H_2/H_∞ FIR Filters for Discrete-time State Space Models

Young Sam Lee, Soo Hee Han, and Wook Hyun Kwon

Abstract: In this paper a new type of filter, called the H_2/H_∞ FIR filter, is proposed for discrete-time state space signal models. The proposed filter requires linearity, unbiased property, FIR structure, and independence of the initial state information in addition to the performance criteria in both H_2 and H_∞ sense. It is shown that H_2, H_∞, and H_2/H_∞ FIR filter design problems can be converted into convex programming problems via linear matrix inequalities (LMIs) with a linear equality constraint. Simulation studies illustrate that the proposed FIR filter is more robust against temporary uncertainties and has faster convergence than the conventional IIR filters.

Keywords: H_2/H_∞ FIR filter, initial state independency, LMI, unbiased property.

1. INTRODUCTION

The estimation problem deals with recovering some unknown parameters or variables from measured information in physical or mathematical models. Among estimation problems, the state estimator, called the filter, has been widely investigated for wide applications. The performance of the filter is measured by stability, small error, and insensitivity or robustness to signal model uncertainties and disturbances.

For a small error, it is usual to require the filter to be unbiased. For stochastic systems, an unbiased filter means that no matter what the real state is, the filter will follow it on the average. This also means that if there is no noise in the systems the filter will follow the real state exactly. In a similar way to the stochastic case, filters for deterministic systems can adopt the unbiased property in a deterministic sense. The unbiasedness for deterministic systems requires the filters to match exactly the real states of systems with zero disturbances. In short, the unbiased property will be used even for deterministic systems throughout this paper. The terminology 'deadbeat' has also been used in other studies instead of 'unbiased' [1,2].

Some prefer finite impulse response (FIR) filters to infinite impulse response (IIR) filters for robustness and stability. FIR filters make use of a finite number of measurements and inputs on the most recent time interval $[k-N,\ k-1]$, called the receding horizon, or the moving window. FIR filters for signal reconstruction have long been researched. However, FIR filters for state reconstruction have recently been investigated [3-6]. It has been generally accepted that the FIR structure is more robust to temporary modeling uncertain parameters and numerical errors than the IIR structure. Additionally, bounded input bounded output (BIBO) stability is always guaranteed for FIR filters.

In conventional filters that estimate states, the initial state information is often assumed to be known, which in practice, is often not the case. Therefore, in this paper the initial state information is assumed to be completely unknown. That is, the suggested filters will be obtained independently of the initial state information.

In this paper a linear FIR filter that is independent of the initial state information is represented by

$$\hat{x}_k = \sum_{i=k-N}^{k-1} H_{k-i}y_i + \sum_{i=k-N}^{k-1} L_{k-i}u_i \tag{1}$$

at time k for some gains H_{k-i} and L_{k-i}. The filter gains H_{k-i} and L_{k-i} are independent of the initial state information.

Filter properties depend heavily on the performance criterion. In the H_2 performance criterion, the H_2 norm of the transfer function from the disturbance to the estimation error is minimized [7-9]. This approach has been widely used and researched because it is tractable mathematically. In the H_∞ performance criterion, the worst case gain between disturbance and estimation error is minimized [10-14]. More recently,

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there have been approaches that consider both of the performance criteria simultaneously [15]. In this paper, we will take those two performance criteria into account to obtain the optimal filter for state space models.

Existing FIR filters are mainly focused on the minimum variance criterion that is a special case of the H_2 performance criterion [3-6]. The H_∞ FIR filtering problem was first considered in [16]. The H_∞ FIR filter presented in [16] is obtained by repeatedly solving a finite horizon H_∞ filtering problem. However, in practice it neither guarantees the H_2 norm bound nor has independence from the initial state. H_∞ FIR filter for signal reconstruction was considered [17]. However, that result is not applicable to state reconstruction problems. To the best of our knowledge, there exists no H_∞ FIR filter for state reconstruction that guarantees the H_∞ norm bound. In this paper, we reformulate the H_2 FIR filter problem in terms of linear matrix inequalities (LMI) for the first time. H_∞ FIR filter that guarantees the H_∞ norm bound is proposed in terms of LMI for the first time. Finally, by combining those two results, mixed H_2/H_∞ FIR filter will be presented.

The proposed H_2/H_∞ FIR filter is both unbiased and optimal by design for the given performance criterion. The phrase 'by design' means that the unbiased property and optimality are simultaneously built into the proposed FIR filter during its design. Actually, the unbiased property of the proposed FIR filter avoids the unnecessary large estimation error.

This paper is organized as follows. In Section 2, preliminaries and the problem statement are presented. In Section 3, H_2, H_∞, and H_2/H_∞ FIR filtering problems are formulated in terms of linear matrix inequalities. In Section 4, a numerical example is given. Finally, the conclusions are presented in Section 5.

Notation: Throughout the paper the superscript T represents matrix transposition and $*$ denotes the symmetric entries of a symmetric matrix, implied by symmetry. The matrix I denotes an identity matrix with appropriate dimensions. R^n denotes any real n-dimensional linear vector space and $R^{m\times n}$ is the set of all $m\times n$ real matrices. The notation $P>0$ ($P<0$) means that P is symmetric and positive-definite (negative-definite).

2. PROBLEM STATEMENT

Consider the following linear discrete-time state space signal model

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Gw_k, \\ y_k &= Cx_k + Dw_k, \end{aligned} \tag{2}$$

where $x_k \in R^n$ is the state, $u_k \in R^l$ is the input, $y_k \in R^q$ is the measured output, and $w_k \in R^p$ is the disturbance input. In the case of no disturbance input, system (2) becomes

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k, \\ y_k &= Cx_k. \end{aligned} \tag{3}$$

The system in (3) will be called the nominal system.

Conventional filters of IIR structure are of the following form:

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + K(y_k - C\hat{x}_k), \tag{4}$$

where K is the filter gain matrix. Estimation error at step k is defined to be $e_k = \hat{x}_k - x_k$. Define $T_K(z)$ to be the transfer function from the disturbance input w to the estimation error e. Then, depending on estimation performance criterion, three filtering problems of the IIR type are formulated as follows:

- H_2 filtering problem: Find the filter (4) that minimizes $\|T_K(z)\|_2$.
- H_∞ filtering problem: Find the filter (4) that minimizes $\|T_K(z)\|_\infty$.
- H_2/H_∞ filtering problem: Find the filter (4) that minimizes $\|T_K(z)\|_\infty$ subject to $\|T_K(z)\|_2 < \beta$ (or minimizes $\|T_K(z)\|_2$ subject to $\|T_K(z)\|_\infty < \beta$).

In a similar fashion to the IIR case we can formulate three different FIR filtering problems depending on the performance criterion. The aim of this paper is to develop design methods for FIR filters with a batch form

$$\hat{x}_k = HY_{k-1} + LU_{k-1} \tag{5}$$

as solutions to those three FIR filtering problems. H and L in (5) are the gain matrices of a linear filter represented by

$$\begin{aligned} H &\triangleq [H_N \quad H_{N-1} \quad \cdots \quad H_1], \\ L &\triangleq [L_N \quad L_{N-1} \quad \cdots \quad L_1]. \end{aligned}$$

U_{k-1} and Y_{k-1} are defined as

$$U_{k-1} \triangleq [u_{k-N}^T \quad u_{k-N+1}^T \quad \cdots \quad u_{k-1}^T]^T, \tag{6}$$

$$Y_{k-1} \triangleq [y_{k-N}^T \quad y_{k-N+1}^T \quad \cdots \quad y_{k-1}^T]^T. \tag{7}$$

u_{k-i} and y_{k-i}, where $i=1,\cdots,N$, are the inputs and outputs, respectively, at time $k-i$. It is noted that the estimate $\hat{x}_k$ in (5) is a linear function of the finite number of inputs and measurements on the most recent time interval $[k-N,\ k-1]$, called the horizon. N, which is a positive integer, is a horizon length.

We require that the filter in (5) be independent of any *a priori* information about the horizon initial state, x_{k-N}, by making a filter of FIR structure. Furthermore, we require an unbiased property that the FIR filter in (5) satisfies the following relation for the nominal system (3):

$$\hat{x}_k = x_k \quad \text{for any} \quad x_{k-N}. \tag{8}$$

To determine the constraint required for (8) to be satisfied, denote the measurements on the most recent time interval $[k-N, k-1]$ in terms of the state x_k at the current time k as

$$Y_{k-1} = \bar{C}_N x_k + \bar{B}_N U_{k-1} + (\bar{G}_N + \bar{D}_N) W_{k-1}, \tag{9}$$

where

$$W_{k-1} \triangleq [w_{k-N}^T \quad w_{k-N+1}^T \quad \cdots \quad w_{k-1}^T]^T. \tag{10}$$

$\bar{C}_N$, $\bar{B}_N$, $\bar{G}_N$, $\bar{D}_N$ are constant matrices obtained as follows:

$$\bar{C}_N \triangleq \begin{bmatrix} CA^{-N} \\ \vdots \\ CA^{-2} \\ CA^{-1} \end{bmatrix}, \tag{11}$$

$$\bar{B}_N \triangleq -\begin{bmatrix} CA^{-1}B & CA^{-2}B & \cdots & CA^{-N}B \\ 0 & CA^{-1}B & \cdots & CA^{-N+1}B \\ 0 & 0 & \cdots & CA^{-N+2}B \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & CA^{-1}B \end{bmatrix}, \tag{12}$$

$$\bar{G}_N \triangleq -\begin{bmatrix} CA^{-1}G & CA^{-2}G & \cdots & CA^{-N}G \\ 0 & CA^{-1}G & \cdots & CA^{-N+1}G \\ 0 & 0 & \cdots & CA^{-N+2}G \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & CA^{-1}G \end{bmatrix}, \tag{13}$$

$$\bar{D}_N \triangleq diag\{\underbrace{D, D, \cdots D}_{N}\}. \tag{14}$$

For a nominal system (3) we obtain, from (9),

$$\hat{x}_k = HY_{k-1} + LU_{k-1} = H\bar{C}_N x_k + H\bar{B}_N U_{k-1} + LU_{k-1}.$$

Therefore, the constraints on H and L required to satisfy (8) are given by

$$H\bar{C}_N = I, \quad H\bar{B}_N = -L. \tag{15}$$

From (15), we rewrite the FIR filter in (5) as

$$\hat{x}_k = H(Y_{k-1} - \bar{B}_N U_{k-1}), \quad H\bar{C}_N = I. \tag{16}$$

The constraint $H\bar{C}_N = I$ will be called the *unbiased constraint* in the sense that it is an unbiased constraint for the nominal system (3) with zero disturbance, but may not be an unbiased constraint for the system (2) with nonzero disturbance input.

Define $T_H(z)$ as the transfer function from the disturbance input w to the estimation error e of an FIR filter (16). Then we can formulate three FIR filtering problems as follows:

- H_2 FIR filtering problem: Find the filter (16) that minimizes $\|T_H(z)\|_2$.
- H_∞ FIR filtering problem: Find the filter (16) that minimizes $\|T_K(z)\|_\infty$.
- H_2/H_∞ FIR filtering problem: Find the filter (16) that minimizes $\|T_H(z)\|_\infty$ subject to $\|T_H(z)\|_2 < \beta$ (or minimizes $\|T_H(z)\|_2$ subject to $\|T_H(z)\|_\infty < \beta$).

In the next section, we present the formulation of the above FIR filtering problems in terms of LMIs.

Remark 1: It is noted that A should be nonsingular to obtain $\bar{C}_N$, $\bar{B}_N$, and $\bar{G}_N$. In case of high-order systems, the system matrix may be a sparse matrix and hence singular. It seems that the restriction of A being nonsingular is somewhat strong. However, if the system matrix is obtained from sampled-time systems, then it is represented as $A = e^{A_c \Delta}$, where A_c is the system matrix for the continuous-time system and Δ is the sampling period. In that case, A is always nonsingular.

3. H_2/H_∞ FIR FILTERING VIA LMIS

3.1. Error dynamics of FIR filters

As a starting point we derive the transfer function $T_H(z)$. The disturbance input w_k satisfies the following state model on W_{k-1}

$$W_k = A_u W_{k-1} + B_u w_k, \tag{17}$$

where

$$A_u = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & I \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in R^{pN \times pN}, \quad B_u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ I \end{bmatrix} \in R^{pN \times p}.$$

It follows from (9) that

$$Y_{k-1} - \bar{B}_N U_{k-1} - \bar{C}_N x_k = (\bar{G}_N + \bar{D}_N) W_{k-1}. \tag{18}$$

Pre-multiply (18) by H. From (16), we obtain

$$e_k = \hat{x}_k - x_k = H(\bar{G}_N + \bar{D}_N)W_{k-1}. \quad (19)$$

From (17) and (19), $T_H(z)$ is given by

$$T_H(z) = H(\bar{G}_N + \bar{D}_N)(zI - A_u)^{-1}B_u. \quad (20)$$

3.2. H_2 FIR Filtering

Given a system transfer function

$$G(z) \triangleq \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] = C(zI - A)^{-1}B,$$

it is well-known that $\|G(z)\|_2$ is given by

$$\|G(z)\|_2 = \sqrt{tr(CPC^T)}, \quad (21)$$

where P is the controllability Grammian given by

$$P = \sum_{i=0}^{\infty} A^i BB^T (A^T)^i,$$

and obtained as the solution to the following Lyapunov equation

$$APA^T - P + BB^T = 0.$$

Therefore, we have the following theorem for the H_2 FIR filter:

Theorem 1: Assume that the following LMI problem is feasible:

$$\min_{F,W} \ tr(W) \ \text{subject to}$$

$$\begin{bmatrix} W & (FM + H_0)(\bar{G}_N + \bar{D}_N) \\ * & I \end{bmatrix} > 0, \quad (22)$$

where $H_0 = (\bar{C}_N^T \bar{C}_N)^{-1} \bar{C}_N^T$ M^T is the bases of the null space of $\bar{C}_N^T$. Then the optimal gain matrix of the H_2 FIR filter of the form (20) is given by

$$H = FM + H_0.$$

Proof: The constraint $H\bar{C}_N = I$ is required for the FIR filter to be of the form (16). H_2 norm of the transfer function $T_H(z)$ in (20) is obtained by

$$\|T_H(z)\|_2^2 = tr(H(\bar{G}_N + \bar{D}_N)P(\bar{G}_N + \bar{D}_N)^T H^T),$$

where $P = \sum_{i=0}^{\infty} A_u^i B_u B_u^T (A_u^T)^i$. Because $A_u^i = 0$ for $i \ge N$, we obtain

$$P = \sum_{i=0}^{\infty} A_u^i B_u B_u^T (A_u^T)^i = \sum_{i=0}^{N-1} A_u^i B_u B_u^T (A_u^T)^i = I.$$

Therefore

$$\|T_H(z)\|_2^2 = tr(H(\bar{G}_N + \bar{D}_N)(\bar{G}_N + \bar{D}_N)^T H^T). \quad (23)$$

Introduce a matrix variable W such that

$$W > H(\bar{G}_N + \bar{D}_N)(\bar{G}_N + \bar{D}_N)^T H^T. \quad (24)$$

Then $tr(W) > \|T_H(z)\|_2^2$. By the Schur complement, (24) is equivalent to

$$\begin{bmatrix} W & H(\bar{G}_N + \bar{D}_N) \\ * & I \end{bmatrix} > 0. \quad (25)$$

Therefore, by minimizing $tr(W)$ subject to the equality constraint $H\bar{C}_N = I$ and the above LMI, we obtain the optimal gain matrix H of the H_2 FIR filter. The equality constraint $H\bar{C}_N = I$ can be eliminated by computing the null space of $\bar{C}_N^T$. All solutions to the equality constraint $H\bar{C}_N = I$ are parameterized by

$$H = FM + H_0, \quad (26)$$

where F is a matrix containing the independent variables. Replacing H by $FM + H_0$, the LMI condition in (25) is changed into (22). This completes the proof. □

Remark 2: $\bar{C}_N^T \bar{C}_N$ should be nonsingular in order to obtain H_0. This, in turn, implies that $\bar{C}_N$ should be full column rank. Assuming the nonsingularity of A, $\bar{C}_N$ is full column rank if and only if $\bar{C}_N A^N$ is full column rank. $\bar{C}_N A^N$ is represented as

$$\bar{C}_N A^N = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{N-1} \end{bmatrix}.$$

If (A, C) is observable, $\bar{C}_N A^N$ is full column rank if $N \ge n$. This signifies that the horizon length N greater than n guarantees H_0 to exist.

Remark 3: Recalling that the square of the H_2 norm is the error variance due to white noise with unit intensity, we show that (23) holds as follows:

$$\begin{aligned} &\|T_H(z)\|_2^2 \\ &= E\{e^T(k)e(k)\} \\ &= tr(E\{e(k)e^T(k)\}) \\ &= tr(H(\bar{G}_N + \bar{D}_N)E\{W_{k-1}W_{k-1}^T\}(\bar{G}_N + \bar{D}_N)^T H^T) \\ &= tr(H(\bar{G}_N + \bar{D}_N)(\bar{G}_N + \bar{D}_N)^T H^T). \end{aligned}$$

Define γ_2^* to be the $\|T_H(z)\|_2^2$ due to the optimal H_2 FIR filter and define Ξ_N as

$$\Xi_N \triangleq (\bar{G}_N + \bar{D}_N)(\bar{G}_N + \bar{D}_N)^T.$$

The following theorem states that the optimal H_2 FIR filter can be obtained analytically.

Theorem 2: The optimal H_2 FIR filter gain is given analytically as

$$H = (\bar{C}_N^T \Xi_N^{-1} \bar{C}_N)^{-1} \bar{C}_N^T \Xi_N^{-1}$$

and therefore we have

$$\gamma_2^* = tr(H \Xi_N H^T). \tag{27}$$

Proof: Construct a Lagrangian as

$$J = tr(H\Xi_N H^T) + tr(\Lambda(H\bar{C}_N - I)),$$

where $\Lambda \in R^{n \times n}$ is a Lagrange multiplier. It is clear from (23) that H minimizing the above Lagrangian is the gain matrix of the optimal H_2 FIR filter of the form (16). For optimality, we require that

$$\frac{\partial J}{\partial H} = 2H\Xi_N + \Lambda^T \bar{C}_N^T = 0, \tag{28}$$

$$\frac{\partial J}{\partial \Lambda} = (H\bar{C}_N - I)^T = 0. \tag{29}$$

From (28), we obtain

$$\bar{C}_N \Lambda = -2\Xi_N H^T. \tag{30}$$

Pre-multiply $\bar{C}_N^T \Xi_N^{-1}$ to the left of both sides in (30). Using $H\bar{C}_N = I$, we have

$$\bar{C}_N^T \Xi_N^{-1} \bar{C}_N \Lambda = -2\bar{C}_N^T \Xi_N^{-1} \bar{C}_N H^T = -2\bar{C}_N^T H^T = -2I.$$

Therefore $\Lambda = -2(\bar{C}_N^T \Xi_N^{-1} \bar{C}_N)^{-1}$ and

$$H = -\frac{1}{2}\Lambda^T \bar{C}_N^T \Xi_N^{-1} = (\bar{C}_N^T \Xi_N^{-1} \bar{C}_N)^{-1} \bar{C}_N^T \Xi_N^{-1}.$$

Substituting H above into (23) yields relation (27). This completes the proof. □

3.3. H_∞ FIR Filtering

For the system transfer function

$$G(z) \triangleq \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = C(zI - A)^{-1}B + D$$

it is well known from the *bounded real lemma* that, given $\gamma > 0$, the following two conditions are equivalent:

(1) $\|G(z)\|_\infty < \gamma$.

(2) There exists an $X > 0$ such that

$$\begin{bmatrix} -X & XA & XB & 0 \\ * & -X & 0 & C^T \\ * & * & -\gamma I & D^T \\ * & * & * & -\gamma I \end{bmatrix} < 0.$$

From this, we obtain the following theorem for the optimal H_∞ FIR filter.

Theorem 3: Assume that the following LMI problem is feasible:

$$\min_{F,X} \ \gamma_\infty \text{ subject to}$$

$$\begin{bmatrix} -X & XA_u & XB_u & 0 \\ * & -X & 0 & (\bar{G}_N + \bar{D}_N)^T (FM + H_0)^T \\ * & * & -\gamma_\infty I & 0 \\ * & * & * & -\gamma_\infty I \end{bmatrix} < 0$$

where $H_0 = (\bar{C}_N^T \bar{C}_N)^{-1} \bar{C}_N^T$ and M^T is the basis of the null space of $\bar{C}_N^T$. Then, the optimal gain matrix of the H_∞ FIR filter of the form (16) is given by

$$H = FM + H_0.$$

Proof: From the bounded real lemma, the condition $\|T_H(z)\|_\infty < \gamma_\infty$ is equivalent to the condition under which there exists $X > 0$ such that

$$\begin{bmatrix} -X & XA_u & XB_u & 0 \\ * & -X & 0 & (\bar{G}_N + \bar{D}_N)^T H^T \\ * & * & -\gamma_\infty I & 0 \\ * & * & * & -\gamma_\infty I \end{bmatrix} < 0.$$

The equality constraint $H\bar{C}_N = I$ can be eliminated in exactly the same way as in H_2 FIR filter. This completes the proof. □

3.4. H_2/H_∞ FIR Filtering

From the previous two subsections, the formulation of the H_2/H_∞ FIR filtering problem via LMIs is obvious. Therefore, we obtain the following theorem for the H_2/H_∞ FIR filter:

Theorem 4: Given $\alpha > 1$, assume that the following LMI problem is feasible:

$$\min_{W,X,F} \ \gamma_\infty \text{ subject to}$$

$$\begin{bmatrix} W & (FM + H_0)(\bar{G}_N + \bar{D}_N) \\ * & I \end{bmatrix} > 0,$$

$$\begin{bmatrix} -X & XA_u & XB_u & 0 \\ * & -X & 0 & (\bar{G}_N + \bar{D}_N)^T (FM + H_0)^T \\ * & * & -\gamma_\infty I & 0 \\ * & * & * & -\gamma_\infty I \end{bmatrix} < 0,$$

where $H_0 = (\bar{C}_N^T \bar{C}_N)^{-1} \bar{C}_N^T$ and M^T is the basis of the null space of $\bar{C}_N^T$. Then, the gain matrix of the H_2/H_∞ FIR filter of the form (16) is given by

$$H = FM + H_0.$$

Proof: The proof is obvious and is omitted.

The above H_2/H_∞ FIR filtering problem allows us to design the optimal FIR filter with respect to the H_∞ norm while assuring a prescribed performance level in the H_2 sense. By adjusting $\alpha > 0$, we can trade off the H_∞ performance against the H_2 performance.

4. NUMERICAL EXAMPLE

To illustrate the characteristics and validity of the proposed FIR filter, a numerical example is presented for a linear discrete-time invariant state space model taken from [3].

$$x_{k+1} = \begin{bmatrix} 0.9950 & 0.0998 \\ -0.0998 & 0.9950 + \delta_k \end{bmatrix} x_k + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} u_k + \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} w_k,$$

$$y_k = [1 \quad 0] x_k + [0 \quad 1] w_k,$$

where δ_k is an uncertain model parameter. Assuming $\delta_k = 0$, we obtained H_2 FIR filters, H_∞ FIR filters, and H_2 / H_∞ FIR filters with $\alpha = 1.05$ for different horizon lengths $(N = 3, 4, \cdots 10)$ using the results in Theorems 1, 3, and 4, respectively. Fig. 1 compares the H_2-norms of H_2 FIR filters and H_2/H_∞ FIR filters with that of H_2 IIR filters. Similarly, Fig. 2 compares the H_∞-norms of H_∞ FIR filters and H_2/H_∞ FIR filters with that of H_∞ IIR filters. Both figures show a similar trend: norms of FIR filters decrease as the horizon length increases. This means that the performances of FIR filters improve with the horizon length. It is naturally expected that the norm of FIR filters will approach the norm of IIR filters as the horizon length approaches infinity. The H_2-norms of H_2/H_∞ FIR filters are always greater than that of H_2 FIR filters. Similarly, the H_∞-norms of H_2/H_∞ FIR filters are always greater than that of H_∞ FIR filters. This is because H_2/H_∞ FIR filters are obtained by taking both performance criteria into account.

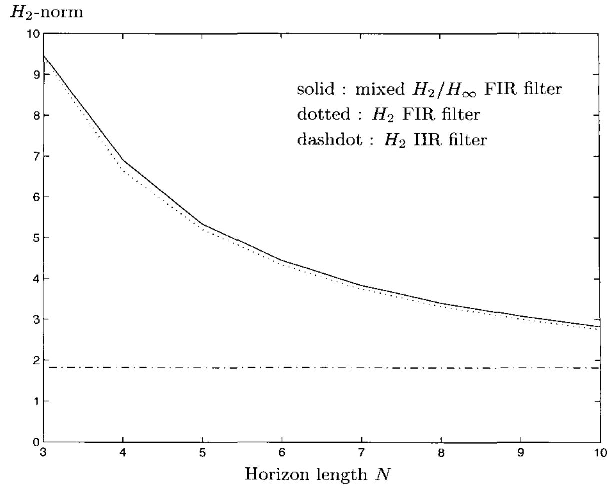

Fig. 1. H_2-norms with respect to different horizon lengths.

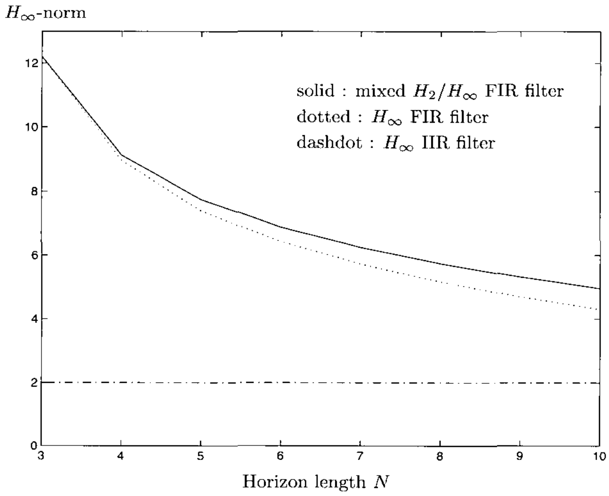

Fig. 2. H_∞-norms with respect to different horizon lengths.

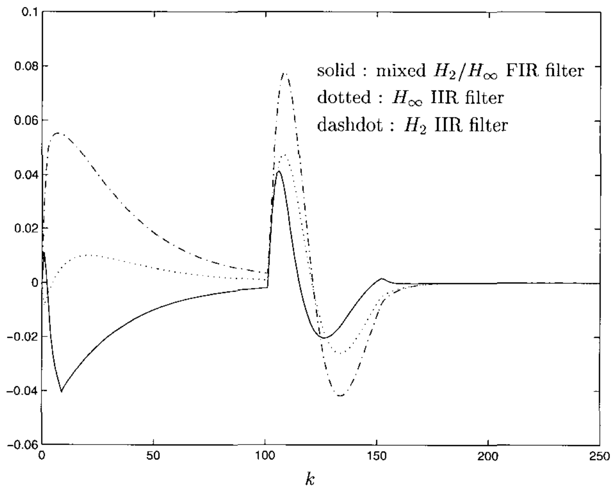

Fig. 3. Estimation error in x_1.

From these figures, we see that the performances of FIR filters can't be better than those of IIR filters in normal situations. However, it is noted that FIR filters can be a better choice than IIR filters in some special cases. One notable case is the situation where the system is subject to temporary parameter variation. As mentioned previously, FIR filters are known to be more robust than IIR filters against temporary modeling uncertainties because they utilize only finite measurements on the most recent horizon. To illustrate this feature, we have designed an H_2/H_∞ FIR filter with $N=10$, $\alpha=1.05$, and $\delta_k=0$ and applied it to a system that is subject to temporary parameter variation as follows:

$$\delta_k = \begin{cases} -0.1, & 100 \le k \le 150 \\ 0, & \text{otherwise.} \end{cases}$$

Fig. 3 compares the estimation error in x_1 of the H_2/H_∞ FIR filter with those of the H_2 and the H_∞ filters of IIR structure where the disturbance input w_k is given by

$$w_k = 0.1\begin{bmatrix} e^{-\frac{k}{30}} \\ e^{-\frac{k}{30}} \end{bmatrix}.$$

Therefore w_k is an exponentially decreasing signal. It is noted that the H_∞ IIR filter demonstrates superior performance for the interval $0 \le k \le 100$. This is because w_k is deterministic disturbance rather than stochastic noise. It is noted that the performance of the H_2/H_∞ FIR filter for $0 \le k \le 10$ is notably inferior. This is because the FIR filter does not have enough data for normal operation. Usually, data corresponding to the horizon length are required for normal operation. For the time interval $100 \le k \le 150$, the system is subject to temporary parameter variation and this in turn leads to temporary modeling uncertainty. The estimation error of the H_2/H_∞ FIR filter is smaller than that of the IIR filters for this interval. For $k \ge 150$ where there is no modeling uncertainty and w_k is very small, the convergence speed of H_2/H_∞ FIR filter to the true state of the system is faster than IIR filters. This example clearly shows the relative merits of proposed FIR filters compared with IIR filters.

5. CONCLUSIONS

In this paper, a new type of filter called the H_2/H_∞ FIR filter is proposed for discrete-time state space signal models. The filtering problem is formulated in terms of linear matrix inequalities. The proposed filter has many desirable properties, that is, the filter is linear with the most recent finite measurements and inputs, does not require *a priori* information of the horizon initial state, and has the unbiased property for zero disturbance. Furthermore, due to the FIR structure, the H_2/H_∞ FIR filter is believed to be robust against temporary modeling uncertainties or numerical errors, while other IIR filters, such as Kalman filters and H_∞ filters, may show poor robustness in these cases. The proposed H_2/H_∞ FIR filter will be useful for many signal processing problems where signals are represented by state space models. In the current paper, we assume that a system matrix is nonsingular. Study on the FIR filters that does not require the nonsingularity of system matrices will be a challenging future work.

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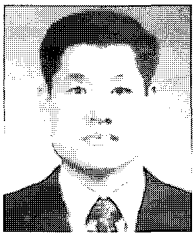

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