

# A Full Rate Quasi-orthogonal STF-OFDM with DAC-ZF Decoder over Wireless Fading Channels

Jiyu Jin, Kwan-Woong Ryu, and Yongwan Park

**ABSTRACT**—In this letter, we propose a quasi-orthogonal space-time-frequency (QOSTF) block coded orthogonal frequency division multiplexing (OFDM) that can achieve full symbol rate with four transmit antennas. Since the proposed QOSTF-OFDM cannot achieve full diversity, we use a diversity advantage collection with zero forcing (DAC-ZF) decoder to compensate the diversity loss at the receiving side. Due to modulation advantage and collected diversity advantage, the proposed scheme exhibits a better bit-error rate performance than other orthogonal schemes.

**Keywords**— MIMO, STBC, OFDM, STFBC, QOSTF-OFDM.

## I. Introduction

Space-time block coded orthogonal frequency division multiplexing (ST-OFDM) suffers performance degradation in environments with high Doppler frequency [1]. On the other hand, performance of space-frequency block coded OFDM (SF-OFDM) degrades in heavily frequency-selective channels [2]. Therefore, a space-time-frequency block coded OFDM (STF-OFDM) [3] was proposed to distribute the elements of the orthogonal design both in time and frequency in order to relax the requirements for constant channel coefficients in both dimensions. This is particularly important for numbers of transmit antennas greater than two, where the channel coefficients have to be constant over four symbols. However, with a transmit dimension beyond 2, only 1/2 or 3/4 rates can be achieved by orthogonal codes over complex constellations [4]. It should be pointed out that [1] through [4] assume that

channels remain static over the length of the codeword. This assumption is not valid for time selective fading or frequency selective fading channels. In this letter, we propose a full-rate quasi-orthogonal STF-OFDM (QOSTF-OFDM). Both the time selective fading and frequency selective fading channel models are considered.

## II. Design of Full-Rate QOSTF-OFDM

The design principle of the orthogonal space-time block code was provided by [4]. With four transmit antennas, the space-time block code was applied to the OFDM by [3]. Based on [3], the 1/2 rate orthogonal space-time block code  $C_{1/2}$  [4] can be used to construct a 1/2 rate STF-OFDM as shown in Figs.1 and 2. Figure 1 shows an STF-OFDM where four information symbols are distributed in four time slots and two tones (T4F2). Figure 2 shows another STF-OFDM where four information symbols are distributed in two time slots and four tones (T2F4). Since with four transmit antennas the full symbol rate cannot be achieved by an orthogonal code over complex

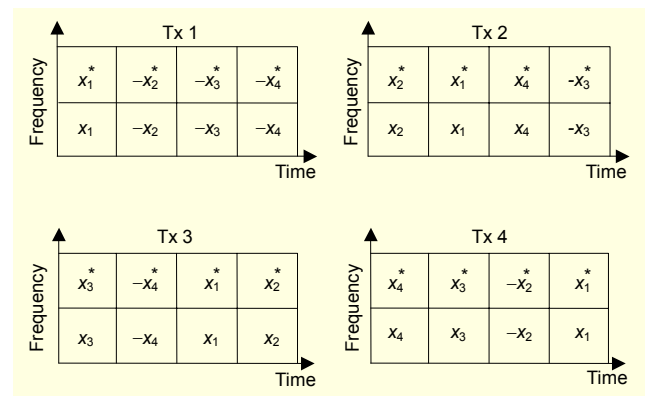


Fig. 1. Coding scheme of the STF-OFDM with  $C_{1/2}$  (T4F2).

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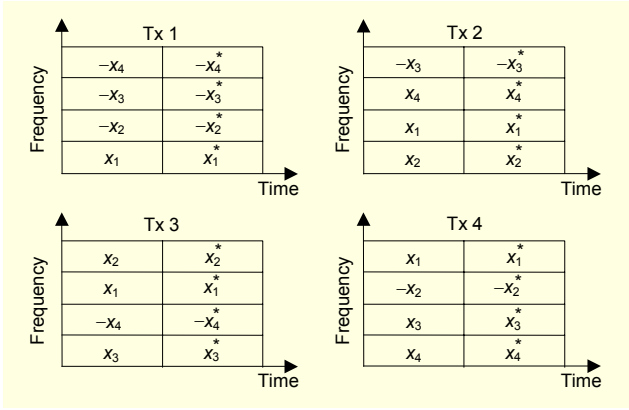


Fig. 2. Coding scheme of the STF-OFDM with  $C_{1/2}$  (T2F4).

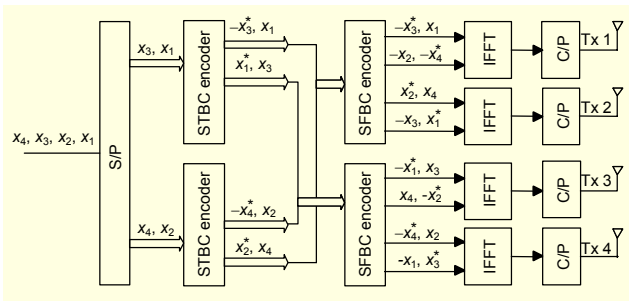


Fig. 3. The proposed QOSTF-OFDM.

constellations, in this section we propose a full-rate QOSTF-OFDM with four transmit antennas. Figure 3 shows an example of the proposed QOSTF-OFDM with four transmit antennas and two tones.

We consider a system with  $N_T$  transmit antennas and  $N_R$  receive antennas. At the transmitter, the input data symbols are modulated and serial-to-parallel converted. The modulated symbols are encoded using two space-time encoders. The outputs of the space-time encoders are interchanged and encoded using two space-frequency encoders. The space-time-frequency coded matrix can be given as

$$X = \begin{bmatrix} x_1(t_k, f_k) & x_4(t_k, f_k) & x_3(t_k, f_k) & x_2(t_k, f_k) \\ -x_3^*(t_{k+1}, f_k) & x_2^*(t_{k+1}, f_k) & x_1^*(t_{k+1}, f_k) & -x_4^*(t_{k+1}, f_k) \\ -x_4^*(t_k, f_{k+1}) & x_1^*(t_k, f_{k+1}) & -x_2^*(t_k, f_{k+1}) & x_3^*(t_k, f_{k+1}) \\ -x_2^*(t_{k+1}, f_{k+1}) & -x_3^*(t_{k+1}, f_{k+1}) & x_4^*(t_{k+1}, f_{k+1}) & x_1^*(t_{k+1}, f_{k+1}) \end{bmatrix}, \quad (1)$$

where  $x_i(t_k, f_k)$  denotes the symbol  $x_i$  ( $i=1, \dots, N_T$ ), which is transmitted on the  $f_k$ -th tone at time  $t_k$ . In (1), rows one and three are transmitted simultaneously with different tones, and rows two and four are transmitted simultaneously with different tones. Since four information symbols are transmitted during two successive time slots and two successive tones, the symbol rate is one. Note that column one is orthogonal to

columns two and three, and column four has the same property as column one. However, columns two and three, and similarly columns one and four, are not orthogonal to each other, that is, the proposed space-time-frequency block code is a quasi-orthogonal code. The space-time-frequency coded symbols are modulated by inverse fast Fourier transform (IFFT) into OFDM symbols. After adding cyclic prefixes, the OFDM symbols are transmitted. At the receiver, after removing the cyclic prefixes and FFT, the received signal vector  $r$  on the  $f_k$ -th and  $f_{k+1}$ -th tones at times  $t_k$  and  $t_{k+1}$  can be given as

$$r = Hx + n, \quad (2)$$

where the channel matrix  $H$  is

$$H = \begin{bmatrix} h_1(t_k, f_k) & h_4(t_k, f_k) & h_3(t_k, f_k) & h_2(t_k, f_k) \\ h_3^*(t_{k+1}, f_k) & -h_2^*(t_{k+1}, f_k) & -h_1^*(t_{k+1}, f_k) & -h_4^*(t_{k+1}, f_k) \\ h_4^*(t_k, f_{k+1}) & -h_1^*(t_k, f_{k+1}) & h_2^*(t_k, f_{k+1}) & -h_3^*(t_k, f_{k+1}) \\ -h_2(t_{k+1}, f_{k+1}) & -h_3(t_{k+1}, f_{k+1}) & h_4(t_{k+1}, f_{k+1}) & h_1(t_{k+1}, f_{k+1}) \end{bmatrix},$$

and  $h_i(t_k, f_k)$  ( $i=1, \dots, N_T$ ) denotes the channel frequency response for the  $f_k$ -th tone at time  $t_k$ . For simplicity, we assume  $N_R=1$ . Also,  $x$  is the code vector  $[x_1, x_2, x_3, x_4]^T$ , and  $[\cdot]^T$  denotes the transpose of matrix  $[\cdot]$ . The additive white Gaussian noise vector  $n$  is  $[n(t_k, f_k), n(t_{k+1}, f_k), n(t_k, f_{k+1}), n(t_{k+1}, f_{k+1})]^T$ . When the channel is time selective or frequency selective, the channel coefficient varies in the time domain or frequency domain, and we then have

$$R = H^H H = \begin{bmatrix} g_1 & g_2 \\ g_3 & g_4 \end{bmatrix} = \begin{bmatrix} a_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & a_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & a_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & a_{44} \end{bmatrix}, \quad (3)$$

where  $g_1 = \begin{bmatrix} a_{11} & b_{12} \\ b_{21} & a_{22} \end{bmatrix}$ ,  $g_2 = \begin{bmatrix} b_{13} & b_{14} \\ b_{23} & b_{24} \end{bmatrix}$ ,  $g_3 = \begin{bmatrix} b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix}$ ,

$g_4 = \begin{bmatrix} a_{33} & b_{34} \\ b_{43} & a_{44} \end{bmatrix}$ ,  $H^H$  denotes the Hermitian of channel matrix  $H$ ,

and  $b_{ij}$  ( $i=1, \dots, 4$  and  $j=1, \dots, 4$ ) with zero mean can be considered as interference. If channel coefficients are static both in time and frequency domain, that is,

$$h_i(t_k, f_k) = h_i(t_k, f_{k+1}) = h_i(t_{k+1}, f_k) = h_i(t_{k+1}, f_{k+1}) = h_i, \quad (4)$$

where  $i=1, \dots, N_T$ , it implies that the channel matrix is quasi-

orthogonal, and (3) can be written as

$$\mathbf{R} = \mathbf{H}^H \mathbf{H} = \begin{bmatrix} a & 0 & 0 & b \\ 0 & a & b & 0 \\ 0 & -b & a & 0 \\ -b & 0 & 0 & a \end{bmatrix}, \quad (5)$$

where  $a = \sum_{i=1}^4 |h_i|^2$ ,  $b = 2 \text{Im}(h_1^* h_2 + h_3 h_4^*)$ , and  $\text{Im}(\cdot)$  denotes the imaginary part of  $(\cdot)$ .

Since the proposed QOSTF-OFDM is a quasi-orthogonal code, full diversity cannot be achieved; thus, the DAC-ZF [5] decoder can be used to remove the interferences in order to provide a diversity advantage for the proposed QOSTF-OFDM at the receiver. The DAC-ZF decoder combines the signals across all receive antennas before the interference cancellation is done by the ZF technique. The combining is accomplished by premultiplying the received signal vector by  $\mathbf{H}^H$ , while interference cancellation is accomplished by premultiplying the result by a diagonalization matrix  $\Phi$ . As shown in (3),  $a_{ij}$ ,  $i=1, \dots, N_T$  and  $j=1, \dots, N_T$ , collect the diversity advantage, which grows with increasing  $N_R$ . At the same time, the summation that yields the combined interference  $b_{ij}$  is proportional to the mean, which tends to zero as  $N_R \rightarrow \infty$ . The decision statistic vector  $\mathbf{Y}$  is then computed as

$$\mathbf{Y} = \Phi \mathbf{H}^H \mathbf{r} = \mathbf{D} \mathbf{x} + \mathbf{v}, \quad (6)$$

where  $\mathbf{D} = \Phi \mathbf{H}^H \mathbf{H} = \text{diag}(d_1, d_2, d_3, d_4)$ ,  $\mathbf{D}$  is a diagonal matrix, and  $\mathbf{v} = \Phi \mathbf{H}^H \mathbf{n}$ . According to [6], the matrix  $\Phi$  that diagonalizes matrix  $\mathbf{R}$  is computed as

$$\Phi = \begin{bmatrix} \theta_1 & \theta_2 \\ \theta_3 & \theta_4 \end{bmatrix}, \quad (7)$$

where  $\theta_1 = -\theta_2 g_4 g_2^{-1}$ ,  $\theta_4 = -\theta_3 g_1 g_3^{-1}$ ,  $\theta_2 = \text{DT}(\mathbf{x}_1)$ ,  $\theta_3 = \text{DT}(\mathbf{x}_2)$ ,  $\mathbf{x}_1 = g_3 - g_4 g_2^{-1} g_1$ ,  $\mathbf{x}_2 = g_2 - g_1 g_3^{-1} g_4$ , and  $\text{DT}(z) = \begin{bmatrix} -z_{31} & z_{12} \\ z_{21} & -z_{11} \end{bmatrix}$

for any  $2 \times 2$  matrix  $z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$ . Also, it can be shown

that  $d_1 = d_2 = -\det(\mathbf{x}_1)$ , and  $d_3 = d_4 = -\det(\mathbf{x}_2)$ , where  $\det(z)$  is the determinant of  $z$ .

The detected signal can be written as

$$\hat{s}_i = \arg \left\{ \min_{x_k \in \mathbb{N}} |[\mathbf{Y}]_i - d_i x_k|^2 \right\}, \quad i = 1, \dots, 4, \quad (8)$$

where  $\mathbb{N}$  is the symbol alphabet and  $[\mathbf{Y}]_i$  is the  $i$ -th row of  $\mathbf{Y}$ .

Note that the proposed QOSTF-OFDM only requires complex symbol-wise decoding, that is, the transmitted symbols can be decoded separately, not jointly. Thus, the decoding complexity increases linearly, not exponentially, with the code size.

### III. Simulation Results

In this section, we demonstrate the performance through computer simulations. An OFDM system with four transmit antennas and four receive antennas is used. The available channel bandwidth is 101.5 MHz, which is divided into 768 tones. 1024-point IFFT is considered in our simulations. A cyclic prefix of  $1.674 \mu\text{s}$  is appended to each frame. We assume that perfect channel state information is available at the receiver. For transmission of ST/SF/STF-OFDM, 16-QAM, and  $C_{1/2}$  are used. Since the symbol rate of the proposed QOSTF-OFDM is one, 4QAM is employed to achieve the same frequency efficiency (2bits/s/Hz) as ST/SF/STF-OFDM. Therefore, compared with ST/SF/STF-OFDM, low level modulation can be employed by the proposed scheme due to its full rate, that is, a modulation advantage can be achieved. At the receiver side, we employ the DAC-ZF decoder to collect the diversity advantage and eliminate interference effects. A key step in the DAC-ZF decoder design is to find the expression of the equivalent channel matrix in relation to the space-time code matrix. This is a straightforward operation in the case of the 1/2 rate space-time code, but not quite so in the case of 3/4 codes. However, the DAC-ZF decoder cannot be used to decode such 3/4 rate STF-OFDM because it is difficult to find the equivalent channel matrix. For convenience, we construct two 1/2 rate STF-OFDMs as shown in Figs. 1 and 2 and compare their performance with the proposed QOSTF-OFDM.

Figure 4 shows the BER versus SNR on quasi-static fading channels (single path) at 2 b/s/Hz. For the single path quasi-static fading channel, the fading coefficients are constant during transmission of the ST/SF/STF-OFDM matrix, and the orthogonality of ST/SF/STF-OFDM can be maintained. Therefore, ST/SF/STF-OFDM show similar performances. Because we can keep the orthogonality of ST/SF/STF-OFDM

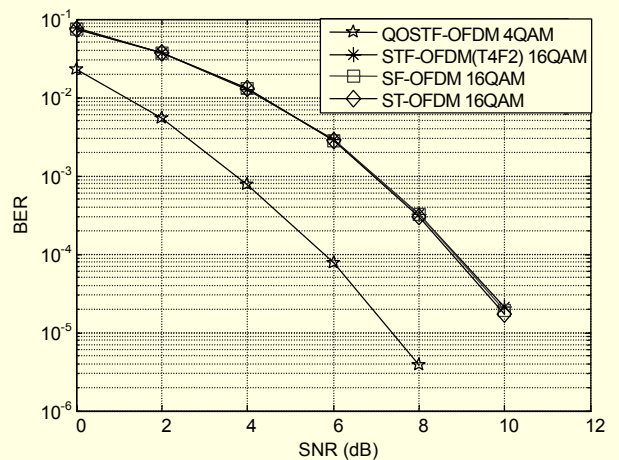


Fig. 4. BER versus SNR at 2 b/s/Hz.

in single quasi-static fading channels, compared with the conventional decoder, the DAC-ZF decoder has no effect. However, the DAC-ZF decoder can mitigate the nonorthogonality effects that are caused by the code structure of the proposed scheme, and about a 3 dB gain can be obtained by the proposed QOSTF-OFDM at the BER of  $10^{-3}$  compared with ST/SF/STF-OFDM.

Figure 5 shows the BER versus normalized Doppler frequency  $f_d T_s$  at SNR = 8 dB under time selective Rayleigh fading channels where  $f_d$  and  $T_s$  denote the Doppler frequency and OFDM symbol duration, respectively. The proposed scheme obtains a better performance than the other schemes because it gets more modulation advantage than the other schemes. Although the proposed scheme suffers diversity loss caused by both code structure and high Doppler shift, the DAC-ZF decoder can compensate the diversity loss. To

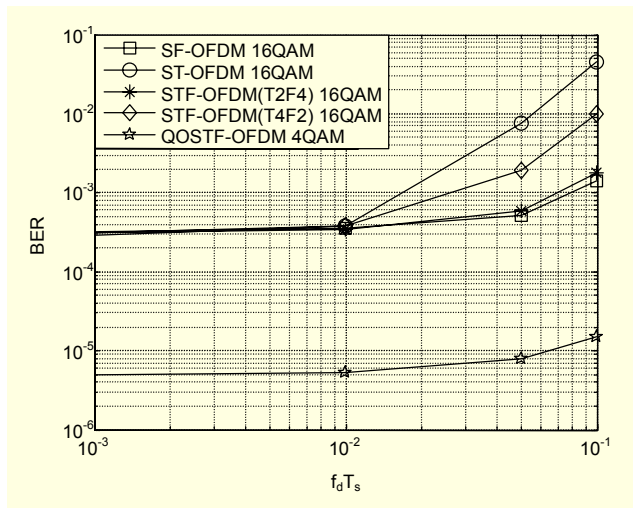


Fig. 5. BER versus the normalized Doppler frequency.

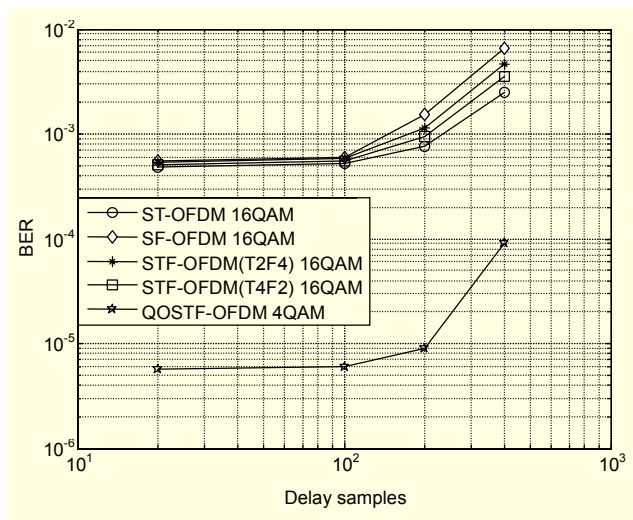


Fig. 6. BER versus delay time.

maintain the orthogonality, the channel coefficients of SF-OFDM, STF-OFDM (T2F4), STF-OFDM (T4F2), and ST-OFDM should be constant during 1, 2, 4, and 8 time slots, respectively. Therefore, the performance degrades as the number of time slots increases for these four schemes.

Figure 6 shows the BER versus delay time at SNR = 8 dB. The 6-path Rayleigh fading with exponential decaying and time nonselective fading channel model is considered. Also the proposed scheme gets better performance than those of other schemes because of its modulation advantage and collected diversity advantage. When the delay spread is small, the channel gains are almost constant on the successive tones, and the condition of the orthogonality for all schemes can almost be satisfied. However, when the delay spread is large, the BER performances of conventional STF-OFDM and SF-OFDM are worse than that of the ST-OFDM.

#### IV. Conclusions

We have introduced a QOSTF-OFDM with the DAC-ZF decoder for four transmit antennas. The proposed scheme exhibits a better BER performance than other orthogonal schemes due to its modulation advantage and collected diversity advantage. Furthermore, the DAC-ZF decoder gets low decoding complexity due to its symbol-wise decoding, especially when high level modulation is used.

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