lead to $\sigma_l^2 \lambda_g << \varpi / (\rho_c M)$. This motivates us to propose an approximated estimator based and low-rank method [6].

$$\breve{\mathbf{H}} = (U \otimes I_{L+1}) \breve{\Theta} (U \otimes I_{L+1}) \hat{\mathbf{H}}, \quad (21)$$

where $\breve{\Theta}$, corresponding to $\overline{\Theta}$, is a diagonal matrix with the ($g(L+1)+l$)th diagonal element

$$\breve{\gamma}_{l,g} = \begin{cases} \overline{\gamma}_{l,g} & \text{for } g \in [0, \kappa] \text{ and } l \in [0, L]; \\ 0 & \text{for } g \in (\kappa, G-1] \text{ and } l \in [0, L]. \end{cases} \quad (22)$$

The least significant values, $\breve{\gamma}_{l,g}$ for $g \in [\kappa, G-1]$, are discarded in the low-rank approximation.

Remark: In contrast to an MMSE estimator, the SVD in the low-rank approximation can be computed off-line, thus resulting in a lower complexity. This low-rank approach is called rank-κ approximation in [6] and [7]. The proper selection of κ is based on the error threshold, the relationship of $\sigma_l^2 \lambda_g$, and interference-noise power $\varpi / (\rho_c M)$. A detailed explanation is given in [7]. The low-rank approximated scheme acts as a channel estimation enhancement, but it requires prior knowledge of noise variance, channel power, and Doppler spread. In a practical system, such knowledge has to be estimated; however, a detailed consideration of this is beyond the focus of this letter.

IV. Simulation Results

In our simulations, the entire system bandwidth, 10 MHz, is divided into K = 2048 subcarriers. The multipath channels satisfy the WSSUS assumption. Each path is simulated with a Jakes Doppler profile and an exponential delay profile. The channel power is selected as $\sigma_l^2 = e^{-l/\kappa}$ for $l \in [0, L]$ (here, $\kappa = (L+1)/\log(2L+2)$ and L=7) and the carrier frequency is chosen as 4.5 GHz. Two cases of mobile speed are considered, namely, 72 km/h and 144 km/h, which give rise to Doppler spreads f_d = 300 Hz and f_d = 600 Hz, respectively. To perform the data detection, the known-training effects should be canceled first from the received signal in the time domain. Then, after converting to the frequency domain, a low complexity algorithm in [8] is adopted for data symbol detection.

The curves of averaged bit error rate (BER) performance against received signal to noise ratio ($SNR = \overline{\rho} \sum_{l=0}^{L} \sigma_l^2 / \sigma_\eta^2$) are shown in Fig. 1. As expected, the low-rank approximated channel estimator achieves better performance than the LS estimator for both Doppler spread cases (for the case of f_d = 300 Hz, we set G=2 and determined the low-rank κ=2; for the case of f_d = 600 Hz, we selected G=4 and the low-rank still as κ=2). For the purpose of comparison, we also simulated the MMSE interpolation-based frequency-multiplexing channel estimation scheme [1], wherein 256 subcarriers are used for pilot tones.

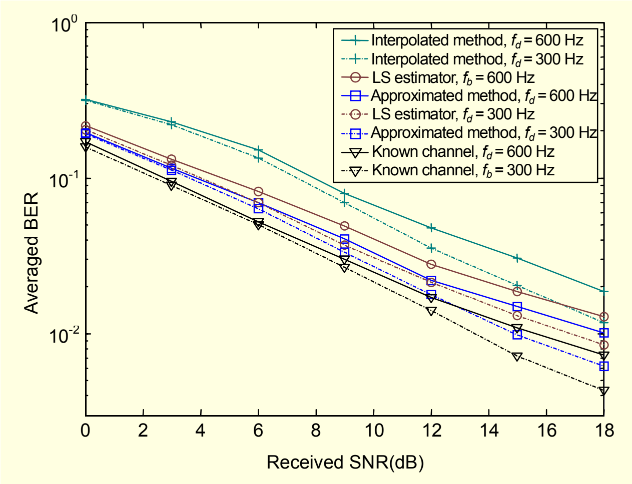

Fig. 1. Averaged BER against received SNR (dB). For the superimposed scheme, the power fraction is chosen as $\alpha = 0.2$.

Our simulations demonstrate that both the proposed low-rank approximated scheme and the LS estimator outperform the approach of [1] for the two Doppler spread cases, respectively. Moreover, given the fact that 256 subcarriers are used for non-data transmission in the scheme of [1], we can conclude that our proposed schemes provide not only better performance but also more favorable bandwidth efficiency in fast fading OFDM channels.

V. Conclusion

In this letter, a time-domain time-varying multipath channel estimation scheme was proposed for OFDM systems using superimposed training. The channel for one OFDM symbol period has been divided into several subchannels, and an LS estimator was developed according to each of the subchannels. We obtained optimal superimposed training sequences with respect to the channel estimate's MSE. In addition, a low complexity channel estimator was deduced with low-rank approximation techniques.

References

[1] A. Stamoulis, S.N. Diggavi, and N. Al-Dhahir, "Intercarrier Interference in MIMO OFDM," *IEEE Trans. on Signal Processing*, vol. 50, Oct. 2002, pp. 2451-2464.

[2] M. Dong, L. Tong, and B.S. Sadler, "Optimal Insertion of Pilot Symbols for Transmissions over Time-Varying Flat Fading Channels," *IEEE Trans. Signal Processing*, vol. 52, May 2004, pp. 1403-1418.

[3] F. Tsuzuki and T. Ohtsuki, "Channel Estimation with Selective Superimposed Pilot Sequences under Fast Fading Environments," *IEICE Trans. Comm.,* vol. E89-B, no. 2, Feb. 2006, pp. 556-563.

[4] J.K. Tugnait and W. Luo, "On Channel Estimation Using Superimposed Training and First Order Statistics," *IEEE Comm. Letters*, vol. 7, Sept. 2003, pp. 413-415.

[5] M. Ghogho, D. McLernon, E.A. Hernandez, and A. Swami, "Channel Estimation and Symbol Detection for Block Transmission Using Data-Dependent Superimposed Training," *IEEE Signal Processing Letters*, vol. 12, Mar. 2005, pp. 226-229.

[6] O. Edfors, M. Sandell, J.V.D. Beek, S.K. Wilson, and P.O. Borjesson, "OFDM Channel Estimation by Singular Value Decomposition," *IEEE Trans. on Comm.*, vol. 46, July 1998, pp. 931-939.

[7] R. Nilsson, F. Sjoberg, and J.P. Leblanc, "A Rank Reduced LMMSE Canceller for Narrowband Interference Suppression in OFDM-Based Systems," *IEEE Trans. on Comm.*, vol. 51, Dec. 2003, pp. 2126-2140.

[8] W. Jeon, K. Chang, and Y. Cho, "An Equalization Technique for Orthogonal Frequency Division Multiplexing Systems in Time-Variant Multipath Channels," *IEEE Trans. on Comm.*, vol. 47, Jan. 1999, pp. 27-32.

[9] W.C. Jakes, *Microwave Mobile Communications*, Piscataway, NJ: IEEE Press, 1993.