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Simple Presentness in Modular Group Algebras over Highly-generated Rings

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Abstract. It is proved that if G is a direct sum of countable abelian p-groups and R is a special selected commutative unitary highly-generated ring of prime characteristic p, which ring is more general than the weakly perfect one, then the group of all normed units $V(RG)$ modulo G, that is $V(RG)/G$, is a direct sum of countable groups as well. This strengthens a result due to W. May, published in (Proc. Amer. Math. Soc., 1979), that treats the same question but over a perfect ring.

1. Introduction

Throughout the text of the paper, let RG be the modular group algebra formed by an abelian p-torsion group G and a commutative ring with identity of characteristic p where p is a prime - since we will be concerned almost exclusively only with p-primary groups, there should be no confusion. For RG such an R-algebra, suppose $V(RG)$ is the group of normalized p-units in RG. As usual, for a subgroup H of G, we define $V(RG; H) = 1 + I(RG; H)$ whenever $I(RG; H)$ is the relative augmentation ideal of RG with respect to H. All other unexplained or not explicitly defined herein notion, notation and terminology are standard and will follow essentially the cited in the bibliography research articles.

The purpose in this exploration, that we pursue, is to find a more large class of commutative unitary rings than both the weakly perfect and the countable ones, over which class the group of normed invertible p-elements in the group algebra is simply presented. Thus we improve almost all principal known classical results in this aspect, as in particular the first one in the literature due to May ([5], [6], [7]) and established for perfect rings. The method for proof used by us is distinct to that in ([5], [6], [7]).

Incidentally, we have argued in [4] the following assertion in this theme, namely:

Theorem 1 ([4]). *Let R be a heightly-additive commutative ring with 1 of characteristic 2 so that $|R^{2^\omega}| = 2$, and let G be a direct sum of countable abelian 2-groups*

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so that $|G^{2^\omega}| = 2$. *Then* $V(RG)/G$ *is a direct sum of cyclic groups, and* $V(RG)$ *is a direct sum of countable groups.*

Here, we will attempt to obtain a result of this type by exploiting of an another technique.

So, we become to the main result quoted in the next section.

2. Commutative modular group algebras over highly-generated rings

Before proving the central attainment, we concentrate our attention on the following ring constructions.

Definition 1. (Weakly Highly Generative) The commutative ring R with identity and prime characteristic p is called *weakly highly-generated* provided the following conditions are satisfied:

(*) $R = \cup_{n<\omega} R_n$, $R_n \subseteq R_{n+1} \subseteq R$ and for every natural number $n \geq 1$ there exists a positive integer k_n with the property $R_n \cap R^{p^{k_n}} \subseteq R^{p^\omega}$.

The above union is in the set-theoretic sense. When it is in a ring-theoretic aspect, we record the following.

Definition 2. (Highly Generative) The commutative ring R with unity of prime characteristic p is said to be *highly-generated* provided that the following conditions are fulfilled:

(**) $R = \cup_{n<\omega} R_n$, $R_n \subseteq R_{n+1} \leq R$ and for each natural number $n \geq 1$ there is a positive integer k_n with the property $R_n \cap R^{p^{k_n}} \subseteq R^{p^\omega}$.

Thus the elements of the subsets (respective the subrings) R_n with finite heights as computed in R are bounded in general at this k_n. Certainly, in the case of subrings, $R_n^{p^{k_n}} \subseteq R^{p^\omega}$ but $R_n^{p^{k_n}} \neq R_n^{p^{k_n+1}}$ (i.e., $R_n^{p^{k_n}} \neq R_n^{p^\omega}$) is possible. To simplify both terms and the computations, we may choose $k_n = n$.

As usual, $N(R)$ shall denote the nil-radical of R.

Properties 1. *If* R *is highly-generated, then*

(a) $R^{p^\omega} = R^{p^{\omega+1}}$ *implies* $R = R^{p^\omega} + N(R)$*;*

(b) $N(R) = 0$ *implies* $R = R^{p^\omega}$.

Indeed, for the first implication, letting $x \in R$ we obtain $x \in R_n$ for some $n < \omega$. Hence $x^{p^k} \in R_n$ for each $k < \omega$, and thus for all $k \geq k_n$ we infer that $x^{p^k} \in R^{p^\omega} = R^{p^{\omega+1}}$. That is why, there is $y \in R^{p^\omega}$ such that $(x-y)^{p^k} = 0$. Finally, $x \in y + N(R) \subseteq R^{p^\omega} + N(R)$ and we are done.

The second inclusion follows immediately with the aid of the preceding one, since for rings R of prime characteristic p with no nilpotents the equality $R^{p^\omega} = R^{p^{\omega+1}}$ is ever valid.

The so-defined ring sorts are quite large and contain many classical kinds of rings. For instance, here are a few:

Examples 1.

(0) Each highly-generated ring is in itself weakly highly-generated, but the converse is untrue.

(1) Any countable ring R, that is, $|R| \leq \aleph_0$, may be represented as a countable ascending chain of finite subsets, namely $R = \cup_{n<\omega} R_n$, $R_n \subseteq R_{n+1}$ and $|R_n| < \aleph_0$. So, evidently, $R_n \cap R^{p^{j_n}} \subseteq R^{p^\omega}$ for some $j_n \in \mathbb{N}$. Hence R is weakly highly-generated, while the reverse is demonstrably false.

When these subsets R_n, that are additive abelian groups, can be chosen to be subrings, the countable ring R is obviously highly-generated. However, this is not the case in general since the polynomial ring $\mathbb{Z}_p[x]$ over the perfect field $\mathbb{Z}_p$ of p-elements is countable without nilpotent elements but not highly-generated (= perfect in this case) as simple technical arguments show.

(2) Every weakly perfect ring R, that is, $R^{p^m} = R^{p^{m+1}}$ for some $m \in \mathbb{N}$ (when $m = 0$ we have $R = R^p$ and such rings are known to be perfect), can be interpreted as $R = \cup_{n<\omega} R_n$ where by setting $R_n = R$ for all $n \in \mathbb{N}$ we have $R_n \cap R^{p^m} = R^{p^{m+1}} = R^{p^\omega}$. Thus R is obviously highly-generated if we take $k_n = m$ for each nonnegative integer n, whereas that the oppositive claim is wrong follows at once since the highly-generated rings may have an infinite number of different finite heights.

Of course, the weakly perfect ring R may be represented as well like this $R = R^{p^m} + R(p^m) = R^{p^\omega} + N(R)$, since $R^{p^m} = R^{p^\omega}$ for some $m \in \mathbb{N}$, whence, once again, it is highly-generated.

(3) The heightly-additive rings from [4] are weakly highly-generated, but the reverse claim is no longer available. If the heightly-additive rings, constructed by us in [4], are the ring-theoretic union of subrings, then they are also highly-generated. Plain examples show that the converse implication fails, i.e., in other words, there are highly-generated rings that are not heightly-additive even in the ring-theoretic sense.

Finally, one can conclude that the class of all (weakly) highly-generated rings properly contains the already described three ring classes.

If $r \in R$ and $g \in G$, then the symbols $|r|_R$ and $|g|_G$ denote the heights of r and g as computed in R and G, respectively. For freely usage in the future, we indicate that $|r_1g_1 + \cdots + r_tg_t|_{RG} = \min_{1\leq i\leq t}\{|r_i|_R, |g_i|_G\}$ whenever $r_1g_1 + \cdots + r_tg_t \in RG$.

For our successful presentation, we need a series of new technical affirmations, distinguished to those from [1]-[3].

Lemma 1. (Intersection) *Assume P is a commutative ring with 1, and A, B_1, B_2 are abelian groups such that $B_1 \cap B_2 = 1$ and $A \cap (B_1 \times B_2) = 1$. Then $V(P(A \times B_1); B_1) \cap V(P(A \times B_1 \times B_2); B_2) = 1$.*

Proof. Given x to belong in the left-hand side. Thus we write in canonical form $x = r_1a_1b_1^{(1)} + \cdots + r_sa_sb_s^{(1)} = e_1a_1'b_1'^{(1)}b_1^{(2)} + \cdots + e_sa_s'b_s'^{(1)}b_s^{(2)}$, where $r_1, \cdots, r_s \in P$ with $r_1 + \cdots + r_s = 1$; $e_1, \cdots, e_s \in P$ with $e_1 + \cdots + e_s = 1$; $a_1, \cdots, a_s \in A$; $a_1', \cdots, a_s' \in A$; $b_1^{(1)}, \cdots, b_s^{(1)} \in B_1$; $b_1'^{(1)}, \cdots, b_s'^{(1)} \in B_1$; $b_1^{(2)}, \cdots, b_s^{(2)} \in B_2$; $s \in \mathbb{N}$.

We therefore have $r_i = e_i$, $a_i = a'_i$, $b_i^{(1)} = b_i'^{(1)}$ and $b_i^{(2)} = 1$ for all $1 \leq i \leq s$. Since $e_1 a'_1 b_1'^{(1)} b_1^{(2)} + \cdots + e_s a'_s b_s'^{(1)} b_s^{(2)}$ lies in $V(P(A \times B_1 \times B_2); B_2)$, everything is done. This ends the proof. □

Lemma 2. (Decomposition) *Assume P is a commutative ring with 1 of $char(P) = p$, $C = C_1 \times C_2$ is an abelian group so that C_1 is p-torsion and A is an abelian group with $A \cap C = 1$. Then $V(P(A \times C); C) = V(P(A \times C_1); C_1) \times V(P(A \times C); C_2)$.*

Proof. Consider the element $fac_1c_2(1 - c'_1c'_2)$, where $f \in P$; $a \in A$; $c_1, c'_1 \in C_1$; $c_2, c'_2 \in C_2$. Taking into account that $1 - uv = (1 - u)v + 1 - v$ for each two elements u and v, we easily derive that $fac_1c_2(1 - c'_1c'_2) = fa(1 - c'_1) - fa(1 - c_1)(1 - c'_1) - fac_1(1 - c'_1)(1 - c_2c'_2) + fac_1c_2(1 - c'_2)$. That is why, by setting $w = 1 + fa(1 - c'_1) - fa(1 - c_1)(1 - c'_1)$, we infer that $1 + fac_1c_2(1 - c'_1c'_2) = w(1 - w^{-1}fac_1(1 - c'_1)(1 - c_2c'_2) + w^{-1}fac_1c_2(1 - c'_2)) \in V(P(A \times C_1); C_1)V(P(A \times C); C_2)$, as desired. That the intersection of the latter two groups is equal to 1 now follows via the Intersection Lemma. Next, because every element of $V(P(A \times C); C)$ is a finite sum of 1 plus elements of the foregoing kind presented, by what we have just obtained, we are done. The proof is over. □

We are now ready to proceed by proving the following statement, which is our goal here.

Theorem 2. (Direct Factor Structure) *Suppose G is an abelian direct sum of countable p-groups and R is a highly-generated commutative ring with identity of prime characteristic p such that $R^{p^\omega} = R^{p^{\omega+1}}$. Then both $V(RG)/G$ and $V(RG)$ are direct sums of countable abelian p-groups.*

Proof. Foremost we decompose $G = G_d \times G_r$, where G_d is the maximal divisible subgroup of G, and G_r is the reduced part of G that is a direct sum of countable p-groups. According to [1], we obtain

($\diamond$) $V(RG)/G \cong V(RG_d)/G_d \times V(RG; G_r)/G_r$.

First of all, we study $V(RG_d)/G_d$. Since $[V(RG_d)/G_d]_d = V(R^{p^\omega}G_d)/G_d$, we detect that $V(RG_d)/G_d \cong V(R^{p^\omega}G_d)/G_d \times V(RG_d)/V(R^{p^\omega}G_d)$. Thus, by supposition, $V(RG_d) = \cup_{n<\omega} V(R_nG_d)$ and $V(RG_d)/V(R^{p^\omega}G_d) = \cup_{n<\omega}[V(R_nG_d)V(R^{p^\omega}G_d) /V(R^{p^\omega}G_d)]$. Moreover, by the modular law, we calculate that $[V(R_nG_d)V(R^{p^\omega}G_d)] \cap V(R^{p^n}G_d) = V(R^{p^\omega}G_d)[V(R_nG_d) \cap V(R^{p^n}G_d)] = V(R^{p^\omega}G_d)V((R_n \cap R^{p^n})G_d) = V(R^{p^\omega}G_d)$. Hence, by virtue of the Kulikov's criterion or its extension archived in [8], $V(RG_d)/V(R^{p^\omega}G_d)$ is a direct sum of cyclics. Finally, we deduce for the first point that $V(RG_d)/G_d$ is a direct sum of cyclic and quasi-cyclic (i.e., of co-cyclic) groups.

Next, we are in a position to explore the structure of the second complementary factor $V(RG; G_r)/G_r$.

In fact, first we presume that $length(G_r) < \Omega = \omega_1$, termed as the first uncountable limit ordinal. Consequently, by exploiting [8], $G_r = \cup_{n<\omega} A_n$ so that $A_n \subseteq A_{n+1}$ and all members A_n are height-finite in G_r, whence in G. Therefore, because the support of each element in the group algebra is finite, $V(RG; G_r)/G_r =$

$\cup_{n<\omega}[V(R_n(G_d\times A_n);A_n)G_r/G_r]$ where $[V(R_n(G_d\times A_n);A_n)G_r/G_r]\cong V(R_n(G_d\times A_n);A_n)/A_n$. We shall show below that for every natural number n the members $V(R_n(G_d\times A_n);A_n)$ are height-finite in $V(RG;G_r)$, and as a consequence that each $V(R_n(G_d\times A_n);A_n)G_r/G_r$ is so in $V(RG;G_r)/G_r$. Indeed, for this aim, we take an arbitrary element x which belongs to $V(R_n(G_d\times A_n);A_n)$ and examine the heights in its canonical record $x=r_1^{(n)}g_{d1}a_1^{(n)}+\cdots+r_s^{(n)}g_{ds}a_s^{(n)}$, where $r_1^{(n)},\cdots,r_s^{(n)}\in R_n$ with $r_1^{(n)}+\cdots+r_s^{(n)}=1$; $g_{d1},\cdots,g_{ds}\in G_d$; $a_1^{(n)},\cdots,a_s^{(n)}\in A_n$; $s\in\mathbb{N}$. If $|r_i^{(n)}|_R\leq n-1$ for some $i=1,\cdots,s$, there is nothing to prove. If now $|r_i^{(n)}|_R\geq n$ for each index i, hence $|r_i^{(n)}|_R=\infty$, we are able to estimate the heights of the group elements from the sum. Since $|g_{di}|_G=\infty$, and $|a_i^{(n)}|_G$ lies in a finite set of ordinals, we obviously detect that $|x|_{V(RG)}=|x|_{V(RG;G_r)}\in\{0,1,\cdots,n-1\}\cup\{heightspectrum_G(A_n)\}$, which substantiates our claim. That $V(R_n(G_d\times A_n);A_n)G_r/G_r$ is with finite height spectrum in $V(RG;G_r)/G_r$ follows now employing the same scheme as in [1] combined with the fact that G_r is nice in $V(RG;G_r)$ (see, for instance, [3]). Finally, owing again to [8], we extract in this situation that $V(RG;G_r)/G_r$ is of countable length a direct sum of countable groups.

Secondly, let now $length(G_r)=\Omega$, hence $G_r=\coprod_{i\in I}G_i$ with $length(G_i)<\Omega$ for every $i\in I$, and $|G_r|=|I|>\aleph_0$. Furthermore, for our convenience, we can write down $G_r=\coprod_{\alpha<\beta}G_\alpha$, where $length(G_\alpha)<\Omega$ and $|G_r|=|\beta|>\aleph_0$. Referring to the Decomposition Lemma and by using of a standard transfinite induction, we infer that

$V(RG;G_r)/G_r\cong\coprod_{\tau<\beta}[V(R(G_d\times\coprod_{\alpha<\tau+1}G_\alpha);G_\tau)/G_\tau]=\coprod_{\tau<\beta}[V(R(G_d\times\coprod_{\alpha\leq\tau}G_\alpha);G_\tau)/G_\tau]$.

Thus, it remains only show that if H is a reduced countable direct factor of the p-primary abelian group G (or, more generally, if H is an isotype reduced countable subgroup of the p-torsion group G), then $V(RG;H)/H$ is a direct sum of countable groups. This is a refinement of the above considered first situation and of a proposition proved by May ([7, Lemmas 6-7]) as our approach is different to that of May. As before, we will use as a back-and-forth argument the generalized version of the Kulikov's criterion documented in [8]. Therefore, as a paralleling argument, the countability of H means that $H=\cup_{n<\omega}H_n$, $H_n\subseteq H_{n+1}$ and H_n is finite, whence height-finite both in H and G, for each $n\geq 1$. Thus $V(RG;H)=\cup_{n<\omega}V(R_nG;H_n)$. We now construct generating subgroups of $V(RG;H)$ in the following manner:

$V_n=\langle v^{(n)}=\sum_{1\leq i\leq s_n}r_i^{(n)}g_i^{(n)}\in V(R_nG;H_n)|g_i^{(n)}\neq g_j^{(n)}$ for $i\neq j$ such that for $1\leq\epsilon_i\leq order(g_i^{(n)})$ it holds $|g_i^{(n)^{\epsilon_i}}|_G>$ maximal height $\in heightspectrum_G(H_n)$ or for all possible heights of $g_i^{(n)}$'s it is fulfilled $|g_i^{(n)^{\pm\epsilon_i}}g_j^{(n)^{\pm\epsilon_j}}\cdots g_k^{(n)^{\pm\epsilon_k}}|_G\in\{0,1,\cdots,n-1\}\cup\{heightspectrum_G(H_n)\}\cup\{\delta_1,\cdots,\delta_n|\geq\omega$ and $\notin heightspectrum_G(H)\}$, $1\leq i\neq j\neq\cdots\neq k\leq s_n\in\mathbb{N}\rangle$. In other words, $\langle g_1^{(n)},\cdots,g_{s_n}^{(n)}\rangle\cap G^{p^n}\subseteq(G^{p^\epsilon}\setminus G^{p^{\epsilon+1}})\cup(G^{p^{\delta_i}}\setminus G^{p^{\delta_i+1}})\cup G^{p^\sigma}:\epsilon\in\{heightspectrum_G(H_n)\};\{\delta_i\geq\omega,\delta_i\notin heightspectrum_G(H)|i=1,\cdots,n\};\sigma>maxheight\in heightspectrum_G(H_n)$.

Apparently, all V_n are correctly defined groups so that $H_n\subseteq V_n\subseteq V_{n+1}\cap$

$V(R_nG;H_n)$, so that $V(RG;H)=\cup_{n<\omega}V_n$ and so that $V(RG;H)/H=\cup_{n<\omega}[V_nH/H]$.

Moreover, since in the support of every element from $V(RG;H_n)$ there is an element of H_n, we observe that (for more details see especially [2] and [3]) the product of generators of V_n is a height-finite element too with height described as in the definition of V_n. In order to establish this claim, we first consider two generators $v^{(1n)}=\sum_{1\leq i\leq s_n} r_i^{(1n)}g_i^{(1n)}$ and $v^{(2n)}=\sum_{1\leq i\leq t_n} r_i^{(2n)}g_i^{(2n)}$ along with their product $v^{(1n)}v^{(2n)}=\sum_{1\leq i\leq s_n}\sum_{1\leq j\leq t_n} r_i^{(1n)}r_j^{(2n)}g_i^{(1n)}g_j^{(2n)}$. With no harm of generality, we assume that in the canonical record of $v^{(1n)}v^{(2n)}$ there exists an element of H_n which is precisely $g_1^{(1n)}g_1^{(2n)}$ where $g_1^{(1n)}\in H_n$ and $g_1^{(2n)}\in H_n$, say. This is possible by multiplying of an appropriate element from H_n. If there are no zero divisors in R, everything is proved because the ring members in the canonical form of $v^{(1n)}v^{(2n)}$ have either finite heights $< n$, or infinite heights ∞ and then we can copy the method used by us in [2].

The crucial moment is when R possesses zero divisor elements. In order to process that situation, we foremost observe that $r_1^{(1n)}=r_1^{(2n)}=1$, whence $\sum_{2\leq i\leq s_n} r_i^{(1n)}=\sum_{2\leq j\leq t_n} r_j^{(2n)}=0$. Thus if $r_i^{(1n)}r_j^{(2n)}=0$ for some indices $i\geq 2$ and $j\geq 2$, it is self-evident that $r_i^{(1n)}r_k^{(2n)}\neq 1$ and $r_k^{(1n)}r_j^{(2n)}\neq 1$ for all other $k\neq i$ and $k\neq j$, since otherwise by multiplying the both sides of the latter equalities with $r_j^{(2n)}$ or $r_i^{(1n)}$ respectively, we detect that $r_i^{(1n)}=r_j^{(2n)}=0$, which leads us to $v^{(1n)}v^{(2n)}\in H_n$, so everything is done. Certainly, the group products $g_i^{(1n)}g_j^{(2n)}$ do not lie in the canonical record of $v^{(1n)}v^{(2n)}$ and also they eventually do not have explicit relationships with the the other group members. Nevertheless, owing to the plain combinatorial arguments that in the support of $v^{(1n)}v^{(2n)}$ there are pairs of indexes $(l_1,l_2),(m_1,m_2)$ and (k_1,k_2) such that $g_{l_1}^{(1n)}{g_{l_2}^{(1n)}}^{-1}\in H_n$, or such that $g_{m_1}^{(2n)}{g_{m_2}^{(2n)}}^{-1}\in H_n$, or such that $g_{k_1}^{(1n)}{g_{k_2}^{(2n)}}^{-1}\in H_n$, combined with the above given dependence on the ring coefficients, we conclude that the approach realized in [2] for computation of the heights $|g_{i'}^{(1n)}g_{j'}^{(2n)}|_G$ for all indices $1\leq i'\leq s_n$ and $1\leq j'\leq t_n$ is successfully applicable in the current situation to obtain that $|v^{(1n)}v^{(2n)}|_{V(RG)}$ belongs to the finite height spectrum described like in the definition of the groups V_n. The case for more than two generating elements follows either by an induction step or in view of a similar procedure as to the preceding one already demonstrated. These derivations established verify our claim.

Thereby, $V(RG;H)$ is really a direct sum of countable groups, whence consulting with ([1], [2]) so does $V(RG;H)/H$. Henceforth, $V(RG;G_r)/G_r$ is also from this group class.

Finally, in conjunction with the formula $(\diamond)$, we claim in general that $V(RG)/G$ is indeed a direct sum of countable groups, as promised.

The proof is finished. $\square$

As an immediate consequence, we obtain the following.

Corollary 1. (Criterion) *Let R be a highly-generated commutative ring with unity of prime characteristic p so that R^{p^ω} is perfect, and let G be an abelian group. Then $V(RG)$ is a direct sum of countable p-groups $\iff$ G is a direct sum of countable p-groups.*

Proof. "$\Rightarrow$". Since $V(RG)/V^{p^\omega}(RG)$ is a direct sum of cyclics, $G/G^{p^\omega} \cong GV^{p^\omega}(RG)/V^{p^\omega}(RG) \subseteq V(RG)/V^{p^\omega}(RG)$ is also from this class of groups as being an isomorphic copy of a subgroup of a direct sum of cyclics.

On the other hand, since $V^{p^\omega}(RG) = V(R^{p^\omega}G^{p^\omega})$ is a direct sum of countable groups and since R^{p^ω} is perfect, a plain appeal to a modified variant of the proof in [6] riches us that G^{p^ω} is a direct sum of countable groups as well.

Finally, combining these two conclusions, we deduce via a classical result of R. J. Nunke that G must be a direct sum of countable groups, as asserted.

"$\Leftarrow$". Follows directly from the Theorem.

So, the corollary is verified in full generality. □

Remark 1. In ([2], p. 26) the equality $G = \oplus_{\beta<\lambda}C_\beta$ should be written and read as $G = \oplus_{\beta<\lambda}G_\beta$.

We close the study with

3. Concluding comments and discussion

In the way of the results established, we pose the following.

Problem 1. *Does it follow that $V(RG)$ is a simply presented p-group if and only if G is a simply presented p-group and R is a weakly highly-generated ring?*

We emphasize that the ideas for an evidence of theorems given by us in [1], [2] and [3] work also for the highly-generated rings. So, the major results from the previous investigations hold valid for such rings, as well.

Well, we come to the

General Direct Factor Structure Problem: *If R is highly-generated and G is a p-group, is then $V(RG)/G$ simply presented?*

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