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A Note on Regular Ternary Semirings

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Abstract. This paper is a sequel of our previous paper [1]. In this paper, we introduce the notions of regular ideal and partial ideal (p-ideal) in a ternary semiring and using these two notions we characterize regular ternary semiring.

1. Introduction

The literature of ternary algebraic system was introduced by D. H. Lehmer [9] in 1932. He investigated certain ternary algebraic systems called triplexes which turn out to be ternary groups. The notion of ternary semigroups was known to S. Banach. He showed by an example that a ternary semigroup does not necessarily reduce to an ordinary semigroup. In [12], M. L. Santiago developed the theory of ternary semigroups and semiheaps. He devoted his attention mainly to the study of regular ternary semigroups, completely regular ternary semigroups, bi-ideals and intersection ideals in ternary semigroups, the standard embedding of a ternary semigroup and a semiheap with some of their applications. In [10], W. G. Lister characterized those additive subgroups of rings which are closed under the triple ring product and he called this algebraic system a ternary ring. He also studied the embedding of ternary rings, representation of ternary rings in terms of modules, semisimple ternary rings with minimum condition and radical theory of such rings.

We consider the ring of integers $\mathbb{Z}$ which plays a vital role in the literature of ring. The subset $\mathbb{Z}^+$ of all positive integers of $\mathbb{Z}$ is an additive commutative semigroup which is closed under the binary product i.e., $\mathbb{Z}^+$ forms a semiring. Now if we consider the subset $\mathbb{Z}^-$ of all negative integers of $\mathbb{Z}$ then we see that $\mathbb{Z}^-$ is an additive commutative semigroup which is closed under the ternary product; however, $\mathbb{Z}^-$ is not closed under the binary product, i.e. $\mathbb{Z}^-$ does not form a semiring. Taking these fact in mind we introduce the notion of ternary semiring in [1] which is a generalization of ternary ring introduced by Lister [10] in 1971. $\mathbb{Z}^-$ is a natural example of a ternary semiring. Thus we see that in the ring of integers $\mathbb{Z}$, $\mathbb{Z}^+$ forms a semiring where as $\mathbb{Z}^-$ forms a ternary semiring. More generally; in

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a partially ordered ring, we can see that its positive cone forms a semiring where as its negative cone forms a ternary semiring. Thus a ternary semiring may be considered as a counterpart of semiring in a partially ordered ring.

In [1], we introduced the notion of ternary semiring, regular ternary semiring and k-regular ternary semiring and studied some of their properties. In [2] and [3], we studied prime and semiprime ideals of ternary semirings respectively. In [4], we introduced and studied the notion of ternary semifield. In [5], [6], [7], we studied Jacobson radical of a ternary semiring.

Our main purpose of this paper is to give a characterization of regular ternary semiring by using the notions of regular ideal and partial ideal (p-ideal). In § 2, we give some basic definitions and examples. In § 3, we introduced the notions of regular ideal and partial ideal (p-ideal) and study regular ternary semiring by using these two notions.

2. Preliminaries

Definition 2.1. A non-empty set S together with a binary operation, called addition and a ternary multiplication, denoted by juxtaposition, is said to be a ternary semiring if S is an additive commutative semigroup satisfying the following conditions:

(i) $(abc)de = a(bcd)e = ab(cde)$,

(ii) $(a+b)cd = acd + bcd$,

(iii) $a(b+c)d = abd + acd$,

(iv) $ab(c+d) = abc + abd$ for all $a, b, c, d, e \in S$.

Example 2.2. Let S be the set of all continuous functions $f : X \longrightarrow \mathbb{R}^-$, where X is a topological space and $\mathbb{R}^-$ is the set of all negative real numbers.
Now we define a binary addition and a ternary multiplication on S in the following way :

i) $(f+g)(x) = f(x) + g(x)$,

ii) $(fgh)(x) = f(x)g(x)h(x)$ for all $f, g, h \in S$ and $x \in X$.

Then together with the binary addition and the ternary multiplication, S forms a ternary semiring.

Remark 2.3. We note that the positive real valued continuous functions form a **Semiring** where as the negative real valued continuous functions form a **Ternary Semiring**.

Definition 2.4. Let S be a ternary semiring. If there exists an element $0 \in S$ such that $0 + x = x$ and $0xy = x0y = xy0 = 0$ for all $x, y \in S$ then '0' is called the zero element or simply the zero of the ternary semiring S. In this case we say that S is a ternary semiring with zero.

We note that a ternary semiring may not contain an identity but there are certain ternary semirings which generate identities in the sense defined below :

Definition 2.5. A ternary semiring S admits an identity provided that there exist elements $\{(e_i, f_i) \in S\ (i = 1, 2, \cdots, n)\}$ such that $\sum_{i=1}^{n} e_i f_i x = \sum_{i=1}^{n} e_i x f_i = \sum_{i=1}^{n} x e_i f_i = x$ for all $x \in S$. In this case, the ternary semiring S is said to be a ternary semiring with identity $\{(e_i, f_i) : i = 1, 2, \cdots, n\}$.

In particular, if there exists an element $e \in S$ such that $eex = exe = xee = x$ for all $x \in S$, then 'e' is called the unital element of the ternary semiring S.

It is easy to see that $xye = (exe)ye = ex(eye) = exy$ and $xye = x(eye)e = xe(yee) = xey$ for all $x, y \in S$. So we have the following result :

Proposition 2.6. *If e is a unital element of a ternary semiring S, then $exy = xey = xye$ for all $x, y \in S$.*

Definition 2.7. An element x of a ternary semiring S is called idempotent if $xxx = x^3 = x$.

Example 2.8. $\mathbb{Z}_0^-$ forms a ternary semiring with zero element '0' and -1 is the unital element of the ternary semiring $\mathbb{Z}_0^-$. -1 is also the idempotent element of $\mathbb{Z}_0^-$.

Example 2.9. Let S be the set of all real numbers and k be a fixed number in S. If we define a binary addition and a ternary multiplication in S by $a + b = 0$ and $abc = a + b + c + k$, then with binary addition and ternary multiplication S is a ternary semiring with $-\frac{k}{2}$ is a unital element.

Throughout this paper S will always denote a ternary semiring with zero and unless otherwise stated a ternary semiring means a ternary semiring with zero.

Definition 2.10. An additive subsemigroup T of S is called a ternary subsemiring of S if $t_1 t_2 t_3 \in T$ for all $t_1, t_2, t_3 \in T$.

Definition 2.11. An additive subsemigroup I of S is called a left (respectively right, lateral) ideal of S if $s_1 s_2 i$ (respectively $i s_1 s_2, s_1 i s_2$) $\in I$ for all $s_1, s_2 \in S$ and $i \in I$. If I is both left and right ideal of S then I is called a two-sided ideal of S. If I is a left, a right, a lateral ideal of S then I is called an ideal of S.

An ideal I of S is called a proper ideal if $I \neq S$.

Definition 2.12. An ideal I of a ternary semiring S is called a k-ideal if $x + y \in I$; $x \in S$, $y \in I$ imply that $x \in I$.

Proposition 2.13. *Let S be a ternary semiring and $a \in S$. Then the principal*

(i) *left ideal generated by a is given by $\langle a \rangle_l = \{ \sum r_i s_i a + na : r_i, s_i \in S; n \in Z_0^+ \}$,*

(ii) *right ideal generated by a is given by $\langle a \rangle_r = \{ \sum a r_i s_i + na : r_i, s_i \in S; n \in Z_0^+ \}$,*

(iii) *two-sided ideal generated by a is given by* $\langle a\rangle_t = \{\sum p_i q_i a + \sum a r_j s_j + \sum p'_k q'_k a r'_k s'_k + na : p_i, q_i, r_j, s_j, p'_k, q'_k, r'_k, s'_k \in S; n \in Z_0^+\}$,

(iv) *lateral ideal generated by a is given by* $\langle a\rangle_m = \{\sum r_i a s_i + \sum p_j q_j a r_j s_j + na : p_j, q_j, r_i, s_i \in S; n \in Z_0^+\}$,

(v) *ideal generated by a is given by* $\langle a\rangle = \{\sum p_i q_i a + \sum a r_j s_j + \sum u_k a v_k + \sum p'_l q'_l a r'_l s'_l + na : p_i, q_i, r_j, s_j, u_k, v_k, p'_l, q'_l, r'_l, s'_l \in S; n \in Z_0^+\}$,

where $\sum$ *denotes the finite sum and* Z_0^+ *is the set of all positive integers with zero.*

Definition 2.14 ([2]). A proper ideal P of a ternary semiring S is called a prime ideal of S if $ABC \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$ for any three ideals A, B, C of S.

Definition 2.15 ([3]). A proper ideal Q of a ternary semiring S is called a semiprime ideal of S if $I^3 \subseteq Q$ implies $I \subseteq Q$ for any ideal I of S.

Definition 2.16 ([3]). A proper ideal I of a ternary semiring S is said to be strongly irreducible if for ideals H and K of S, $H \bigcap K \subseteq I$ implies that $H \subseteq I$ or $K \subseteq I$.

3. Regular ternary semiring

Definition 3.1 ([1]). An element a in a ternary semiring S is called regular if there exists an element x in S such that $axa = a$. A ternary semiring is called regular if all of its elements are regular.

Example 3.2. Let $S = \{(m, n) : m, n$ are non-positive rational numbers $\}$. Then it can be easily verified that w. r. t. componentwise usual binary addition and ternary multiplication of rational numbers S is a regular ternary semiring.

Proposition 3.3. *A ternary semiring S is regular if and only if for each $a \in S$, there exist $x_1, x_2, y_1, y_2, z_1, z_2 \in S$ such that $a = (ax_1x_2)(y_1ay_2)(z_1z_2a)$.*

Now we have the following characterization theorem for regular ternary semiring:

Theorem 3.4. *The following conditions in a ternary semiring S are equivalent*:

(i) *S is regular*;

(ii) *For any right ideal R, lateral ideal M and left ideal L of S, $RML = R \cap M \cap L$*;

(iii) *For $a, b, c \in S$, $\langle a\rangle_r \langle b\rangle_m \langle c\rangle_l = \langle a\rangle_r \cap \langle b\rangle_m \cap \langle c\rangle_l$*;

(iv) *For $a \in S$, $\langle a\rangle_r \langle a\rangle_m \langle a\rangle_l = \langle a\rangle_r \cap \langle a\rangle_m \cap \langle a\rangle_l$.*

Proof. (i) $\Longrightarrow$ (ii). Suppose S is a regular ternary semiring.

Let R, M and L be a right ideal, a lateral ideal and a left ideal of S respectively. Then clearly, $RML \subseteq R \cap M \cap L$. Now for $a \in R \cap M \cap L$, we have $a = axa$ for some $x \in S$. This implies that $a = axa = (axa)(xax)(axa) \in RML$. Thus we have $R \cap M \cap L \subseteq RML$. So we find that $RML = R \cap M \cap L$.

Clearly, (ii) $\Longrightarrow$ (iii) and (iii) $\Longrightarrow$ (iv).

To complete the proof, it remains to show that (iv) $\Longrightarrow$ (i).

Let $a \in S$. Clearly, $a \in \langle a \rangle_r \cap \langle a \rangle_m \cap \langle a \rangle_l = \langle a \rangle_r \langle a \rangle_m \langle a \rangle_l$. Then we have, $a \in (aSS + na)(SaS + SSaSS + na)(SSa + na) \subseteq aSa$. So we find that $a \in aSa$ and hence there exists an elements $x \in S$ such that $a = axa$. This implies that a is regular and hence S is regular. □

We note that every left and right ideal of a regular ternary semiring may not be a regular ternary semiring; however, for a lateral ideal of a regular ternary semiring, we have the following result :

Lemma 3.5 ([1]). *Every lateral ideal of a regular ternary semiring S is a regular ternary semiring.*

Definition 3.6. An ideal I of a ternary semiring S is called a regular ideal if $I + RML = R \cap M \cap L$ for any right ideal $R \supseteq I$, lateral ideal $M \supseteq I$ and left ideal $L \supseteq I$.

Remark 3.7. From Definition 3.6, it follows that S is always a regular ideal and any ideal that contains a regular ideal is also a regular ideal. Now if for any right ideal R, lateral ideal M and left ideal L; RML contains a regular ideal, then $RML = R \cap M \cap L$.

Proposition 3.8. *A ternary semiring S is a regular ternary semiring if and only if $\{0\}$ is a regular ideal of S.*

Let N be the nuclear ideal of a ternary semiring S i.e., the intersection of all non-zero ideals of S, N_r the intersection of all non-zero right ideals of S, N_m the intersection of all non-zero lateral ideals of S and N_l the intersection of all non-zero left ideals of S. Now if $N = \{0\}$, then clearly $N = N_r = N_m = N_l$.

Theorem 3.9. *Let S be a ternary semiring and $N = N_r = N_m = N_l$. Then S is a regular ternary semiring if and only if N is a regular ideal of S.*

Proof. If $N = N_r = N_m = N_l = \{0\}$, then proof follows from Proposition 3.8. So we suppose that $N = N_r = N_m = N_l \neq \{0\}$.

Let S be a regular ternary semiring. Then from Proposition 3.8, it follows that $\{0\}$ is a regular ideal of S. Now, $\{0\} \subseteq N = N_r = N_m = N_l$ implies that N is a regular ideal of S, by using Remark 3.7.

Conversely, let N be a regular ideal of S. Then $N + RML = R \cap M \cap L$ for any right ideal $R \supseteq N$, lateral ideal $M \supseteq N$ and left ideal $L \supseteq N$ of S. Since NNN is a right ideal of S and $N = N_r$, we have $N = N_r \subseteq NNN \subseteq RML$. Consequently, $N + RML = RML$. So $RML = R \cap M \cap L$ and hence from Theorem 3.4, it follows that S is a regular ternary semiring. □

Corollary 3.10. *Let S be a ternary semiring and $N = N_r = N_m = N_l$. Then S is a regular ternary semiring if and only if every ideal of S is regular.*

Proof. Suppose S is a regular ternary semiring. Then from Theorem 3.9, it follows that N is a regular ideal of S. Now $N = N_r = N_m = N_l$ implies that every non-zero ideal of S contains the regular ideal N of S. Consequently, by using Remark 3.7, we find that every ideal of S is regular.

Conversely, if every ideal of S is regular, then N is a regular ideal of S and hence from Theorem 3.9, it follows that S is a regular ternary semiring. □

Theorem 3.11. *The following conditions in a ternary semiring S are equivalent* :

(i) *I is a regular ideal of S;*

(ii) *For $a, b, c \in S$, $I + \langle a\rangle_r\langle b\rangle_m\langle c\rangle_l = I + (\langle a\rangle_r \cap \langle b\rangle_m \cap \langle c\rangle_l)$;*

(iii) *For each $a \in S$, $I + \langle a\rangle_r\langle a\rangle_m\langle a\rangle_l = I + (\langle a\rangle_r \cap \langle a\rangle_m \cap \langle a\rangle_l)$;*

(iv) *For each $a \in S \setminus I = I^c$, $a = x + \sum_{i=1}^{n} ap_iaq_ia + \sum_{i=1}^{n} ar_is_iau_iv_ia$, for some $x \in I$ and $p_i, q_i, r_i, s_i, u_i,\ \ v_i \in S$.*

Proof. (i) $\Longrightarrow$ (ii). Suppose I is a regular ideal of S.

We note that for $a, b, c \in S$, $I \subseteq (I + \langle a\rangle_r), (I + \langle b\rangle_m), (I + \langle c\rangle_l)$.

Now

$$\begin{aligned} I + (\langle a\rangle_r \cap \langle b\rangle_m \cap \langle c\rangle_l) &\subseteq (I + \langle a\rangle_r) \cap (I + \langle b\rangle_m) \cap (I + \langle c\rangle_l) \\ &= I + (I + \langle a\rangle_r)(I + \langle b\rangle_m)(I + \langle c\rangle_l) \text{ (Since } I \text{ is regular)} \\ &\subseteq I + III + I\langle b\rangle_m I + I\langle b\rangle_m\langle c\rangle_l + II\langle c\rangle_l + \langle a\rangle_r II \\ &\quad + \langle a\rangle_r I\langle c\rangle_l + \langle a\rangle_r\langle b\rangle_m I + \langle a\rangle_r\langle b\rangle_m\langle c\rangle_l \\ &\subseteq I + \langle a\rangle_r\langle b\rangle_m\langle c\rangle_l. \end{aligned}$$

Again, $\langle a\rangle_r\langle b\rangle_m\langle c\rangle_l \subseteq \langle a\rangle_r \cap \langle b\rangle_m \cap \langle c\rangle_l$ implies that $I + \langle a\rangle_r\langle b\rangle_m\langle c\rangle_l \subseteq I + \langle a\rangle_r \cap \langle b\rangle_m \cap \langle c\rangle_l$. So we find that $I + \langle a\rangle_r\langle b\rangle_m\langle c\rangle_l = I + \langle a\rangle_r \cap \langle b\rangle_m \cap \langle c\rangle_l$.

Clearly, (ii)$\Longrightarrow$ (iii).

(iii) $\Longrightarrow$ (iv). We first note that $\langle I + \langle a\rangle_r\rangle_r = I + \langle a\rangle_r = I + \langle a\rangle_r \cap S \cap S$ $= I + \langle a\rangle_r SS = I + (aSS + na)SS = I + aSSSS + naSS = I + \langle aSS\rangle_r = I + aSS$. Similarly, we have $\langle I + \langle a\rangle_m\rangle_m = I + SaS + SSaSS$ and $\langle I + \langle a\rangle_l\rangle_l = I + SSa$.

Now

$$\begin{aligned} \langle a\rangle_r \cap \langle a\rangle_m \cap \langle a\rangle_l &\subseteq \langle I + \langle a\rangle_r\rangle_r \cap \langle I + \langle a\rangle_m\rangle_m \cap \langle I + \langle a\rangle_l\rangle_l \\ &\subseteq I + (\langle I + \langle a\rangle_r\rangle_r \cap \langle I + \langle a\rangle_m\rangle_m \cap \langle I + \langle a\rangle_l\rangle_l) \\ &= I + (\langle I + \langle a\rangle_r\rangle_r\langle I + \langle a\rangle_m\rangle_m\langle I + \langle a\rangle_l\rangle_l) \\ &= I + (I + aSS)(I + SaS + SSaSS)(I + SSa) \\ &\subseteq I + (aSaSa + aSSaSSa). \end{aligned}$$

Since $a \in \langle a\rangle_r \cap \langle a\rangle_m \cap \langle a\rangle_l$, there exist $x \in I$ and $p_i, q_i, r_i, s_i, u_i, v_i \in S$ such that $a = x + \sum_{i=1}^{n} ap_iaq_ia + \sum_{i=1}^{n} ar_is_iau_iv_ia$.

(iv)$\Longrightarrow$ (i). Let R, M and L be any right, lateral and left ideal of S respectively such that $R, M, L \supseteq I$. Then clearly, $I + RML \subseteq R \cap M \cap L$. Again, let $a \in R \cap M \cap L$. Then by using condition (iv), we have $a = x + \sum_{i=1}^{n} ap_iaq_ia + \sum_{i=1}^{n} ar_is_iau_iv_ia$ for some $x \in I$ and $p_i, q_i, r_i, s_i, u_i, v_i \in S$. Since $\sum_{i=1}^{n} ap_iaq_ia, \sum_{i=1}^{n} ar_is_iau_iv_ia \in RML$, $a \in I + RML$ and hence $R \cap M \cap L \subseteq I + RML$. Thus $I + RML = R \cap M \cap L$. Consequently, I is a regular ideal. $\square$

Theorem 3.12. *Let I be a regular ideal of a ternary semiring S. For any right ideal R, lateral ideal M and left ideal L of S, if $RML \subseteq I$ then $R \cap M \cap L \subseteq I$.*

Proof. Suppose for any right ideal R, lateral ideal M and left ideal L of S, $RML \subseteq I$, where I is a regular ideal of S. Then $I \subseteq (I + R)$, $(I + M)$, $(I + L)$.

Now

$$\begin{aligned} R \cap M \cap L &\subseteq (I + R) \cap (I + M) \cap (I + L) \\ &= I + ((I + R)(I + M)(I + L)) \text{ (Since } I \text{ is regular)} \\ &\subseteq I + III + IIL + IMI + IML + RII + RIL + RMI + RML \\ &\subseteq I. \end{aligned}$$ $\square$

From Theorem 3.12, we have the following results :

Corollary 3.13. *A regular and strongly irreducible ideal of a ternary semiring S is a prime ideal of S.*

Corollary 3.14. *Every regular ideal of a ternary semiring S is a semiprime ideal of S.*

Let $(S, +, .)$ be a ternary semiring with unital element e.

We define a binary operation '$\circ$' on S by $a \circ b = aeb (= abe = eab)$. Then $(S, +, \circ)$ is a semiring with identity e.

It can be easily verified that $(S, +, .)$ is a regular ternary semiring if and only if $(S, +, \circ)$ is a regular semiring.

Throughout the rest of the paper $(S, +, .)$ denotes a ternary semiring with unital element e and $(S, +, \circ)$ denotes a semiring with identity e.

Definition 3.15. Let S be a ternary semiring with unital element e. An additive subsemigroup I of S is called a

(i) left partial ideal (shortly, left p-ideal) if $sie\ (= esi = sei) \in I$,

(ii) right partial ideal (shortly, right p-ideal) if $ise\ (= ies = eis) \in I$ for all $i \in I$ and $s \in S$.

A left (right) p-ideal of a ternary semiring S with unital element e is called a left (right) p_k-ideal of S if it is a k-ideal of S .

For a ternary semiring S with unital element e, let us put : $aS = aSe$ and $Sa = Sae$ for $a \in S$.

Lemma 3.16. *For any $a \in S$, the set aS is a right p-ideal and the set Sa is a left p-ideal of the ternary semiring $(S, +, .)$.*

Definition 3.17. The p-ideals aS and Sa are respectively called right principal p-ideal and left principal p-ideal generated by 'a'.

Theorem 3.18. *A ternary semiring $(S, +, .)$ is regular if and only if every left (right) principal p-ideal is generated by an idempotent.*

Theorem 3.19. *If a ternary semiring $(S, +, .)$ has the property that every left (right) principal p_k-ideal has its complement which is also a left (right) principal p_k-ideal, then $(S, +, .)$ is a regular ternary semiring.*

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