# Superimposed Pilot Aided Multiuser Channel Estimation for MIMO-OFDM Uplinks

Qinghai Yang and Kyung Sup Kwak

*ABSTRACT—This letter addresses the superimposed pilot aided multiuser channel estimation for the uplinks of multi-input multi-output orthogonal frequency-division multiplexing systems. To mitigate the embedded-data effects on the performance of channel estimation, a novel combining algorithm is proposed. Optimal pilot symbols are developed with respect to the least square channel estimate's mean square error. The averaged sum-capacity lower bound is derived and simulated. Simulation results show that on a low signal-to-noise ratio regime, our proposed scheme achieves better performance and higher capacity than the conventional pilot aided approach.*

*Keywords—MIMO-OFDM, multiuser channel estimation, LS, superimprosed pilot.*

## I. Introduction

To obtain channel state information (CSI) in multi-input multi-output (MIMO) orthogonal frequency-division multiplexing (OFDM) systems, a common approach [1] is to use frequency-multiplex pilots (FMP) with data symbols. Since some subcarriers must be assigned for pilots, it may be argued that this approach is bandwidth inefficient. Superimposed pilot (SIP) aided channel estimation attracts interest due to its potential spectral efficiency. Unfortunately, its channel estimation performance is affected by the embedded data. To cancel these effects, a data-dependent sequence is derived in [2] for single-carrier scenarios. The algorithm in [3] reduces the impact by using the first order statistics of the data.

In this letter, an SIP aided multiuser channel estimation scheme is developed for MIMO-OFDM uplinks. To decrease the unknown-data effects, a new signal combining algorithm is proposed. Optimal pilot symbols are derived with respect to the mean square error (MSE) of the least square (LS) channel estimate. Averaged sum capacity is resorted to find some insights on spectral efficiency.

**Notation:** Vec(A) represents stacking the columns of matrix A. Tr(A) is the trace of matrix A. Superscripts $T$, $*$, $H$ denote transpose, conjugate and Hermitian, respectively; $\otimes$ represents the Kronecker product; and $\mathbf{I}_N$ is an $N\times N$ matrix.

## II. Signal Model

The multiuser MIMO-OFDM system consists of $U$ user and one base station (BS) as shown in Fig. 1. Each of the users is equipped with $N_T$ transmit antennas and the BS has $N_R$ antennas. All users share the same frequency band, which is split into $K$ subcarriers via the OFDM modulation.

The symbol $s_{u,t}(k, n)$ is transmitted with power $\rho_s$ from the $u$-th

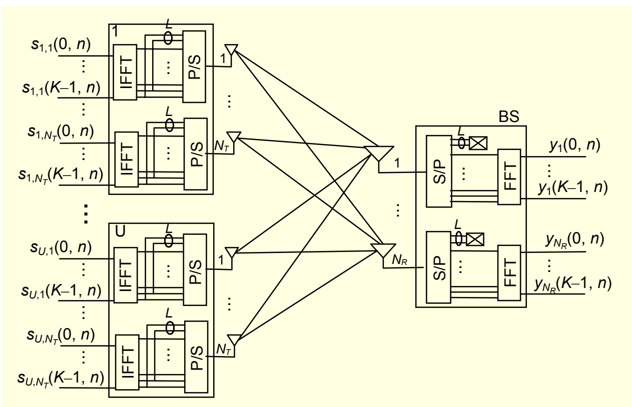


Fig. 1. Uplink of multiuser MIMO-OFDM systems.

Manuscript received Mar. 16, 2006; revised June 7, 2006.

This research was supported by the University IT Research Center Project (INHA UWB-ITRC), Korea.

Qinghai Yang (phone: +82 32 864 8935, email: yangqing_hai@hotmail.com) and Kyung Sup Kwak (email: kskwak@inha.ac.kr) are with Graduate School of Information Technology and Telecommunications, Inha University, Incheon, Korea.

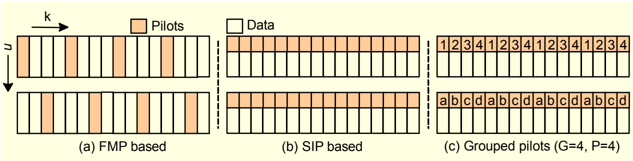


Fig. 2. Placement of data and pilot tones ($K = 16$, $N = 2$).

user's $t$-th antenna over the $k$-th subcarrier at time $n$ ($n \in [0, N-1]$). In FMP schemes [1], symbols $s_{u,t}(k, n), \forall u, t$, on some subcarriers are known pilots, as shown in Fig. 2(a), whereas, for SIP schemes in Fig. 2(b), $s_{u,t}(k, n)$ can be expressed as

$$s_{u,t}(k,n) = d_{u,t}(k,n) + c_{u,t}(k,n), \quad (1)$$

where $d_{u,t}(k, n)$ and $c_{u,t}(k, n)$ are data and pilot symbols with powers $\rho_d = (1\text{-}\alpha)\rho_s$ and $\rho_c = \alpha\rho_s$ ($0<\alpha<1$), respectively. A block of $K$ symbols from collecting $s_{u,t}(k, n)$, $\forall k$, forms one OFDM symbol, which is computed by the inverse fast Fourier transform (IFFT) process. As in common practice, a cyclic prefix (CP) of length $L$ is added to eliminate inter-symbol interference where $L$ denotes the length of all channels. At the BS, the signal vector for the $k$-th tone at time $n$ is given by

$$\mathbf{y}(k,n) = \sum_{u=1}^{U} \mathbf{\Pi}_u(k,n)[\mathbf{d}_u(k,n) + \mathbf{c}_u(k,n)] + \mathbf{z}(k,n), \quad (2)$$

where $\mathbf{y}(k,n) = (y_1(k,n), \cdots, y_{N_R}(k,n))^T$, $\mathbf{d}_u(k,n) = (d_{u,1}(k,n), \cdots, d_{u,N_T}(k,n))^T$ and $\mathbf{c}_u(k,n) = (c_{u,1}(k,n), \cdots, c_{u,N_T}(k,n))^T$; $\mathbf{z}(k,n)$ is an AWGN vector with elements of zero mean and variance $\sigma_z^2$; $\mathbf{\Pi}_u(k,n)$ ($N_R \times N_T$) is the channel frequency response, which is

$$\mathbf{\Pi}_u(k,n) = \sum_{l=0}^{L} \mathbf{H}_u(l,n) e^{-j2\pi lk/K}, \quad (3)$$

where matrix $\mathbf{H}_u(l, n)$ ($N_R \times N_T$) represents the channel impulse response. With (3), we can rewrite (2) into

$$\begin{aligned} \mathbf{y}(k,n) &= [\mathbf{e}_k \otimes \mathbf{d}^T(k,n) \otimes \mathbf{I}_{N_R}]\mathbf{h}(n) \\ &\quad + [\mathbf{e}_k \otimes \mathbf{c}^T(k,n) \otimes \mathbf{I}_{N_R}]\mathbf{h}(n) + \mathbf{z}(k,n), \end{aligned} \quad (4)$$

where $\mathbf{e}_k = (1, \cdots, e^{-j2\pi Lk/K})$, $\mathbf{d}(k,n) = (\mathbf{d}_1^T(k,n), \cdots, \mathbf{d}_U^T(k,n))^T$, $\mathbf{c}(k,n) = (\mathbf{c}_1^T(k,n), \cdots, \mathbf{c}_U^T(k,n))^T$; and $\mathbf{h}(n) = [\mathbf{h}_1^T(n), \cdots, \mathbf{h}_U^T(n)]^T$ with $\mathbf{h}_u(n) = (\text{Vec}(\mathbf{H}_u(0,n))^T, \cdots, \text{Vec}(\mathbf{H}_u(L,n))^T)^T$. Note that $\mathbf{h}(n)$ is a $UN_R N_T(L+1) \times 1$ vector.

We assume that the channel is spatially correlated with $\mathbf{R}_{Tx}^{u,l}(N_T \times N_T)$ and $\mathbf{R}_{Bs}^{l}(N_R \times N_R)$, the transmit and receive correlation matrices, respectively. From the intelligent multi-element transmit and receive antennas (I-METRA) model, the spatial correlation matrix of the MIMO channel can be expressed as the Kronecker product of $\mathbf{R}_{Tx}^{u,l}$ and $\mathbf{R}_{Bs}^{l}$, that is, $\mathbf{R}_{mimo}^{u,l} = \mathbf{R}_{Tx}^{u,l} \otimes \mathbf{R}_{Bs}^{l}$. Considering a wide sense stationary uncorrelated scattering (WSSUS) model, the $u$-th user's channel autocorrelation matrix is given by

$$\mathbf{R}_{\mathbf{h}_u} = E\{\mathbf{h}_u(n)\mathbf{h}_u^H(n)\} = \text{bdiag}(\sigma_{u,0}^2 R_{mimo}^{u,0}, \cdots, \sigma_{u,L}^2 \mathbf{R}_{mimo}^{u,L}), \quad (5)$$

where $\sigma_{u,l}^2$ denotes the corresponding channel power and bdiag(.) is the block-diagonal process. As the distances between users in a cell are generally large enough, the channels are spatially independent between users' antennas. It thus yields

$$\mathbf{R}_{\mathbf{h}} = E\{\mathbf{h}(n)\mathbf{h}\}^H(n)\} = \text{bdiag}(\mathbf{R}_{\mathbf{h}_1}, \cdots, \mathbf{R}_{\mathbf{h}_U}). \quad (6)$$

## III. Channel Estimation

### 1. Proposed Combining Algorithm

Under the SIP scheme, the $K$ subcarriers within one OFDM symbol are arranged into $G$ groups of $P$ size, that is, $k = gP+p$ $g \in [0, G-1]$ and $p \in [0, P-1]$. For a certain index $p$, stacking of vectors $Y(gP+P, n), \forall g$, yields a $GN_R \times 1$ vector $\mathbf{y}_{p,n} = (\mathbf{y}(0P+p,n)^T, \cdots, \mathbf{y}(PG-P+p,n)^T)^T$:

$$\mathbf{y}_{p,n} = (\mathbf{A}_{p,n} \otimes \mathbf{I}_{N_R})\mathbf{h}(n) + (\mathbf{B}_{p,n} \otimes \mathbf{I}_{N_R})\mathbf{h}(n) + \mathbf{z}_{p,n}, \quad (7)$$

where $\mathbf{A}_{p,n}$, $\mathbf{B}_{p,n}$ and $\mathbf{z}_{p,n}$ are obtained from the stacking of $\mathbf{e}_{gP+p} \otimes \mathbf{c}^T(gP+p,n)$, $\mathbf{e}_{gP+p} \otimes \mathbf{d}^T(gP+p,n)$, and $\mathbf{z}(gP+p,n)$, $\forall g$, respectively. Now, we define a $UN_T \times 1$ pilot vector $\boldsymbol{\varphi}(p,n)$ as $\boldsymbol{\varphi}(p,n) = (\boldsymbol{\varphi}_1^T(p,n), \cdots, \boldsymbol{\varphi}_U^T(p,n))^T$ with $\boldsymbol{\varphi}_u(p,n) = (\phi_{u,1}(p,n), \cdots, \phi_{u,N_T}(p,n))^T$. In our scheme, pilots of all $G$ tones related to $\mathbf{A}_{p,n}$ are designed equal to $\boldsymbol{\varphi}(p, n)$; that is, $\mathbf{c}(gP+p,n) = \boldsymbol{\varphi}(p,n), \forall g$. Such an example is shown in Fig. 2 (c). Thus, matrix $\mathbf{A}_{p,n}$ can be further written as

$$\mathbf{A}_{p,n} = \mathbf{F}_G \mathbf{W}_p \otimes \boldsymbol{\varphi}(p,n)^T, \quad (8)$$

where $\mathbf{F}_G$ is a $G \times (L+1)$ matrix with the $(g, l)$th element $[\mathbf{F}_G]_{g,l} = e^{-j2\pi lg/G}$; $\mathbf{W}_p$ is an $(L+1) \times (L+1)$ diagonal matrix with the $(l, l)$th element $[\mathbf{W}_p]_{l,l} = e^{-j2\pi lp/K}$. We can observe

$$\mathbf{W}_p^H \mathbf{F}_G^H \mathbf{F}_G \mathbf{W}_p = G\mathbf{I}_{L+1}. \quad (9)$$

Left multiplying $\frac{1}{G}(\mathbf{W}_p^H \mathbf{F}_G^H \otimes \mathbf{I}_{N_R})$ to $\mathbf{y}_{p,n}$ yields

$$\bar{\mathbf{y}}_{p,n} = (\mathbf{I}_{L+1} \otimes \boldsymbol{\varphi}(p,n)^T \otimes \mathbf{I}_{N_R})\mathbf{h}(n) + \boldsymbol{\upsilon}_{p,n} \quad (10)$$

where $\boldsymbol{\upsilon}_{p,n} = \frac{1}{G}(\mathbf{W}_p^H \mathbf{F}_G^H \otimes \mathbf{I}_{N_R})[(\mathbf{B}_{p,n} \otimes \mathbf{I}_{N_R})\mathbf{h}(n) + \mathbf{z}_{p,n}]$. The auto-correlation matrix of $\boldsymbol{\upsilon}_{p,n}$ can be derived as

$$\mathbf{R}_{\upsilon} = E\{\boldsymbol{\upsilon}_{p,n}\boldsymbol{\upsilon}_{p,n}^H\} = \frac{1}{G}(\mathbf{I}_{L+1} \otimes \mathbf{\Omega} + \sigma_z^2 \mathbf{I}_{(L+1)N_R}), \quad (11)$$

where matrix $\mathbf{\Omega} = (\rho_d N_T \sum_u \sum_l \sigma_{u,l}^2 \mathbf{R}_{Bs}^l)$.

2. LS Channel Estimator

Further assuming channel variation is negligible within $N$ OFDM symbol periods, the index $n$ of $\mathbf{h}(n)$ can be omitted. We then stack $\bar{\mathbf{y}}_{p,n}$ , $\forall p,n,$ and obtain a $PNN_R(L+1)\times 1$ vector $(\bar{\mathbf{y}}_{0,0}^T,\cdots,\bar{\mathbf{y}}_{P-1,0}^T,\cdots,\bar{\mathbf{y}}_{0,N-1}^T,\cdots,\bar{\mathbf{y}}_{P-1,N-1}^T)^T$, which can be reshaped to a vector:

$$\hat{\mathbf{y}} = (\mathbf{I}_{L+1} \otimes \mathbf{\Phi} \otimes \mathbf{I}_{N_R})\mathbf{h} + \boldsymbol{\upsilon}, \tag{12}$$

where $\mathbf{\Phi}$ is obtained from stacking of $\boldsymbol{\varphi}(p,n)^T$ and $\boldsymbol{\upsilon}$ corresponds to $\boldsymbol{\upsilon}_{p,n}$. So far, the linear LS channel estimate can be obtained as

$$\hat{\mathbf{h}} = \{\mathbf{I}_{L+1} \otimes [(\mathbf{\Phi}^H\mathbf{\Phi})^{-1}\mathbf{\Phi}^H] \otimes \mathbf{I}_{N_R}\}\hat{\mathbf{y}}. \tag{13}$$

The MSE matrix is defined as the auto-correlation matrix of the channel estimate error $\tilde{\mathbf{h}} = \mathbf{h} - \hat{\mathbf{h}}$, that is, $\tilde{\mathbf{R}}_{\mathbf{h}} = E\{\tilde{\mathbf{h}}\tilde{\mathbf{h}}^{\mathrm{H}}\}$:

$$\tilde{\mathbf{R}}_{\mathbf{h}} = \frac{1}{G}[\mathbf{I}_{L+1} \otimes (\mathbf{\Phi}^H\mathbf{\Phi})^{-1} \otimes (\mathbf{\Omega} + \sigma_z^2 \mathbf{I}_{N_R})]. \tag{14}$$

3. Optimal Pilot Symbols

In this subsection, optimal multiuser pilot symbols are proposed with minimizing the trace of the MSE matrix, denoted as $\mathrm{Tr}(\tilde{\mathbf{R}}_{\mathbf{h}})$. By SVD, let $\mathbf{\Omega} = \mathbf{V}\mathbf{\Lambda}_{\mathbf{\Omega}}\mathbf{V}^H$. Thus,

$$\mathrm{Tr}(\tilde{\mathbf{R}}_{\mathbf{h}}) = \frac{L+1}{G}\mathrm{Tr}((\mathbf{\Phi}^H\mathbf{\Phi})^{-1}) \cdot \mathbf{V}\,\mathrm{Tr}(\mathbf{\Lambda}_{\mathbf{\Omega}} + \sigma_z^2\mathbf{I}_{N_R})\mathbf{V}^H. \tag{15}$$

Based on the majorization theory, minimization of $\mathrm{Tr}(\tilde{\mathbf{R}}_{\mathbf{h}})$ only requires designing the matrix $\mathbf{\Phi}^H\mathbf{\Phi}$ diagonally.

**Proposition.** The optimal multiuser pilot symbols are proposed as

$$\phi_{u,t}(p,n) = \sqrt{\rho_c}\exp\{-j2\pi[(t-1)+(u-1)N_T](n+pN)/PN\}$$
$$\text{for } u \in [1,U], t \in [1,N_T], p \in [0,P-1] \text{ and } n \in [0,N-1]. \tag{16}$$

*Proof.* The $(u,u')$th inner block matrix of $\mathbf{\Phi}^H\mathbf{\Phi}$ is

$$[\mathbf{\Phi}^H\mathbf{\Phi}]_{u,u} = \sum_{n=0}^{N-1}\sum_{p=0}^{P-1}\mathbf{X}_u(p,n)^H\mathbf{X}_{u'}(p,n), (N_T \times N_T), \tag{17}$$

where the $(t,t')$th element of $\mathbf{X}_u(p,n)^H\mathbf{X}_{u'}(p,n)$ is $\phi_{u,t}^*(p,n)\phi_{u',t'}(p,n)$. With the proposition, we obtain

$$\sum_{n=0}^{N-1}\sum_{p=0}^{P-1}\phi_{u,t}^*(p,n)\phi_{u',t'}(p,n) = \rho_c\sum_{n=0}^{N-1}\sum_{p=0}^{P-1}e^{j2\pi[N_T(u-u')+(t+t')](n+pN)/PN}. \tag{18}$$

Let us constrain that $PN \ge UN_T$. Equation (18) results in zero in any cases of $\{u \ne u'\}$ or $\{t \ne t'\}$; if and only if the two couples satisfy $\{u = u'\}$ and $\{t = t'\}$ jointly, (18) is nonzero $(PN\rho_c)$. Here, $u,u' \in [1,U]$ and $t,t' \in [1,N_T]$. This demonstrates that $\mathbf{\Phi}^H\mathbf{\Phi}$ is diagonal.

Accordingly, the MSE matrix can be rewritten as

$$\tilde{\mathbf{R}}_{\mathbf{h}} = \frac{1}{KN\rho_c}[\mathbf{I}_{UN_T(L+1)} \otimes (\mathbf{\Omega} + \sigma_z^2\mathbf{I}_{N_R})]. \tag{19}$$

## IV. Sum-Capacity Lower Bound

Let us form a matrix $\mathbf{\Pi}(k,n) = (\mathbf{\Pi}_1(k,n),\cdots,\mathbf{\Pi}_U(k,n))$ as in (3). Once $\hat{\mathrm{h}}$ is obtained, the $\hat{\mathbf{\Pi}}(k,n)$ (estimate of $\mathbf{\Pi}$ $(k, n)$) can be computed using (3). As the distribution of $\mathbf{\Pi}$ $(k, n)$ is independent of indices $k$ or $n$, it is reasonable to assume that the distribution of $\hat{\mathbf{\Pi}}(k,n)$ is independent of indices $k$ or $n$ as well. Thus, we assume a random matrix $\hat{\mathbf{\Psi}}$ $(N_R\times UN_T)$ which has the same distribution as $\hat{\mathbf{\Pi}}(k,n)$:

$$\hat{\mathbf{\Psi}} \sim \hat{\mathbf{\Pi}}(k,n). \tag{20}$$

For SIP scheme with the estimated channel, we rewrite (7) to

$$\begin{aligned}\mathbf{y}_{p,n} &= (\mathbf{B}_{p,n} \otimes \mathbf{I}_{N_R})\hat{\mathbf{h}} + (\mathbf{A}_{p,n} \otimes \mathbf{I}_{N_R})\hat{\mathbf{h}} + \mathbf{v}_{p,n},\\ \mathbf{v}_{p,n} &= (\mathbf{A}_{p,n} \otimes \mathbf{I}_{N_R})\tilde{\mathbf{h}} + (\mathbf{B}_{p,n} \otimes \mathbf{I}_{N_R})\tilde{\mathbf{h}} + \mathbf{z}_{p,n}.\end{aligned} \tag{21}$$

In addition to the LS channel estimate, it gives

$$\begin{aligned}\mathbf{R}_{\mathbf{v}} &= E\{\mathbf{v}_{p,n}\mathbf{v}_{p,n}^H\}\\ &= \sigma_z^2\mathbf{I}_{GN_R} + \frac{UN_T(L+1)}{KN\rho_c}[\frac{2\rho_c\sigma_\eta^2}{L+1}(\mathbf{F}_G\mathbf{F}_G^H)\otimes\mathbf{I}_{N_R}\\ &\quad + (\rho_d\mathbf{I}_G + \frac{\rho_c}{L+1}\mathbf{F}_G\mathbf{F}_G^H) \otimes (\mathbf{\Omega} + \sigma_\eta^2\mathbf{I}_{N_R})].\end{aligned} \tag{22}$$

The averaged sum capacity lower bound is derived in [1] as

$$C_{si} = \frac{P}{K}E\{\log\det(\mathbf{I}_{GN_R} + \rho_d\mathbf{R}_{\mathbf{v}}^{-1}(\mathbf{I}_G \otimes \hat{\mathbf{\Psi}}_{si}\hat{\mathbf{\Psi}}_{si}^H)\}, \tag{23}$$

where $\hat{\mathbf{\Psi}}_{si}$ denotes the $\hat{\mathbf{\Psi}}$ in (20) under the SIP scheme.

For FMP schemes, the LS channel estimate's MSE matrix is $\sigma_z^2/QN\rho_s\mathbf{I}_{UN_RN_T(L+1)}$ [4], where $Q$ is the number of pilot tones within one OFDM symbol. Unlike the requirement for $PN \ge UN_T$ in the SIP scheme, $QN \ge UN_T(L+1)$ is necessary for the FMP method. Given the estimated channel, we can derive the sum-capacity lower bound as

$$C_{fm} = \frac{K-Q}{K}\log\det(\mathbf{I}_{N_R} + \frac{\rho_s}{\mu}\hat{\mathbf{\Psi}}_{fm}\hat{\mathbf{\Psi}}_{fm}^H), \tag{24}$$

where $\mu = (\frac{UN_R N_T(L+1)}{N_R QN}+1)\sigma_z^2$; $\hat{\mathbf{\Psi}}_{fm}$ denotes the $\hat{\mathbf{\Psi}}$ in the case of the FMP. According to [1], maximization of the lower bound $C_{fm}$ requires $QN=UN_T(L+1)$.

## V. Simulations

The simulation parameters were selected as: $K=1024$, $N=2$, $U=2$, $L=3$, $N_T=2$, and $N_R=4$. A simple spatial correlation case was considered: $\mathbf{R}_{Tx}^{u,l}=[1, 0.2; 0.2, 1]$, $\forall u, l$, and $[\mathbf{R}_{Bs}^{l}]_{r,r'}=0.35^{|r-r'|}$, $\forall l$ and $r, r' \in [1, N_R]$. The averaged SNR was defined as $SNR = 10\log((\rho_s N_T/\sigma_z^2)\cdot\sum_{u,l}\sigma_{u,l}^2)$ and the normalized channel estimate's MSE as $MSE = \mathrm{Tr}(\tilde{\mathbf{R}}_{\mathbf{h}})/\mathrm{Tr}(\mathbf{R}_{\mathbf{h}})$. We chose $Q=8$.

### 1. Performance

It is shown in Fig. 3 that for the SIP scheme, better BER performance is obtained for a larger $\alpha$ value. The SIP approach outperforms the FMP one on a low SNR regime, but it performs worse for high SNRs. Thus the adoption of the SIP scheme is preferred in the case of low SNRs.

As the SNR increases, the unknown-data effects become significant and thus the slope of the BER curve becomes smoother than that of the FMP scheme, resulting in a cross point at the BER slightly greater than $10^{-3}$. If the data effects can be further mitigated, for example by resorting to an iterative algorithm to reduce the trace of $\mathbf{\Omega}$ or by increasing the pilot power, a sharper curve-slope is expected for the SIP scheme. This makes it feasible for practical applications.

### 2. Spectral Efficiency.

Figure 4 shows that on a low SNR regime, the proposed SIP scheme achieves a higher capacity bound than the FMP case. As the SNR increases, the bound of the SIP scheme increases more slowly than that of the FMP scheme, and finally, the FMP scheme surpasses the SIP scheme on a high SNR regime. Therefore, from the spectral-efficiency point of view, the proposed SIP scheme is still preferred for the low SNRs. For the SIP scheme on a low SNR regime, the smaller $\alpha$ the higher capacity bound is obtained.

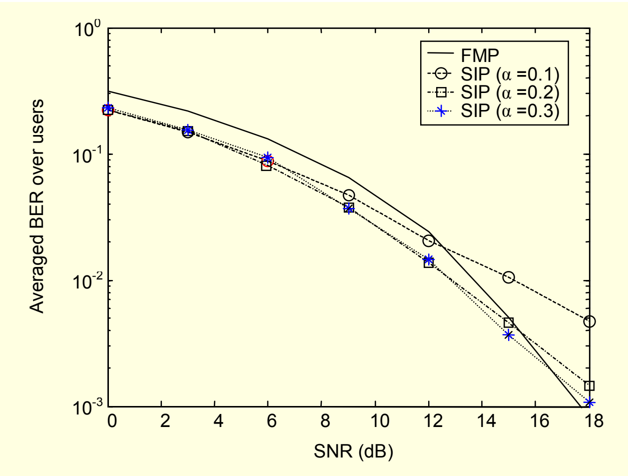


Fig. 3. BER performance against SNR.

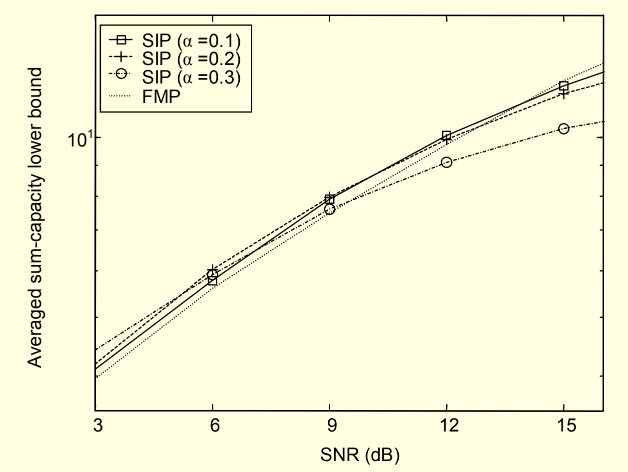


Fig. 4. Averaged sum-capacity lower bound against SNR.

## VI. Conclusions

A novel multiuser channel estimation approach based on SIP was proposed. A new combining algorithm was developed to reduce the embedded-data effects. With the minimization of the channel estimate's MSE, optimal pilot symbols were designed. In contrast to the FMP scheme, our simulations showed that the SIP scheme benefits not only BER performance but also spectral efficiency on low SNR regimes.

## References


[1] X. Ma, L. Yang, and G. B. Giannakis, "Optimal Training for MIMO Frequency-Selective Fading Channels," *IEEE Tran. on Wireless Comm.*, vol. 4, Mar. 2005, pp. 453-466.

[2] M. Ghogho, D.McLernon, E. A. Hernandez, and A. Swami, "Channel Estimation and Symbol Detection for Block Transmission Using Data-Dependent Superimposed Training," *IEEE Signal Processing Letters*, vol. 12, Mar. 2005, pp. 226-229.

[3] J.K.Tugnait and W. Luo, "On Channel Estimation Using Superimposed Training and First Order Statistics," *IEEE Comm. Letters*, vol. 7, Sept. 2003, pp.413-415.

[4] Q. Yang, H. Zhu, J. K. Lim, and K. S. Kwak, "Pilot-Aided Channel Estimation for Multiuser MIMO-OFDM Systems," *ITC-CSCC 2005*, Jeju, Korea, vol. 3, July 2005, pp. 1195-1196.