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Weighted Sharing of Two Sets

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Abstract. Using the notion of weighted sharing of sets we improve two results of H. X. Yi on uniqueness of meromorphic functions.

1. Introduction, definitions and results

Let f and g be two nonconstant meromorphic functions defined in the open complex plane $\mathbb{C}$. If for some $a \in \mathbb{C} \cup \{\infty\}$, f and g have the same set of a-points with same multiplicities then we say that f and g share the value a CM (counting multiplicities). If we do not take the multiplicities into account, f and g are said to share the value a IM (ignoring multiplicities).

Let S be a set of distinct elements of $\mathbb{C} \cup \{\infty\}$ and $E_f(S) = \cup_{a \in S}\{z : f(z) - a = 0\}$, where each zero is counted according to its multiplicity. If we do not count the multiplicity the set $\cup_{a \in S}\{z : f(z) - a = 0\}$ is denoted by $\overline{E}_f(S)$.

In the paper we denote by S_1 and S_2 the following sets $S_1 = \{1, \omega, \omega^2, \cdots, \omega^{n-1}\}$ and $S_2 = \{\infty\}$, where $\omega = cos\frac{2\pi}{n} + isin\frac{2\pi}{n}$ and n is a positive integer.

Yi ([6], [8]), Song-Li ([5]) and other authors investigate the problem of uniqueness of two meromorphic functions f, g for which $E_f(S_i) = E_g(S_i)$ or $\overline{E}_f(S_i) = \overline{E}_g(S_i)$, where $i = 1,\ 2$.

In 1997 H. X. Yi and L. Z. Yang proved the following two results.

Theorem A ([10]). *Let f and g be two nonconstant meromorphic functions such that $E_f(S_1) = E_g(S_1)$ and $\overline{E}_f(S_2) = \overline{E}_g(S_2)$. If $n \geq 6$ then one of the following hold*:

(1) $$f \equiv tg,$$

where $t^n = 1$,

(2) $$f.g \equiv s,$$

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where $s^n = 1$ *and* 0, ∞ *are lacunary values of* f *and* g.

Theorem B ([10]). *Let* f *and* g *be two nonconstant meromorphic functions such that* $\overline{E}_f(S_1) = \overline{E}_g(S_1)$ *and* $E_f(S_2) = E_g(S_2)$. *If* $n \geq 10$ *then* f *and* g *satisfy* (1) *or* (2).

In the paper, we investigate the possibility of improving Theorem A and B by relaxing the nature of sharing the sets. To this end we employ the idea of weighted sharing of values and sets introduced in [2], [3] which measures how close a shared value is to being shared IM or to being shared CM. In the following definition we explain this notion.

Definition 1 ([2], [3]). Let k be a nonnegative integer or infinity. For $a \in \mathbb{C} \cup \{\infty\}$ we denote by $E_k(a; f)$ the set of all a-points of f, where an a-point of multiplicity m is counted m times if $m \leq k$ and $k+1$ times if $m > k$. If $E_k(a; f) = E_k(a; g)$, we say that f, g share the value a with weight k.

We write f, g share (a, k) to mean that f, g share the value a with weight k. Clearly if f, g share (a, k) then f, g share (a, p) for any integer p, $0 \leq p < k$. Also we note that f, g share a value a IM or CM if and only if f, g share $(a, 0)$ or (a, ∞) respectively.

Definition 2 ([3]). Let S be a set of distinct elements of $\mathbb{C} \cup \{\infty\}$ and k be a nonnegative integer or ∞. We denote by $E_f(S, k)$ the set $\cup_{a \in S} E_k(a; f)$.

Clearly $E_f(S) = E_f(S, \infty)$ and $\overline{E}_f(S) = \overline{E}_f(S, 0)$.

We now state the main results of the paper.

Theorem 1. *If* $E_f(S_1, 2) = E_g(S_1, 2)$, $E_f(S_2, 0) = E_g(S_2, 0)$ *and* $n \geq 6$ *then* f, g *satisfy one of* (1) *and* (2).

Theorem 2. *If* $E_f(S_1, 0) = E_g(S_1, 0)$, $E_f(S_2, 3) = E_g(S_2, 3)$ *and* $n \geq 10$ *then* f, g *satisfy one of* (1) *and* (2).

Though for the standard definitions and notations of the value distribution theory we refer to [1], we now explain some notations which are used in the paper.

Definition 3 ([2], [3]). We denote by $N(r, a; f| = 1)$ the counting function of simple a-points of f.

Definition 4 ([2], [3]). If s is a positive integer, we denote by $\overline{N}(r, a; f| \geq s)$ the reduced counting function of those a points of f whose multiplicities are not less than s.

Definition 5 ([2], [3]). Let f, g share a value a IM. We denote by $\overline{N}_*(r, a; f, g)$ the reduced counting function of those a-points of f whose multiplicities differ from the multiplicities of the corresponding a-points of g.

Clearly $\overline{N}_*(r, a; f, g) \equiv \overline{N}_*(r, a; g, f)$.

Definition 6 ([10]). Let f, g share a value a IM. Let z_0 be an a-point of f with multiplicity p and an a-point of g with multiplicity q. We denote by $\overline{N}_L(r,a;f)$ the reduced counting function of those a-points of f where $p>q$ and by $N_E^{1)}(r,a;f)$ the counting function of those a-points of f where $p=q=1$. Also by $N_E^{(2}(r,a;f)$ we denote the counting function of those a-points of f where $p=q\geq 2$.

$\overline{N}_L(r,a;g)$, $N_E^{1)}(r,a;g)$ and $N_E^{(2}(r,a;g)$ are defined analogously.

Clearly $\overline{N}_*(r,a;f,g)=\overline{N}_L(r,a;f)+\overline{N}_L(r,a;g)$.

Definition 7. Let $a,\ b\in\mathbb{C}\cup\{\infty\}$. We denote by $N(r,a;f|\ g=b)$ the counting function of those a-points of f, counted according to multiplicity, which are b-points of g.

Definition 8. Let $a,b_1,b_2,\cdots,b_q\in\mathbb{C}\ \cup\{\infty\}$. We denote by $N(r,a;f|\ g\neq b_1,b_2,\cdots,b_q)$ the counting function of those a-points of f, counted according to multiplicity, which are not the b_i-points of g for $i=1,2,\cdots,q$.

2. Lemmas

In this section we present some lemmas which will be needed in the sequel. Let F and G be two nonconstant meromorphic functions defined in $\mathbb{C}$. Henceforth we shall denote by H and V the following two functions

$$H=\left(\frac{F''}{F'}-\frac{2F'}{F-1}\right)-\left(\frac{G''}{G'}-\frac{2G'}{G-1}\right).$$

and

$$V=\left(\frac{F'}{F-1}-\frac{F'}{F}\right)-\left(\frac{G'}{G-1}-\frac{G'}{G}\right)=\frac{F'}{F(F-1)}-\frac{G'}{G(G-1)}.$$

Lemma 1 ([10]). *If F, G share $(1,0)$ and $H\not\equiv 0$ then*

$$N_E^{1)}(r,1;F)\leq N(r,H)+S(r,F)+S(r,G).$$

Lemma 2 ([4]). *The following holds*

$$N(r,0;F'|F\neq 0)\leq \overline{N}(r,\infty;F)+\overline{N}(r,0;F)+S(r,F).$$

Lemma 3. *If F and G share $(1,0)$ then*

$$\begin{aligned}T(r,F)\ \leq\ & N_E^{1)}(r,1;F)+2\,\overline{N}(r,0;F)+2\,\overline{N}(r,\infty;F)+\overline{N}(r,0;G)\\ &+\overline{N}(r,\infty;G)-2\,N_0(r,0;F')-N_0(r,0;G')+S(r,F)+S(r,G),\end{aligned}$$

where $N_0(r,0;F')$ is the counting function of those zeros of F' which are not the zeros of $F(F-1)$ and $N_0(r,0;G')$ is similarly defined.

Proof. In view of Lemma 2 we get

$$\begin{aligned}\overline{N}(r,1;F) &= N_E^{1)}(r,1;F)+\overline{N}_L(r,1;F)+\overline{N}_E^{(2}(r,1;F)+\overline{N}_L(r,1;G)\\ &\le N_E^{1)}(r,1;F)+\overline{N}(r,1;F\mid\ge 2)+\overline{N}(r,1;G\mid\ge 2)\\ &\le N_E^{1)}(r,1;F)+N(r,0;F'\mid F=1)+N(r,0;G'\mid G=1)\\ &\le N_E^{1)}(r,1;F)+N(r,0;F'\mid F\neq 0)+N(r,0;G'\mid G\neq 0)\\ &\quad -N_0(r,0;F')-N_0(r,0;G')\\ &\le N_E^{1)}(r,1;F)+\overline{N}(r,0;F)+\overline{N}(r,\infty;F)+\overline{N}(r,0;G)\\ &\quad +\overline{N}(r,\infty;G)-N_0(r,0;F')-N_0(r,0;G')+S(r,F)\\ &\quad +S(r,G).\end{aligned}$$

So by the second fundamental theorem we obtain

$$\begin{aligned}T(r,F) &\le \overline{N}(r,0;F)+\overline{N}(r,\infty;F)+\overline{N}(r,1;F)-N_0(r,0;F')+S(r,F)\\ &\le N_E^{1)}(r,1;F)+2\,\overline{N}(r,0;F)+2\,\overline{N}(r,\infty;F)+\overline{N}(r,0;G)\\ &\quad +\overline{N}(r,\infty;G)-2\,N_0(r,0;F')-N_0(r,0;G')+S(r,F)\\ &\quad +S(r,G).\end{aligned}$$

This proves the lemma. □

Lemma 4. *If F, G share $(1,0)$, $(\infty,0)$ and $H\not\equiv 0$ then*

$$\begin{aligned}N(r,H) &\le \overline{N}(r,0;F\mid\ge 2)+\overline{N}(r,0;G\mid\ge 2)+\overline{N}_*(r,1;F,G)\\ &\quad +\overline{N}_*(r,\infty;F,G)+\overline{N}_0(r,0;F')+\overline{N}_0(r,0;G'),\end{aligned}$$

where $\overline{N}_0(r,0;F')$ is the reduced counting function of those zeros of F' which are not the zeros of $F(F-1)$ and $\overline{N}_0(r,0;G')$ is similarly defined.

Proof. We can easily verify that possible poles of H occur at

(i) multiple zeros of F and G,

(ii) those poles of F and G whose multiplicities are distinct from the multiplicities of the corresponding poles of G and F respectively,

(iii) those 1-points of F and G whose multiplicities are distinct from the multiplicities of the corresponding 1-points of G and F respectively,

(iv) zeros of F' which are not the zeros of $F(F-1)$,

(v) zeros of G' which are not zeros of $G(G-1)$.

Since H has only simple poles, the lemma follows from above. This proves the lemma. □

Lemma 5 ([7]). *If $H \equiv 0$ then $T(r,G) = T(r,F) + O(1)$. Also if $H \equiv 0$ and*

$$\limsup_{\substack{r\longrightarrow\infty \\ r\in I}} \frac{\overline{N}(r,0;F)+\overline{N}(r,\infty;F)+\overline{N}(r,0;G)+\overline{N}(r,\infty;G)}{T(r,F)} < 1$$

where $I \subset (0,\infty)$ is a set of infinite linear measure, then $F \equiv G$ or $F.G \equiv 1$.

Remark 1. Let $F = f^n$ and $G = g^n$, where n (≥ 5) is an integer. If $H \equiv 0$ then from Lemma 5 it follows that f and g satisfy one of (1) and (2).

Lemma 6 ([9]). *If F, G share $(\infty,0)$ and $V \equiv 0$ then $F \equiv G$.*

Lemma 7. *Let $F = f^n$, $G = g^n$ and $V \not\equiv 0$. If f, g share (∞,k), where $0 \leq k < \infty$, then the poles of F and G are the zeros of V and*

$$(nk+n-1)\,\overline{N}(r,\infty;f|\geq k+1) \leq N(r,\infty;V) + S(r,f) + S(r,g).$$

Proof. Since f, g share (∞,k), it follows that F, G share (∞,nk) and so a pole of F with multiplicity p $(\geq nk+1)$ is a pole of G with multiplicity r $(\geq nk+1)$ and vice-versa. Noting that F and G have no pole of multiplicity q where $nk < q < nk+n$, we get from the definition of V

$$\begin{aligned}(nk+n-1)\,\overline{N}(r,\infty;f|\geq k+1) &= (nk+n-1)\,\overline{N}(r,\infty;F|\geq nk+n)\\ &\leq N(r,0;V)\\ &\leq N(r,\infty;V)+S(r,f)+S(r,g).\end{aligned}$$

□

Lemma 8. *Let $F = f^n$, $G = g^n$ and $V \not\equiv 0$. If f,g share (∞,k), where $0 \leq k < \infty$, and F, G share $(1,0)$ then*

$$\begin{aligned}(nk+n-1)\,\overline{N}(r,\infty;f|\geq k+1) \leq\; & 2\,\overline{N}(r,0;f)+2\,\overline{N}(r,0;g)+2\,\overline{N}(r,\infty;f)\\ & -N(r,0;f'|f\neq 0,1,\omega,\cdots,\omega^{n-1})\\ & -N(r,0;g'|g\neq 0,1,\omega,\cdots,\omega^{n-1})\\ & +S(r,f)+S(r,g).\end{aligned}$$

Proof. From the definition of V and Lemma 2 it follows that

$$\begin{aligned}
N(r,\infty;V) &\leq \overline{N}(r,0;F)+\overline{N}(r,0;G)+\overline{N}_*(r,1;F,G)\\
&\leq \overline{N}(r,0;F)+\overline{N}(r,0;G)+\overline{N}_L(r,1;F)+\overline{N}_L(r,1;G)\\
&\leq \overline{N}(r,0;f)+\overline{N}(r,0;g)+\overline{N}(r,1;F|\geq 2)+\overline{N}(r,1;G|\geq 2)\\
&\leq \overline{N}(r,0;f)+\overline{N}(r,0;g)+N(r,0;F^{'}\big|\,F=1)+N(r,0;G^{'}\big|\,G=1)\\
&\leq \overline{N}(r,0;f)+\overline{N}(r,0;g)+N(r,0;F^{'}\big|\,F\neq 0)+N(r,0;G^{'}\big|\,G\neq 0)\\
&\quad -N_0(r,0;F')-N_0(r,0;G^{'})\\
&\leq 2\,\overline{N}(r,0;f)+2\,\overline{N}(r,0;g)+2\,\overline{N}(r,\infty;f)-N_0(r,0;F')\\
&\quad -N_0(r,0;G^{'})+S(r,f)+S(r,g).
\end{aligned}$$

Noting that $N_0(r,0;F^{'})=N(r,0;f^{'}|\ f\neq 0,1,\omega,\cdots,\omega^{n-1})$ and $N_0(r,0;G^{'})=N(r,0;g^{'}|\,g\neq 0,1,\omega,\cdots,\omega^{n-1})$, the lemma follows from above and Lemma 7. This proves the lemma. □

Lemma 9. *Let $F=f^n$, $G=g^n$ and $V\not\equiv 0$. If f, g share $(\infty,0)$ and F, G share $(1,k)$, where $1\leq k\leq\infty$, then*

$$\begin{aligned}
(n-1-\frac{1}{k})\,\overline{N}(r,\infty;f) &\leq \frac{k+1}{k}\,\overline{N}(r,0;f)+\overline{N}(r,0;g)\\
&\quad -\frac{1}{k}N(r,0;f^{'}\Big|\,f\neq 0,1,\omega,\cdots,\omega^{n-1})+S(r,f)+S(r,g).
\end{aligned}$$

Proof. From the definition of V and Lemma 2 we get

$$\begin{aligned}
N(r,\infty;V) &\leq \overline{N}(r,0;F)+\overline{N}(r,0;G)+\overline{N}_*(r,1;F,G)\\
&\leq \overline{N}(r,0;f)+\overline{N}(r,0;g)+\overline{N}(r,1;F|\geq k+1)\\
&\leq \overline{N}(r,0;f)+\overline{N}(r,0;g)+\frac{1}{k}\,N(r,0;F^{'}\Big|\,F=1)\\
&\leq \overline{N}(r,0;f)+\overline{N}(r,0;g)+\frac{1}{k}\,N(r,0;F^{'}\Big|\,F\neq 0)-\frac{1}{k}N_0(r,0;F^{'})\\
&\leq \frac{k+1}{k}\,\overline{N}(r,0;f)+\overline{N}(r,0;g)+\frac{1}{k}\,\overline{N}(r,\infty;f)\\
&\quad -\frac{1}{k}\,N(r,0;f^{'}\Big|\,f\neq 0,1,\omega,\cdots,\omega^{n-1})+S(r,f).
\end{aligned}$$

Combining this with Lemma 7 and noting that f,g share $(\infty,0)$, the lemma is proved. This proves the lemma. □

Lemma 10 ([2]). *If F, G share $(1,2)$ then*

$$\begin{aligned}
&\overline{N}_0(r,0;G^{'})+\overline{N}(r,1;G|\geq 2)+\overline{N}_*(r,1;F,G)\\
\leq\ &\overline{N}(r,0;G)+\overline{N}(r,\infty;G)+S(r,G).
\end{aligned}$$

3. Proofs of the theorems

Proof of Theorem 1. Let $F = f^n$, $G = g^n$ and f, g do not satisfy (1). Since $E_f(S_1, 2) = E_g(S_1, 2)$ and $E_f(S_2, 0) = E_g(S_2, 0)$, it follows that F, G share $(1, 2)$ and $(\infty, 0)$. If possible, we suppose that $H \not\equiv 0$. Then by the second fundamental theorem, Lemma 1, 4 and 10 and noting that $N_E^{1)}(r, 1; F) = N(r, 1; F| = 1)$ we obtain

$$
\begin{aligned}
(3)\quad T(r,F) &\leq \overline{N}(r,\infty;F) + \overline{N}(r,0;F) + \overline{N}(r,1;F) - N_0(r,0;F^{'}) + S(r,F)\\
&\leq \overline{N}(r,\infty;f) + \overline{N}(r,0;f) + \overline{N}(r,0;F| \geq 2) + \overline{N}(r,0;G| \geq 2)\\
&\quad + \overline{N}_*(r,1;F,G) + \overline{N}_*(r,\infty;F,G) + \overline{N}(r,1;G| \geq 2)\\
&\quad + \overline{N}_0(r,0;F^{'}) + \overline{N}_0(r,0;G^{'}) - N_0(r,0;F^{'}) + S(r,F)\\
&\leq 2\,\overline{N}(r,\infty;f) + 2\,\overline{N}(r,0;f) + \overline{N}(r,0;g) + \overline{N}(r,0;G)\\
&\quad + \overline{N}(r,\infty;G) + S(r,F) + S(r,G)\\
&= 3\,\overline{N}(r,\infty;f) + 2\,\overline{N}(r,0;f) + 2\,\overline{N}(r,0;g) + S(r,f) + S(r,g).
\end{aligned}
$$

Since $F \not\equiv G$ we get by Lemma 6 that $V \not\equiv 0$. So by Lemma 9 for $k = 2$ we get from (3)

$$
\begin{aligned}
(4)\qquad n\,T(r,f) &\leq \frac{6}{2n-3}\,[\frac{3}{2}\,\overline{N}(r,0;f) + \overline{N}(r,0;g)] + 2\,\overline{N}(r,0;f)\\
&\quad + 2\,\overline{N}(r,0;g) + S(r,f) + S(r,g)\\
&\leq \frac{4n+3}{2n-3}\,T(r,f) + \frac{4n}{2n-3}\,T(r,g) + S(r,f) + S(r,g).
\end{aligned}
$$

Similarly we obtain

$$
(5)\qquad n\,T(r,g) \leq \frac{4n}{2n-3}\,T(r,f) + \frac{4n+3}{2n-3}\,T(r,g) + S(r,f) + S(r,g).
$$

Adding (4) and (5) we get

$$
\frac{2n^2 - 11n - 3}{2n-3}\{T(r,f) + T(r,g)\} \leq S(r,f) + S(r,g),
$$

which is a contradiction for $n \geq 6$.

Hence $H \equiv 0$ and so by Lemma 5 and Remark 1 the theorem is proved. This proves the theorem. □

Proof of Theorem 2. Let $F = f^n$, $G = g^n$ and f, g do not satisfy (1). Since $E_f(S_1, 0) = E_g(S_1, 0)$ and $E_f(S_2, 3) = E_g(S_2, 3)$, it follows that F, G share $(1, 0)$ and $(\infty, 3n)$. If possible, we suppose that $H \not\equiv 0$. Then by Lemmas 1, 2, 3 and 4 we get

$$\begin{aligned}
T(r,F) \leq\ & N_E^{1)}(r,1;F) + 2\,\overline{N}(r,0;f) + 2\,\overline{N}(r,\infty;f) + \overline{N}(r,0;g) + \overline{N}(r,\infty;g) \\
& -2\,N_0(r,0;F^{'}) - N_0(r,0;G^{'}) + S(r,F) + S(r,G) \\
\leq\ & \overline{N}(r,0;f) + \overline{N}(r,0;g) + \overline{N}(r,\infty;f| \geq 4) + \overline{N}_*(r,1;F,G) + 2\,\overline{N}(r,0;f) \\
& +2\,\overline{N}(r,\infty;f) + \overline{N}(r,0;g) + \overline{N}(r,\infty;g) + S(r,f) + S(r,g) \\
\leq\ & 3\,\overline{N}(r,0;f) + 2\,\overline{N}(r,0;g) + 3\,\overline{N}(r,\infty;f) + \overline{N}(r,\infty;f| \geq 4) \\
& +\overline{N}(r,1;F| \geq 2) + \overline{N}(r,1;G| \geq 2) + S(r,f) + S(r,g) \\
\leq\ & 3\,\overline{N}(r,0;f) + 2\,\overline{N}(r,0;g) + 3\,\overline{N}(r,\infty;f) + \overline{N}(r,\infty;f| \geq 4) \\
& +N(r,0;F^{'}|F \neq 0) + N(r,0;G^{'}|G \neq 0) + S(r,f) + S(r,g) \\
\leq\ & 4\,\overline{N}(r,0;f) + 3\,\overline{N}(r,0;g) + 5\,\overline{N}(r,\infty;f) + \overline{N}(r,\infty;f| \geq 4) \\
& +S(r,f) + S(r,g).
\end{aligned}$$

Since $F \not\equiv G$, by Lemma 6 we get $V \not\equiv 0$. So by Lemma 8 for $k = 3$ we get from above

$$\begin{aligned}
(6)\ n\,T(r,f) \leq\ & 4\,\overline{N}(r,0;f) + 3\,\overline{N}(r,0;g) + 5\,\overline{N}(r,\infty;f) \\
& +\frac{1}{4n-1}\{2\,\overline{N}(r,0;f) + 2\,\overline{N}(r,0;g) + 2\,\overline{N}(r,\infty;f)\} \\
& +S(r,f) + S(r,g) \\
\leq\ & (4 + \frac{2}{4n-1})\,\overline{N}(r,0;f) + (3 + \frac{2}{4n-1})\,\overline{N}(r,0;g) \\
& +\frac{2}{n-3}\,(5 + \frac{2}{4n-1})\,\{\overline{N}(r,0;f) + \overline{N}(r,0;g)\} \\
& +S(r,f) + S(r,g) \\
\leq\ & \{4 + \frac{42n-12}{(n-3)(4n-1)}\}\,T(r,f) + \{3 + \frac{42n-12}{(n-3)(4n-1)}\}\,T(r,g) \\
& +S(r,f) + S(r,g).
\end{aligned}$$

Similarly we obtain

$$\begin{aligned}
(7)\ n\,T(r,g) \leq\ & \{3 + \frac{42n-12}{(n-3)(4n-1)}\}\,T(r,f) + \{4 + \frac{42n-12}{(n-3)(4n-1)}\}\,T(r,g) \\
& +S(r,f) + S(r,g).
\end{aligned}$$

Adding (6) and (7) we get

$$\{n - 7 - \frac{84n-24}{(n-3)(4n-1)}\}\,\{T(r,f) + T(r,g)\} \leq S(r,f) + S(r,g),$$

which is a contradiction for $n \geq 10$.

Hence $H \equiv 0$ and so by Lemma 5 and Remark 1 the theorem follows. This proves the theorem. □

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