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The Inverse Laplace Transform of a Wide Class of Special Functions

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Abstract. The aim of the present work is to obtain the inverse Laplace transform of the product of the factors of the type $s^{-\rho}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-\sigma_i}$, a general class of polynomials and the multivariable H-function. The polynomials and the functions involved in our main formula as well as their arguments are quite general in nature. On account of the general nature of our main findings, the inverse Laplace transform of the product of a large variety of polynomials and numerous simple special functions involving one or more variables can be obtained as simple special cases of our main result. We give here exact references to the results of seven research papers that follow as simple special cases of our main result.

1. Introduction

The Laplace transform of the function $f(t)$ is defined in the following usual manner

$$F(s)=L\{\,f(t)\;;s\,\}=\int_0^\infty e^{-st}f(t)\;dt. \tag{1.1}$$

The function $f(t)$ is called the inverse Laplace transform of $F(s)$ and will be denoted by $L^{-1}\{F(s)\}$ in the paper.

Also, $S_n^m[x]$ occurring in the sequel denotes the general class of polynomials introduced by Srivastava [10, p.1, Eq.(1)]

$$S_n^m[x]=\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}x^k, \qquad n=0,\;1,\;2,\;\cdots, \tag{1.2}$$

where m is an arbitrary positive integer and the coefficients $A_{n,k}$ $(n,k\geq 0)$ are arbitrary constants, real or complex. On suitably specializing the coefficients $A_{n,k}$, $S_n^m[x]$ yields a number of known polynomials as its special cases. These include, among others, the Hermite polynomials, the Jacobi polynomials, the Laguerre polynomials, the Bessel polynomials, the Gould-Hopper polynomials, the Brafman polynomials and several others [14, pp. 158-161].

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The H-function of r complex variables $z_1, \cdots, z_r$ was introduced by Srivastava and Panda [13]. We shall define and represent it in the following form [12, p. 251, Eq. (C.1)]

$$
\begin{aligned}
(1.3)\qquad & H[z_1, \cdots, z_r] \\
= \; & H^{0,N\,:\,M',N';\cdots;\,M^{(r)},N^{(r)}}_{P,Q\,:\,P',Q'\;;\cdots;\,P^{(r)}\;,Q^{(r)}} \\
& \left[\begin{array}{c|l} z_1 & \left(a_j;\, \alpha'_j, \cdots, \alpha_j^{(r)}\right)_{1,\,P} : \left(c'_j,\, \gamma'_j\right)_{1,P'}; \cdots; \left(c_j^{(r)},\, \gamma_j^{(r)}\right)_{1,P^{(r)}} \\ \vdots & \\ z_r & \left(b_j;\, \beta'_j, \cdots, \beta_j^{(r)}\right)_{1,\,Q} : \left(d'_j,\, \delta'_j\right)_{1,Q'}; \cdots; \left(d_j^{(r)},\, \delta_j^{(r)}\right)_{1,Q^{(r)}} \end{array}\right] \\
= \; & \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi_1(\xi_1) \cdots \phi_r(\xi_r) \psi(\xi_1, \cdots, \xi_r)\, z_1^{\xi_1} \cdots z_r^{\xi_r}\, d\xi_1 \cdots d\xi_r,
\end{aligned}
$$

where $\omega = \sqrt{-1}$,

(1.4)

$$
\phi_i(\xi_i) = \frac{\prod_{j=1}^{M^{(i)}} \Gamma\left(d_j^{(i)} - \delta_j^{(i)}\xi_i\right) \prod_{j=1}^{N^{(i)}} \Gamma\left(1 - c_j^{(i)} + \gamma_j^{(i)}\xi_i\right)}{\prod_{j=M^{(i)}+1}^{Q^{(i)}} \Gamma\left(1 - d_j^{(i)} + \delta_j^{(i)}\xi_i\right) \prod_{j=N^{(i)}+1}^{P^{(i)}} \Gamma\left(c_j^{(i)} - \gamma_j^{(i)}\xi_i\right)}, \quad \forall i \in \{1, \cdots, r\}
$$

$$
(1.5)\qquad \psi(\xi_1, \cdots, \xi_r) = \frac{\prod_{j=1}^{N} \Gamma\left(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(i)}\xi_i\right)}{\prod_{j=N+1}^{P} \Gamma\left(a_j - \sum_{i=1}^{r} \alpha_j^{(i)}\xi_i\right) \prod_{j=1}^{Q} \Gamma\left(1 - b_j + \sum_{i=1}^{r} \beta_j^{(i)}\xi_i\right)}.
$$

The nature of contours $L_1, \cdots, L_r$ in (1.3), the various special cases and other details of the above function can be found in the book referred to above. It may be remarked here that all the Greek letters occurring in the left-hand side of (1.3) are assumed to be positive real numbers for standardization purposes; the definition of this function will, however, be meaningful even if some of these quantities are zero. Again, it is assumed that the various multivariable H-functions occurring in the paper always satisfy their appropriate conditions of convergence [12, pp. 252-253, Eqs. (C.4-C.6)].

2. Main result

$$
\begin{aligned}
(2.1)\qquad & L^{-1}\Bigg\{ s^{-\rho}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-\sigma_i}\prod_{j=1}^{\varsigma} S_{n_j}^{m_j}\left[e_j s^{-\lambda_j}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-\eta_i^{(j)}}\right] \\
& H\left[z_1 s^{-u_1}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-v_i'},\cdots,z_r s^{-u_r}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-v_i^{(r)}}\right]\Bigg\} \\
= \; & t^{\rho+l_1\sigma_1+\cdots+l_\tau\sigma_\tau-1}\sum_{k_1=0}^{[n_1/m_1]}\cdots\sum_{k_\varsigma=0}^{[n_\varsigma/m_\varsigma]}\frac{(-n_1)_{m_1k_1}\cdots(-n_\varsigma)_{m_\varsigma k_\varsigma}}{k_1!\cdots k_\varsigma!} \\
& A'_{n_1,k_1}\cdots A^{(\varsigma)}_{n_\varsigma,k_\varsigma}\left(e_1 t^{\lambda_1+\eta_1' l_1+\cdots+\eta_\tau' l_\tau}\right)^{k_1}\cdots\left(e_\varsigma t^{\lambda_\varsigma+\eta_1^{(\varsigma)}l_1+\cdots+\eta_\tau^{(\varsigma)}l_\tau}\right)^{k_\varsigma} \\
& H^{0,N+\tau\;:M',N';\cdots;\,M^{(r)},N^{(r)};1,0\;;\cdots;\;1,0}_{p+\tau,Q+\tau+1:P',Q'\;;\cdots;\,P^{(r)},Q^{(r)}\;;0,1\;;\cdots;\;0,1}\left[\begin{array}{l} z_1 t^{u_1+v_1' l_1+\cdots+v_\tau' l_\tau} \\ \vdots \\ z_r t^{u_r+v_1^{(r)}l_1+\cdots+v_\tau^{(r)}l_\tau} \\ \alpha_1 t^{l_1} \\ \vdots \\ \alpha_\tau t^{l_\tau}\end{array}\right| \\
& \left(1-\sigma_1-\eta_1'k_1-\cdots-\eta_1^{(\varsigma)}k_\varsigma;v_1',\cdots,v_1^{(r)},1,\frac{0,\cdots,0}{\tau-1}\right),\cdots, \\
& \left(1-\sigma_\tau-\eta_\tau'k_1-\cdots-\eta_\tau^{(\varsigma)}k_\varsigma;v_\tau',\cdots,v_\tau^{(r)},\frac{0,\cdots,0}{\tau-1},1\right), \\
& \left(1-\sigma_1-\eta_1'k_1-\cdots-\eta_1^{(\varsigma)}k_\varsigma;v_1',\cdots,v_1^{(r)},\frac{0,\cdots,0}{\tau}\right),\cdots, \\
& \left(1-\sigma_\tau-\eta_\tau'k_1-\cdots-\eta_\tau^{(\varsigma)}k_\varsigma;v_\tau',\cdots,v_\tau^{(r)},\frac{0,\cdots,0}{\tau}\right), \\
& \left(a_j\;;\;\alpha_j',\cdots,\alpha_j^{(r)},\frac{0,\cdots,0}{\tau}\right)_{1,P} \qquad\qquad : \\
& (1-\rho-l_1\sigma_1-\cdots-l_\tau\sigma_\tau-(\lambda_1+\eta_1' l_1+\cdots+\eta_\tau' l_\tau)\;k_1-\cdots \\
& -\left(\lambda_\varsigma+\eta_1^{(\varsigma)}l_1+\cdots+\eta_\tau^{(\varsigma)}l_\tau\right)\;k_\varsigma;u_1+v_1' l_1+\cdots+v_\tau' l_\tau,\cdots, \\
& u_r+v_1^{(r)}l_1+\cdots+v_\tau^{(r)}l_\tau,l_1,\cdots,l_\tau\Big),\left(b_j;\;\beta_j',\cdots,\beta_j^{(r)},\frac{0,\cdots,0}{\tau}\right)_{1,\,Q} \quad : \\
& \left.\begin{array}{l}\left(c_j',\;\gamma_j'\right)_{1,P'};\cdots;\left(c_j^{(r)},\;\gamma_j^{(r)}\right)_{1,P^{(r)}};\text{---};\cdots;\text{---} \\ \left(d_j',\;\delta_j'\right)_{1,Q'};\cdots;\left(d_j^{(r)},\;\delta_j^{(r)}\right)_{1,Q^{(r)}};\frac{(0,1)\;;\cdots;(0,1)}{\tau}\end{array}\right],
\end{aligned}
$$

where the function occurring on the right-hand side of (2.1) is the H-function of $r+\tau$ variables and the following conditions are satisfied

The quantities $\lambda_1,\ \eta'_1,\ \cdots,\ \eta'_\tau,\ \cdots,\ \lambda_\varsigma,\ \eta_1^{(\varsigma)},\ \cdots,\ \eta_\tau^{(\varsigma)},\ u_1,\ v'_1,\ \cdots,\ v'_\tau,\ u_r,\ v_1^{(r)},\ \cdots,\ v_\tau^{(r)}$ are all positive, $\mathrm{Re}(s) > 0$,

$$\mathrm{Re}\,(\rho + l_1\sigma_1 + \cdots + l_\tau\sigma_\tau)$$
$$+ \min_{1\le j\le M^{(i)}} \left[\mathrm{Re}\ \left(u_i + v_1^{(i)} l_1 + \cdots + v_\tau^{(i)} l_\tau\right)\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] > 0,$$

$$\sum_{j=1}^{P} \alpha_j^{(i)} + \sum_{j=1}^{P^{(i)}} \gamma_j^{(i)} - \sum_{j=1}^{Q} \beta_j^{(i)} - \sum_{j=1}^{Q^{(i)}} \delta_j^{(i)} - \left(u_i + v_1^{(i)} l_1 + \cdots + v_\tau^{(i)} l_\tau\right) < 0,$$
$$i = 1, \cdots, r;$$

$$U_i = -\sum_{j=N+1}^{P} \alpha_j^{(i)} + \sum_{j=1}^{N^{(i)}} \gamma_j^{(i)} - \sum_{j=N^{(i)}+1}^{P^{(i)}} \gamma_j^{(i)} - \sum_{j=1}^{Q} \beta_j^{(i)} - \sum_{j=1}^{M^{(i)}} \delta_j^{(i)}$$
$$- \sum_{j=M^{(i)}+1}^{Q^{(i)}} \delta_j^{(i)} - \left[u_i + v_1^{(i)}\,(l_1+1) + \cdots + v_\tau^{(i)}\,(l_\tau+1)\right] > 0,$$

$$|\arg\ z_i| < \left(\frac{1}{2}\right) U_i\pi \quad \text{or} \quad U_i = 0 \quad \text{and} \quad z_i > 0, \quad i = 1, \cdots, r\ ;$$

$$0 < l_j < 1, \quad |\arg\ \alpha_j| < (1 - l_j)\,\frac{\pi}{2}, \quad j = 1, \cdots, \tau \quad \text{or} \quad l_1 = \cdots = l_\tau = 1$$
$$\text{and} \quad \alpha_1 > 0, \cdots, \alpha_\tau > 0.$$

It may be remarked here that some of the exponents $l_1, \cdots, l_\tau,\ \lambda_1, \eta'_1, \cdots, \eta'_\tau, \cdots, \lambda_\varsigma,\ \eta_1^{(\varsigma)}, \cdots, \eta_\tau^{(\varsigma)},\ u_1, v'_1, \cdots, v'_\tau,\ u_r, v_1^{(r)}, \cdots, v_\tau^{(r)}$ in (2.1) can also decrease to zero provided that both sides of the resulting equation have a meaning. Also the number occurring below the line at any place on the right-hand side of (2.1) indicates the total number of zeros/ ones/ pairs covered by it. Thus $\frac{0, \cdots, 0}{r}$, $\frac{1, \cdots, 1}{r}$, $\frac{(0,1)\,; \cdots ;\,(0,1)}{r}$ would mean r zeros/ r ones / r pairs, and so on.

Proof. We first express the product of a general class of polynomials occurring on the left-hand side of (2.1) in the series form given by (1.2), replace the multivariable H-function occurring therein by its well known Mellin-Barnes contour integral with the help of (1.3). Now we interchange the orders of summations and integration (which is permissible under the conditions stated with (2.1)), find the inverse Laplace transform of the result thus obtained by making use of the following known formula [4, p. 12, Eq. (12)]

$$(2.2)\ L^{-1}\left\{ s^{\sum\limits_{i=1}^{\tau} l_i a_i - \lambda} \prod_{i=1}^{\tau} \left(s^{l_i} + \lambda_i\right)^{-a_i} \right\}$$

$$= \frac{t^{\lambda-1}}{\prod\limits_{i=1}^{\tau} \Gamma(a_i)} H^{0,0:1,1;\cdots;1,1}_{0,1\,:1,1;\cdots;1,1} \left[\begin{array}{c} \lambda_1 t^{l_1} \\ \vdots \\ \lambda_\tau t^{l_\tau} \end{array} \middle| \begin{array}{c} \text{-----} : (1-a_1, 1); \cdots ; (1-a_\tau, 1) \\ \\ (1-\lambda; l_1, \cdots, l_\tau) : (0,1) \quad ; \cdots ; (0,1) \end{array} \right],$$

where $\mathrm{Re}(s) > 0$, $\mathrm{Re}(\lambda) > 0$, $0 < l_i < 2$, $|\arg \lambda_i| < (2 - l_i)\ \frac{\pi}{2}$ or $l_i = 2$ and $\lambda_i > 0$, $i = 1, 2, \cdots, \tau$, express the multivariable H-function thus obtained in terms of Mellin-Barnes contour integral with the help of (1.3) and reinterpret the resulting contour integral thus obtained in terms of the multivariable H-function. We arrive at the desired result (2.1) after a little simplification.

3. Special cases and applications

On account of the most general nature of our main result, several known and new results, follow as its special cases. The results given herein and many others which are not recorded here specifically can find important applications in boundary value problems occurring in certain fields of science and engineering. The inverse Laplace transform formula (2.1) established here is unified in nature and acts as a key formula. Thus the general class of polynomials involved in the formula (2.1) reduce to a large spectrum of polynomials listed by Srivastava and Singh [14, pp. 158-161], and so from the formula(2.1) we can further obtain various inverse Laplace transform involving a number of simpler polynomials. Again, the multivariable H-function occurring in this formula can be suitably specialized to a remarkably wide variety of useful functions (or product of several such functions) which are expressible in terms of E, F, G and H-functions of one, two or more variables. For example if $N = P = Q = 0$, the multivariable H-function occurring in the left-hand side of the formula (2.1) would reduce immediately to the product of r different H-functions of Fox [1]; thus the table listing various special cases of the H-function [6, pp. 145-159] can be used to derive from this inverse Laplace transform formula a number of other inverse Laplace transform formulae involving any of these simpler special functions. Thus if we take $\tau = \varsigma = 2$ in (2.1), we get a known result due to Gupta and Soni [3, pp.21-22, Eq. (2)]. If we take $\tau = \varsigma = 1$, $\rho = \lambda_1 = 0$, $l_1 = \eta_1' = 1$, $u_i = 0$, $i = 1, 2, \cdots, r$ in (2.1), we get a known result due to Srivastava and Singh [14, p.169, Eq. (2.10)] expressed in a different form. Again, if in the main result (2.1), we reduce the multivariable H-function to the product of the Whittaker function and the Meijer's G-function and take the other factors /functions occurring therein to be unity, we arrive after a little simplification at a result which is in essence the formula obtained earlier by Srivastava [9, p. 827, Eq. (2.4)], which on further specialization easily yields a result obtained by Rathie [7, p. 368, Eq. (3.2)]. Also, if in the left hand side of (2.1), we take $\varsigma = 2$ and reduce the

H-function of r variables occurring therein to the H-function of Fox [1], we arrive at a result obtained by Soni and Singh [8, Eq. (2.1)]. Finally, if in the left-hand side of (2.1), we take $\tau = \varsigma = 2$ and put $N = P = Q = 0$, $M^{(i)} = Q^{(i)} = 1$, $N^{(i)} = P^{(i)} = 0$, $d_1^{(i)} = 0$, $\delta_1^{(i)} = 1$, $u_i = 1$, $v_1^{(i)} = v_2^{(i)} = 0$ and let z_i tends to zero $(i = 2, \cdots, r)$, the H-function of r variables occurring therein reduces to the H-function of Fox [1], and we arrive at another result obtained by Gupta and Soni [2, pp.2-3. Eq.(2.1)], which on further specialization gives a result obtained by Srivastava [10, pp. 1-2, Eq. (2)] but expressed in a different form.

Now, we give two new and interesting inverse Laplace transforms that follow as special cases of (2.1)

(i) If in the left-hand side of the (2.1), we take $\varsigma = 2$ and reduce the general class of polynomials $S_{n_1}^{m_1}$ and $S_{n_2}^{m_2}$so obtained into the Konhauser biorthogonal polynomials [11, p.225, Eq. (3.23); 5, p.304, Eq. (5)] and to the Hermite polynomials [14, 158, Eq. (1.4)] respectively and also reduce the multivariable H-function occurring in its left-hand side into the product of r distinct (Fox) H-functions of one variable, we get the following result after a little simplification

$$(3.1)\quad L^{-1}\left\{ s^{-\left(\rho+\frac{n_1}{2}\right)} \prod_{i=1}^{\tau} \left(s^{l_i}+\alpha_i\right)^{-\sigma_i} Z_{n_1}^{\alpha}\left[(\frac{1}{s})^{1/\beta};\ \beta \right] H_{n_2}\left[\frac{\sqrt{s}}{2}\right] \prod_{i=1}^{r} H_{P^{(i)},Q^{(i)}}^{M^{(i)},N^{(i)}}\left[z_i s^{-u_i} \middle| \begin{array}{l} \left(c_j^{(i)},\gamma_j^{(i)}\right)_{1,P^{(i)}} \\ \left(d_j^{(i)},\delta_j^{(i)}\right)_{1,Q^{(i)}} \end{array} \right] \right\}$$

$$= \frac{t^{\rho+l_1\sigma_1+\cdots+l_\tau\sigma_\tau-1}}{\Gamma(\sigma_1)\cdots\Gamma(\sigma_\tau)} \sum_{k_1=0}^{n_1} \sum_{k_2=0}^{[n_2/2]} \frac{(-n_1)_{k_1}(-n_2)_{2k_2}}{k_1!k_2!} \frac{\Gamma(1+\alpha+\beta n_1)}{n_1!\Gamma(1+\alpha+\beta k_1)} (-1)^{k_2} t^{k_1+k_2}$$

$$H_{0,1\ :P',Q';\cdots;P^{(r)},Q^{(r)};1,1\ ;\cdots;\ 1,1}^{0,0\ :M',N';\cdots;M^{(r)},N^{(r)};1,1\ ;\cdots;\ 1,1} \left[\begin{array}{l} z_1 t^{u_1} \\ \vdots \\ z_r t^{u_r} \\ \alpha_1 t^{l_1} \\ \vdots \\ \alpha_\tau t^{l_\tau} \end{array} \middle| \begin{array}{l} \text{------------} : \\ (1-\rho-l_1\sigma_1-\cdots-\ l_\tau\sigma_\tau-k_1-k_2; u_1,\cdots,u_r,l_1,\cdots,l_\tau) : \end{array} \right.$$

$$\left. \begin{array}{l} \left(c_j',\ \gamma_j'\right)_{1,P'};\cdots;\left(c_j^{(r)},\ \gamma_j^{(r)}\right)_{1,P^{(r)}};(1-\sigma_1,1)\ ;\cdots;(1-\sigma_\tau,1) \\ \left(d_j',\ \delta_j'\right)_{1,Q'};\cdots;\left(d_j^{(r)},\ \delta_j^{(r)}\right)_{1,Q^{(r)}}\ ;\quad (0,1)\quad ;\cdots;(0,1) \end{array} \right]$$

provided that the conditions easily obtainable from (2.1) are satisfied. Further, on taking $\beta = 1$ in (3.1), the Konhauser biorthogonal polynomials involved therein reduces to the Laguerre polynomials $L_{n_1}^{(\alpha)}\left(\frac{1}{s}\right)$and we get the corresponding inverse Laplace transform involving this polynomials.

(ii) On reducing the product of r distinct (Fox) H-functions of one variable occurring in the left-hand side of (3.1) to the product of r different modified Bessel functions of the second kind [12, p.18, Eq.(2.6.6)], we arrive at the following result after a little simplification

$$
\begin{aligned}
(3.2) \qquad & L^{-1}\left\{ s^{-\left(\rho+\frac{n_1}{2}+\frac{r}{2}\right)} \prod_{i=1}^{\tau} \left(s^{l_i}+\alpha_i\right)^{-\sigma_i} Z_{n_1}^{\alpha}\left[\left(\frac{1}{s}\right)^{1/\beta};\beta\right] \right. \\
& \left. H_{n_2}\left[\frac{\sqrt{s}}{2}\right] \prod_{i=1}^{r} K_{\nu_i}\left[\frac{z_i}{s}\right] \right\} \\
= \; & 2^{-\frac{3r}{2}} \prod_{i=1}^{r} (z_i)^{-1/2} \frac{t^{\rho+l_1\sigma_1+\cdots+l_\tau\sigma_\tau-1}}{\Gamma(\sigma_1)\cdots\Gamma(\sigma_\tau)} \sum_{k_1=0}^{n_1} \sum_{k_2=0}^{[n_2/2]} \frac{(-n_1)_{k_1}(-n_2)_{2k_2}}{k_1!k_2!} \\
& \frac{\Gamma(1+\alpha+\beta n_1)}{n_1!\Gamma(1+\alpha+\beta k_1)} (-1)^{k_2} t^{k_1+k_2} \\
& H_{0,1\ :0,2\ ;\cdots;0,2;1,1\ ;\cdots;1,1}^{0,0\ :2,0;\cdots;\ 2,0;1,1\ ;\cdots;1,1} \left[\begin{array}{l} z_1 t/2 \\ \vdots \\ z_r t/2 \\ \alpha_1 t^{l_1} \\ \vdots \\ \alpha_\tau t^{l_\tau} \end{array} \right| \\
& \qquad \qquad \text{--------------} \qquad : \\
& \left(1-\rho-l_1\sigma_1-\cdots-\ l_\tau\sigma_\tau-k_1-k_2;\frac{1,\cdots,1}{r},l_1,\cdots,l_\tau\right): \\
& \begin{array}{lllll} \text{-------} & ;\cdots; & \text{-------} & ;(1-\sigma_1,1) \;\; ;\cdots; & (1-\sigma_\tau,1) \\ \left(\frac{1}{4}\pm\frac{\nu_1}{2},\frac{1}{2}\right) & ;\cdots; & \left(\frac{1}{4}\pm\frac{\nu_r}{2},\frac{1}{2}\right) & ;(0,1) \;\; ;\cdots; & (0,1) \end{array} \Bigg],
\end{aligned}
$$

where $\mathrm{Re}(s)>0$, $z_i>0$, $\mathrm{Re}(\rho+l_1\sigma_1+\cdots+l_\tau\sigma_\tau\pm\nu_i+\frac{1}{2})>0$, $i=1,\cdots,r$ $0<l_j<1$, $|\arg\ \alpha_j|<(1-l_j)\frac{\pi}{2}$, $j=1,\cdots,\tau$ or $l_1=\cdots=l_\tau=1$ and $\alpha_1>0,\ \cdots,\ \alpha_\tau>0$.

The inverse Laplace transform of the product of a wide variety of polynomials (which are special cases of $S_{n_1}^{m_1},\cdots,S_{n_\varsigma}^{m_\varsigma}$) and numerous other simple special functions involving one or more variables (which are particular cases of the multivariable H-function) can also be obtained from our main result but we do not record them here for lack of space.

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