H_2-optimal Control with Regional Pole Assignment via State Feedback

Guo-Sheng Wang, Bing Liang, and Guang-Ren Duan

Abstract: The design of H_2-optimal control with regional pole assignment via state feedback in linear time-invariant systems is investigated. The aim is to find a state feedback controller such that the closed-loop system has the desired eigenvalues lying in some desired stable regions and attenuates the disturbance between the output vector and the disturbance vector. Based on a proposed result of parametric eigenstructure assignment via state feedback in linear systems, the considered H_2-optimal control problem is changed into a minimization problem with certain constraints, and a simple and effective algorithm is proposed for this considered problem. A numerical example and its simulation results show the simplicity and effectiveness of this proposed algorithm.

Keywords: Eigenstructure assignment, H_2 control, linear systems, optimization, state feedback.

1. INTRODUCTION

In the past years, the H_2-optimal control theory has attracted much attention, see e.g., [1-3] and the references therein. The well known LQG (Linear Quadratic Gaussian) and LQR (Linear Quadratic Regulator) designs in control are examples of the H_2 synthesis procedure. The objective of the H_2 optimal problem is to find a controller that minimizes a quadratic performance index (the H_2 norm) of the system and offers a way of combining the design criteria of quadratic performance and disturbance attenuation. But such a controller design method cannot guarantee that the closed-loop systems have good transient responses.

It is well known that the systems' transient responses are determined mainly by the locations of the systems' eigenvalues. As an important design method associated with eigenvalues and eigenvectors in control theory, eigenstructure assignment has attracted much attention of many researchers, such as [4-21]. One type of approach for eigenstructure assignment is the parametric approach, which parameterizes all the solutions to the problem, such as [12-16]. This method presents complete, explicit and parametric expressions of all the feedback gain matrices and the closed-loop eigenvector matrices. Moreover, this method offers all the design degrees of freedom, which can be further utilized to satisfy some additional performances, such as robustness [17-19].

In this paper, we will consider the application of the parametric eigenstructure assignment approach to the H_2-optimal control with regional pole assignment. There are a few publications on the similar kind of this problem, such as [22-25]. Yang *et al.* [22] investigate the H_∞ design with pole placement constraints via an LMI approach in uncertainty linear systems and propose a necessary and sufficient condition for the solvability of the problem are given in terms of a set of feasible LMIs. Apkarian *et al.* [23] consider the problem of eigenstructure assignment, and H_2 synthesis with enhanced LMI, and the proposed methods involve a specific transformation on the Lyapunov variables and a reciprocal variant of the Projection Lemma. Chilali *et al.* [24] addresses the design of state- or output-feedback H_∞ controllers that satisfy additional constraints on the closed-loop pole location and sufficient conditions for feasibility are derived for a general class of convex regions of the complex plane. Lam *et al.* [25] present a computation method for pole assignment with eigenvalue and stability robustness and the robustness measure is constructed to balance the tradeoff between an eigenvalue sensitivity measure and a stability robustness measure, both defined in terms of the non-differentiable spectral norm. In this paper, the design degrees offered by the parametric eigenstructure assignment method in [12] are utilized to consider the design of H_2-optimal control with regional pole assignment in linear time-invariant systems. The aim is to design a state feedback controller such that the

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closed-loop system has the desired closed-loop poles lying in some desired regions and the disturbance attenuation performance. By utilizing a parametric solution for state feedback eigenstructure assignment proposed, the disturbance attenuation index is parameterized and the considered H_2-optimal control with regional pole assignment is changed into a minimization problem with certain constraints. And then an effective and simple algorithm is proposed.

This paper is organized as follows. The next section gives the description of H_2-optimal control with regional pole assignment. Section 3 proposes the parametric result of eigenstructure assignment via state feedback in linear time-invariant systems. Based on the proposed parametric method of state feedback eigenstructure assignment, solutions to the considered problem are proposed, and a simple and effective algorithm is developed in Section 4. Section 5 presents an illustrative example to show the simplicity and effectiveness of the proposed algorithm. Concluding remarks are drawn in Section 6.

2. PROBLEM FORMULATION

Consider a linear time-invariant continuous system in the form of

$$\begin{cases} \dot{x} = Ax + Bu + Fw \\ y = Cx + Du, \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$ is the state vector, $u \in \mathbf{R}^r$ is the control vector, $w \in \mathbf{R}^l$ is the exogenous input vector and $y \in \mathbf{R}^m$ is the output vector, respectively; A, B, C and D are known matrices with appropriate dimensions with $\operatorname{rank}(B) = r$, and satisfy the following assumption:

Assumption A: The matrix pair (A,B) is controllable, that is,

$$\operatorname{rank}\left(\begin{bmatrix} A - sI_n & B \end{bmatrix}\right) = n, \quad \forall s \in \mathbf{C}.$$

Applying the following state feedback controller

$$u = Kx, \quad K \in \mathbf{R}^{r \times n}, \tag{2}$$

to system (1), obtains the closed-loop system as

$$\begin{cases} \dot{x} = (A + BK)x + Fw \\ y = (C + DK)x. \end{cases} \tag{3}$$

Recall the fact that non-defective matrices possess eigenvalues which are less insensitive with respect to parameter perturbations, in this paper we only consider that the eigenvalues of the closed-loop system (3) are distinct and self-conjugate. Let the closed-loop eigenvalues of system (3) be $s_i \in C$, $i = 1, 2, \cdots, n$, and their corresponding eigenvectors be $v_i \in C^n$, $i = 1, 2, \cdots, n$. Then there hold

$$(A + BK)v_i = s_i v_i, \; i = 1, 2, \cdots, n. \tag{4}$$

From system (3), the closed-loop system transfer function from w to y can be given by

$$T_{yw}(K) = (C + DK)(sI_n - A - BK)^{-1} F. \tag{5}$$

Let P be the positive semi-definite solution of the equation

$$(A + BK)^T P + P(A + BK) + (C + DK)^T (C + DK) = 0. \tag{6}$$

It can be shown that

$$\left\| T_{yw}(K) \right\|_2 = \sqrt{\operatorname{trace}(F^T PF)}, \tag{7}$$

where P is the positive semi-definite solution to (6). Then the problem to be considered in this paper can be described as follows.

Problem H_2: Given system (1) satisfying Assumption A, a group of self-conjugate and distinct scalars $s_i \in \mathbf{C}$, $i = 1, 2, \cdots, n$ and a group of stable regions S_i, $i = 1, 2, \cdots, n$ on the left complex plane. The design objective is to find a state feedback controller (2) to achieve the minimization of $\left\| T_{yw}(K) \right\|_2$ in (7), i.e.

$$\min_K \left\| T_{yw}(K) \right\|_2, \tag{8}$$

which subject to the following conditions:

1) Equations in (4) hold and $\det(V) \neq 0$,
2) $s_i \in S_i$, $i = 1, 2, \cdots, n$.

3. CLOSED-LOOP EIGENSTRUCTURE ASSIGNMENT

Set

$$\Lambda = \operatorname{diag}[s_1 \;\; s_2 \cdots s_n], \; V = [v_1 \;\; v_2 \cdots v_n].$$

Then (4) is equivalent with

$$AV + BKV = V\Lambda. \tag{9}$$

Denote

$$W = KV, \tag{10}$$

there holds

$$AV + BW = V\Lambda. \tag{11}$$

Because Assumption A is satisfied, applying a series of element matrix transformations to $[A - sI_n \;\; B]$, we can obtain a pair of unimodular matrices

$P(s) \in \mathbf{R}^{n\times n}[s]$ and $Q(s) \in \mathbf{R}^{(n+r)\times(n+r)}[s]$ satisfying

$$P(s)\ [A - sI_n \quad B]\ Q(s) = [0 \quad I_n],\ \forall s \in \mathbf{C}. \quad (12)$$

Partition $Q(s)$ into the following form

$$Q(s) = \begin{bmatrix} Q_{11}(s) & Q_{12}(s) \\ Q_{21}(s) & Q_{22}(s) \end{bmatrix},\ Q_{11}(s) \in \mathbf{R}^{n\times r}[s]. \quad (13)$$

Based on the above reasoning, we can give the following theorem which offers the parametric solutions to state feedback eigenstructure assignment for system (1).

Theorem 1 [12]: Given matrices $A \in \mathbf{R}^{n\times n}$ and $B \in \mathbf{R}^{n\times r}$ with full rank, if the matrix pair (A, B) is controllable, then the parametric expressions of all the state feedback gain matrices K in (9) can be given as follows

$$K = WV^{-1}, \quad (14)$$

where

$$V = [v_1 \quad v_2 \quad \cdots \quad v_n],\ v_i = Q_{11}(s_i)f_i, \quad (15)$$

and

$$W = [w_1 \quad w_2 \quad \cdots \quad w_n],\ w_i = Q_{21}(s_i)f_i, \quad (16)$$

where $f_i \in \mathbf{C}^r$, $i = 1, 2, \cdots, n$, are a group of free parametric vectors, and satisfy the following constraints:

Constraint 1: $s_i = \bar{s}_j \Leftrightarrow f_i = \overline{f_j}$, $i, j = 1, 2, \cdots, n$;

Constraint 2: $\det(V) \neq 0$.

From the above Theorem 1, we can find this parametric eigenstructure assignment has the following advantages:

Remark A: The above general parametric expressions (15) for the closed-loop eigenvectors associated with the assigned closed-loop eigenvalues are in a direct closed explicit parametric form, and are thus simpler and more convenient to use. They can be immediately written out as soon as the pair of right coprime polynomial matrices $Q_{11}(s)$ and $Q_{21}(s)$ satisfying (12) are obtained.

Remark B: Both the free parametric vectors $f_i \in C^r$, $i = 1, 2, \cdots, n$ and the undetermined closed-loop eigenvalues $s_i \in C$, $i = 1, 2, \cdots, n$ can be regarded as the design freedom offered by this parametric method. When more requirements beyond the basic closed-loop eigenstructure are imposed on the closed-loop system, we can first turn these requirements into some additional constraints on the closed-loop eigenvalues or/and the parameters $f_i \in C^r$, $i = 1, 2, \cdots, n$, and then solve from (14)-(16) the required solution to the problem by restricting parameters $f_i \in C^r$ and $s_i \in C$, $i = 1, 2, \cdots, n$ to satisfy the set of additional constraints.

4. SOLUTION TO PROBLEM H_2

Based on the parametric results in Theorem 1, we can obtain the following lemma, which gives the parametric solution to (6).

Lemma 1: Given matrices $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times r}$, $C \in \mathbf{R}^{m\times n}$, and $D \in \mathbf{R}^{m\times r}$, where the matrix B is full rank. If the matrix pair (A, B) is controllable, then all the solutions of P in (6) can be given as

$$P = -V^{-T}\left[\frac{(v_i^T C^T + w_i^T D^T)(Cv_j + Dw_j)}{s_i + s_j}\right]_{n\times n} V^{-1}, (17)$$

where

$$V = [v_1 \quad v_2 \quad \cdots \quad v_n],\ W = [w_1 \quad w_2 \quad \cdots \quad w_n]$$

are determined by (15) and (16), respectively.

Proof: Noticing (9), which is equivalent to the following equation

$$A + BK = V\Lambda V^{-1}. \quad (18)$$

Substituting (18) into (6), obtains

$$(V\Lambda V^{-1})^T P + PV\Lambda V^{-1} = -(C + DK)^T (C + DK). \ (19)$$

Again denote

$$\tilde{P} = V^T PV \ \text{ or } \ P = V^{-T}\tilde{P}V^{-1}. \quad (20)$$

Then (19) can be changed into

$$\Lambda\tilde{P} + \tilde{P}\Lambda = -V^T (C + DK)^T (C + DK)V. \quad (21)$$

Denote $\tilde{P} = [\tilde{p}_{ij}]_{n\times n}$, noticing $\tilde{p}_{ij} = \tilde{p}_{ji}$, $i, j = 1, 2, \cdots, n$, from (21) we can obtain

$$\tilde{p}_{ij} = -\frac{(v_i^T C^T + w_i^T D^T)(Cv_j + Dw_j)}{s_i + s_j}. \quad (22)$$

From $P = V^{-T}\tilde{P}V^{-1}$, it is clear to see that (17) holds.

According to Theorem 1 and Lemma 1, we can set

$$P = P(s_i, f_i,\ i = 1, 2, \cdots, n), \quad (23)$$

which denotes that the matrix P in (7) is parameterized by $s_i \in \mathbf{C}$ and $f_i \in \mathbf{C}^r$, $i = 1, 2, \cdots, n$. From Theorem 1, Lemma 1 and the above reasoning, we can obtain the following theorem, which gives the solutions to Problem H_2.

Theorem 2: Given matrices $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times r}$, $C \in \mathbf{R}^{m\times n}$, $D \in \mathbf{R}^{m\times r}$, and $F \in \mathbf{R}^{n\times l}$ in system (1), where the matrix B is full rank. If the matrix pair (A, B) is controllable, the desired state feedback gain matrix K in Problem $\boldsymbol{H_2}$ can be given by (14), where the parameters $s_i \in \mathbf{C}$ and $f_i \in \mathbf{C}^r$, $i=1, 2, \cdots, n$ are determined by the following minimization problem:

$$\min_{\{s_i\}\subseteq C, \{f_i\}\subseteq \mathbf{C}^r} \sqrt{\operatorname{trace}(F^T P(f_i, s_i) F)}, \qquad (24)$$

subject to Constraints 1, 2, and

Constraint 3: $s_i \in S_i$, $i=1, 2, \cdots, n$.

Denote the real finite eigenvalues s_i by δ_i, and the corresponding parameter f_i by h_i; denote a pair of self-conjugate eigenvalues s_i and s_l by $s_i = \overline{s_l} = \delta_i + \delta_l j$, and the corresponding parameter $f_i = \overline{f_l} = h_i + h_l j$, where δ_i and h_i, $i=1, 2, \cdots, n$ are real. Then Constraint 1 automatically holds and Constraint 3 is changed into

Constraint 3': $a_i \le \delta_i \le b_i$, $i=1, 2, \cdots, n$, where a_i and b_i, $i=1, 2, \cdots, n$, are some specified real numbers.

With the above denotations, the minimization index (24) in Theorem 2 can be simplified into the following problem

$$\min_{\{\delta_i\}\subseteq R, \{h_i\}\subseteq R^r} \sqrt{\operatorname{trace}(F^T P(\delta_i, h_i) F)}, \qquad (25)$$

s. t. Constraint 2 and 3'.

Based on Theorem 2, we can develop the following algorithm, which give the detail steps to solve Problem H_2.

Algorithm $\boldsymbol{H_2}$:

1. Compute a pair of unimodular matrices $P(s)$ and $Q(s)$ satisfying (12), and partition $Q(s)$ as in (13).
2. Set the parametric expressions of free vectors f_i, $i=1, 2, \cdots, n$, and compute the parametric expressions of matrices V and W from (15) and (16).
3. From (17), compute the parametric expressions of P.
4. Determine $s_i \in \mathbf{C}$ and $f_i \in \mathbf{C}^r$, $i=1, 2, \cdots, n$ satisfying Constraints 1-3 or $\delta_i \in \mathbf{R}$ and $h_i \in \mathbf{R}^r$, $i=1, 2, \cdots, n$ satisfying constraint 2 and 3', by solving the minimization problem (24) or (25).
5. Compute the gain matrix K, from (14) and the obtained matrices V and W.

Obviously, the above algorithm is in sequential order, while no 'going back' procedures are involved. Further, because of the completeness of the eigenstructure assignment approach used, the optimality of the solution to Problem H_2 is totally dependent on the solution to the optimization problem (24) or (25). For solution to these minimization problems, there are many software packages that can be used, such as

1) We have found that the Matlab package FMINCON is very reliable and suitable for solving these minimization problems, where FMINCON finds a constrained minimum of a function of several variables and solves problems of the form:

$$\min_X F(X)$$

subject to $AX \le B$ or/and $AX = B$ (linear constraints); $C(X) \le 0$ or/and $C(X) = 0$ (nonlinear constraints); $LB \le X \le UB$.

2) We can also solve the optimal problem by the gradient method. Denote

$$J_1 = \sqrt{\operatorname{trace}(F^T P(f_i, s_i) F)},$$

and

$$J_2 = \sqrt{\operatorname{trace}(F^T P(\delta_i, h_i) F)},$$

then the necessary conditions for the optimal problem (24) or (25) are, respectively,

$$\frac{\partial J_1}{\partial s_i} = 0, \quad \frac{\partial J_1}{\partial f_i} = 0, \quad i=1, 2, \cdots, n,$$

where the parameters $s_i \in \mathbf{C}$ and $f_i \in \mathbf{C}^r$, $i=1, 2, \cdots, n$ satisfy Constraints 1-3, and

$$\frac{\partial J_2}{\partial \delta_i} = 0, \quad \frac{\partial J_2}{\partial h_i} = 0, \quad i=1, 2, \cdots, n,$$

where the parameters $\delta_i \in \mathbf{R}$ and $h_i \in \mathbf{R}^r$, $i=1, 2, \cdots, n$ satisfy Constraint 2 and 3'.

5. AN ILLUSTRATIVE EXAMPLE

Consider a linear system in the form of (1) with the following matrices

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix},$$

$$D = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

It is easy to find that the matrix pair (A, B) is controllable. Assume that the closed-loop eigenvalues are

$$-3 < s_1 < -1, \quad -7 \le re(s_{2,3}) \le -4, \quad -8 \le im(s_{2,3}) \le 8.$$

Thus from Algorithm H_2, we have the following steps:

1) By applying a series of element matrix transformations to $[A - sI_3 \;\; B]$, we can obtain the unimodular matrix $Q(s)$ and partition it as follows

$$Q(s) = \left[\begin{array}{cc|ccc} 1 & 0 & \times & \times & \times \\ s & 0 & \times & \times & \times \\ 0 & 1 & \times & \times & \times \\ \hline 0 & s-1 & \times & \times & \times \\ s^2 & -1 & \times & \times & \times \end{array}\right].$$

2) Set $f_i = [a_i \;\; b_i]$, $i = 1, 2, 3$. From (15) and (16), we can obtain

$$V = \begin{bmatrix} a_1 & a_2 & a_3 \\ s_1 a_1 & s_2 b_2 & s_3 b_3 \\ b_1 & b_2 & b_3 \end{bmatrix},$$

$$W = \begin{bmatrix} (s_1 - 1)b_1 & (s_2 - 1)b_2 & (s_3 - 1)b_3 \\ s_1^2 a_1 - b_1 & s_2^2 a_2 - b_2 & s_3^2 a_3 - b_3 \end{bmatrix}.$$

3) From (17), we can easily obtain the parametric expression of P.

4) By utilizing the function FMINCON in toolbox of Matlab, we can solve the minimization problem (25) and obtain the minimization value as 2.7424. In this case,

$$s_1 = -1, \quad s_{2,3} = -4 \pm 0.1783i,$$

$$f_1 = \begin{bmatrix} 4.1987 \\ -3.3710 \end{bmatrix}, \quad f_{2,3} = \begin{bmatrix} -0.1439 \mp 0.3184i \\ 1.7984 \mp 0.3101i \end{bmatrix}.$$

5) From (14), we obtain the desired gain matrix as

$$K = \begin{bmatrix} -3.5074 & -1.1046 & -4.9927 \\ -3.9225 & -5.0073 & -0.8945 \end{bmatrix}.$$

Arbitrarily choosing

$$s_1^0 = -1, \quad s_2^0 = -4 \pm 0.1783i,$$

$$f_1^0 = \begin{bmatrix} 5 \\ 20 \end{bmatrix}, \quad f_{2,3}^0 = \begin{bmatrix} 2 \pm 2i \\ 3 \pm 4i \end{bmatrix},$$

obtains the value of (25) as 11.8867 and the arbitrarily choosing gain matrix as

$$K^0 = \begin{bmatrix} 32.5117 & 6.6196 & -8.4730 \\ 17.9392 & -1.5270 & -5.6166 \end{bmatrix}.$$

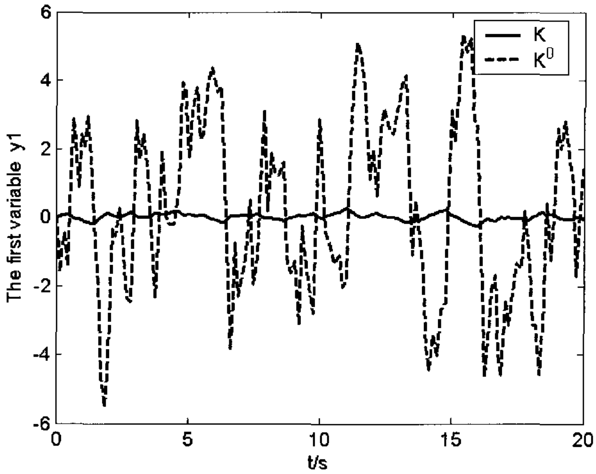

Fig. 1. Comparisons of the first output errors under K and K^0.

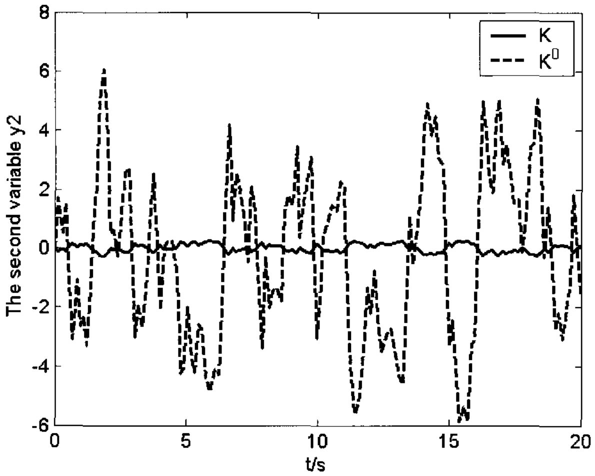

Fig. 2. Comparisons of the second output errors under K and K^0.

In order to further show the effect of Algorithm H_2, we select the random numbers in $[-1, 1]$ as the disturbance. In Figs. 1 and 2, "K" represents the output errors between the system without disturbances and the system with disturbances under the desired gain K, and "K^0" represents the output errors between the system without disturbances and the system with disturbances under the arbitrarily choosing gain K^0. Then we can find that the outputs of the system with no disturbances are very close to those of the system with disturbances under the desired gain K, while the outputs of the system with no disturbances are far from those of the system with disturbances under the arbitrarily choosing gain K^0.

6. CONCLUSION

By utilizing a parametric method of state feedback eigenstructure assignment, a new design method for

H_2–optimal control with regional pole assignment via state feedback in linear time-invariant systems is proposed in this paper. By using this proposed method, the closed-loop system has the desired eigenvalues lying in some desired stable regions and the H_2 norm is minimized. Thus this method can guarantee that the closed-loop systems have good transient responses and the disturbances can be attenuated from the outputs. An illustrative example and the simulation results show the benefits of this proposed method.

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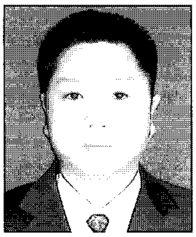

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