Journal of the Korea Society of Mathematical Education Series D:
Research in Mathematical Education
Vol. 10, No. 3, September 2006, 215–227

시리즈 D:
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제 10 권 제 3 호 2006 년 9 월, 215–227

Application and Consideration on Open-ended Problems in Mathematics Tests of Shanghai Senior Middle School Entrance Examination

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(Received August 8, 2005)

After reviewing China's appearance and research on the Math open-ended problems, together with the application of those problems in mathematics test of Shanghai Senior Middle School Entrance Exams (SSMSEE) since 1999, this paper points out the difficulty in establishing an evaluation system for such problem.
Through comparative study, the paper gives an operational definition of open-ended problem, and it attempts to establish an evaluation system and non-systematic competence targets that are appropriate to Math open-ended problems. Meanwhile, it describes the performance feature of those targets. By applying the standard international grading system of difficulty, it discusses the elements of difficulty in Math open-ended problems, and puts forward an evaluation as well as a level-of-difficulty forecasting system that is appropriate to the Middle School Entrance Exam.

Keywords: open-ended problems, validity, level-of-difficulty
ZDM Classification: D53, C73
MSC2000 Classification: 97D50

BACKGROUND

In the past twenty years, due to different kinds of tests in China for selection purposes, such as entrance exams for the middle school and college, and Mathematics Olympic contests, students at all levels were educated in the "exercise-immersion approach." The students are trained to be only "tests-handling machines" that can give remarkably quick responses to all types of test questions. In order to save Mathematics education in China from the mistake of "Exercise-immersion" teaching approach, an increasing number of scholars in China put forward an idea of using "Problem" to supplement, alter, and influence the test papers, so as to further the reform of Mathematics education in China

by realizing the hope that China's Mathematics education can be problem-solving oriented with only the necessary amount of exercise.

In 1971, with the lead of Dao Tian-mao in Japan, Open-ended problems were studied (*cf.* Dai 2000). And in 1980, Ze (*cf.* Ze 1999) first introduced to China's Mathematics educators the research done on Open-ended problems in Japan, creating an interest in China's educational circle. From 1980, Dai Zai-ping was the first scholar who applied open-ended problems in tests in China. After the teaching experiment done on a small scale in 1993, the new project named "Open-ended problems — a New Model in Mathematics Education" was established as one of the key projects in the Ninth-Five-Year Plan of China National Teaching Science (Pedagogies).

During the "Mathematics Education Curriculum Reform Seminar" held at Teaching Material Research Center of China Primary Education in China State Education Committee in 1993, scholars agreed open-ended problems "with more than one solution, require students to analyze the problem, establish a model, and then solve the problem." This approach should be advocated because it is an important way to cultivate students' abilities, improve their sense of application and cooperation.

From the perspective of the testing process, reports, or theses (Silver 1995; Pehkonen 1995; Qiao 1997; Dai 1999; Zhen 2001a, 2001b, 2001c), it marked the beginning of widespread adoption of open-ended problems in testing. However, experiments and research in this field are not so satisfied. At the same time, the validity and effectiveness of the approach have not been yet conducted in Mathematics test. The main reason is the lack of an easily operational evaluation system, thus raising the question on the effectiveness and validity of Open-ended problems in Mathematics test. Meanwhile, due to the complexity and difficulty in controlling the mental processes, open-ended problems faced a serious challenge in their definition, applications, and evaluations. At least, this type of problems follow less rules, regulations, and methods in the preparation of tests and evaluation used in conventional tests or questions.

Applying the current status of Open-ended problems in the Shanghai Senior Middle School Entrance Examination (SSMSEE)

From 1999 till now, every year nearly 110,000 students in Shanghai graduate from primary schools and enter the secondary ones through the SSMSEE. Even though there have been increasing rates of the students' entrance into college, there still exist the partition between the key middle schools and the ordinary ones at both city and district levels. This situation, together with parents' high expectations for their children, propels the middle school candidates to score high as much as possible. It is a common belief that every mark in the exam counts in the candidate's fate. Under such circumstances, the

demand for the fairness and objectivity of the test papers is high. Contrasted with other major subjects like Chinese and English, due to its objectivity in paper-making, Mathematics plays a more important role in score differentiation.

During the past years, due to the concerns about causing social instability, the scale of difficulty in test papers or items was under control in the process of making them out. Because there is no ready-made set of techniques in controlling the difficulty scale, and because it is not easy to formulate the marking criteria for open-ended problems, the introduction of open-ended problem into SSMSEE, which is "high-risked" and "high-staked," should be very much careful. Consequently, open-ended problems were only "superficially" used, *i.e.*, in contrast to the traditional close-ended questions, this type of open-ended problems can have only more answers and can be approached in different ways.

According to the relevant documentation of SMSEE papers in Shanghai, Open-ended problems made their first appearance in 1999. However, to what extent these open-ended problems have assisted the learners to achieve this higher order thinking process. Let us consider some examples of the open-ended problems form the SMSEE as given below:

Example 1 (Shanghai Municipal, 1999)

Assume quadrilateral $ABCD$ satisfies _____ (Fill in a set of conditions you think appropriate.), this quadrilateral's diagonal lines are perpendicular to each other.

Example 2 (Shanghai Municipal, 2000)

The two given numbers are 3 and 6, please fill in a number so that one among the three numbers is the mean terms of proportion of the other two. The number is _____. (Please fill in only one number.)

Example 3 (Shanghai Municipal, 2001)

Refer to figure 1, the three points A, B, C of $\triangle ABC$ are located at the apexes of unit squares. Please draw $\triangle A'B'C'$ in a 4×4 large square which satisfies

$$\triangle ABC \backsim \triangle A'B'C'.$$

(The similarity ratio is not equal to one, and the points $A'B'C'$ are all located at the apexes of unit squares.)

Example 4 (Shanghai, 2004)

Line AD is $\triangle ABC$'s angular bisector, and the points E is the mid-point of segment AB, and the points F is the mid-points of segment AC. Now join D to point E, and D to point F respectively. Please add one condition so that the quadrilateral $AEDF$ is a diamond without joining any other points. This additional condition is _____.

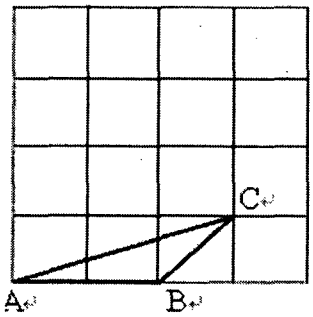

Figure 1. Similar triangles.

The four questions *cf.* above are abstracted from the modified textbooks or improved exercises. In testing the learners' understanding of the geometric mean or in testing their application of judgments in similar triangles (as in 2001), or in a diamond (as in 2002), the emphasis of test is on the abilities of the learners to apply their mathematics conception or mathematics theorem, different thinking processes can cause different results. This emphasis is a remarkable feature of open-ended problems in Shanghai SMSEE.

Take another question in year 2000 as an example. Traditional test questions often ask in the following way:

If the numbers 3 and 6 are known, then the mean terms of proportion of two numbers is _____.

The knowledge tested is the same, but open-ended problems are much more demanding and required carefulness of reasoning process.

Another example is in year 2001. In the textbook, students are only required to prove the similarity between the two triangles. Therefore, questions in the textbook merely require students to find out the conditions that suit the judgments, like the corresponding lines being proportional, and then draw the conclusions with the knowledge of relationship between the condition and the conclusion. From the students' feedback, we are surprised to find that many students are only focused on the proportional relationship among corresponding lines, but neglected the condition of equality in corresponding angles. This reveals the lack of self-evaluation after they finished the item. After the examination, this test question caused a stir in the grass-root level teachers because these effects cannot be achieved by using the traditional textbook questions easily.

The application of Open-ended problems in test papers after several years not only provides much experience but also leads to the diversity in teaching approaches and methods. Careful analysis of applying Open-ended problems in SSMSEE reveals some problems encountered. For example, Open-ended problems, if applied in the type of item named with 'Fill-in the Blanks', require students to fill only one answer in the blank, thus

limit the purpose of open-ended problems due to the restraining effects on students' performance, even though these questions are easy to mark and easy to control in terms of their level of difficulty. Besides, Open-ended problems are faced with several problems as follows:

a) The workload of marking test papers, as well as the difficulty in differentiating the degree or extent of difficulty in Open-ended problems, is increased.

b) In terms of test paper formulation process, these problems are only complementary to close-ended problems with an emphasis on applying knowledge, which obviously contrary to the principle and purpose of open-ended problems which measure learners' psychology process and quality.

After reviewing the background and analyzing the current situation and practice of Open-ended problems in SSMSEE, concluding series of difficulties in applying open-ended problems in SSMSEE, we fetch three results.

First, Open-ended problems should be added to the test paper as an alternative to the former test problems.

Second, multiple definitions of open-ended problems lead to the unclear connotation and denotation of the term; and its difficulty in operationalization puts the formulation of the so-called "high risk" test papers, like SSMSEE, in a preliminary state of careful experimentation.

Third, the essential feature of Open-ended problems is to allow learners to provide various solutions to the problems of different complexity so as to show their respective level of performance or ability. However, the foundation of the classification of levels of performance or ability is in itself lacking, thus leading to the unpredictability and uncertainty of the level of difficulty. Meanwhile, vague test targets of open-ended problems also have a negative effect on improving the validity of the whole test papers.

SOME CONSIDERATIONS ON OPEN-ENDED PROBLEMS

Up till now in China, the study of open-ended problems in Mathematics is mainly on the four aspects as follows:

a. Definition or conception of open-ended problems in Mathematics;
b. Types of open-ended problems;
c. Significance of Math open-ended problems in education;
d. Formulating or making out open-ended problems.

As far as this research project is concerned, it mainly studies the application aspect of open-ended problems. Since open-ended problems are used in SSMSEE, this paper is

more concerned with how open-ended problems can be put into use in such exams. In this paper we focus on the discussion on *a*, *b*, *d*.

Definition of Open-ended problems

Up till now, there is no internationally accepted definition of open-ended problems, so here listed as follows are some definitions from relevant references, including reference article (*cf.* Silver 1995) and article (*cf.* Pehkonen 1995).

a) Open-ended problems are questions or problems which have no single definite answers or solutions to them.

Typical opinions are:

1) "Questions or problems to which several correct answers or right solutions can be obtained from the pre-suppositions are called open-ended problems. ... The purpose of such questions is to make problem-solvers to explore the way and process of solving problems through pondering ... The motive of such questions is to forge one's aptitude and mindset of thinking over and solving problems in the mathematical way."(*cf.* Ze 1999)

2) "Questions or problems which have more than one answer or solution to them are called open-ended problems. The outstanding feature is multiple answers or solutions."(*cf.* Yu 1999)

b) Problems or questions that demand problem-solvers to draw their own conclusions from incomplete pre-suppositions or make choice among a variety of answers due to the problems' indefinite conclusions are called open-ended problems.

Typical opinions are as follows:

1) "Open-ended problems are those problems or questions that require the problem-solvers to obtain their own answers or make choice among many possible answers because these problems are essentially incomplete in premises and/or indefinite in conclusions or answers."(*cf.* Sun 1996)

2) "Open-ended problems, in contrast to traditional close-ended problems with complete pre-suppositions and definite answers, are those that have incomplete presuppositions and indefinite answers."(*cf.* Liu)

c) Open-ended problems refers to those problems or questions that have "open presuppositions" (continuously changing premises), "open conclusions" (various conclusions or no conclusions), and "open strategies" (various ways to solve the problems). (*cf.* Fan 1998)

d) Open-ended problems refer to those problem-solvers-oriented problems which explain only the requirements in solving the problems in general principle and which have multiple solutions. (*cf.* Zhang 2002)

However, definitions outlined above have flaws as follows:

1) The meaning of "No single answer" or "multiple solutions" is vague in whether two answers or solutions are multiple.

2) "No definite answer or solution" is ambiguous in differentiating the meaning between the "conclusions" (solutions) of open-ended problems and the "conclusions" in making out Mathematics open-ended problems.

3) Those definitions may cause controversy in whether one question or problem with multiple methods is really open-ended problems.

4) Mr. Zhang's definition, to some extent, reveals the solution (answer) characteristic of open-ended problems, uncover the fundamental cause for the uncertainty or indefiniteness of solutions or answers. Nevertheless, it is difficult to put it into use when the making-out of test paper is guided under his definition.

In view of the novelty of open-ended problems in mathematics instructions to enhance higher order thinking process of the learners and also noted the weaknesses in the problems together with its implementation, we propose a few considerations below:

One major concern is the lack of clear definition of open-ended problems that had led to the formation of undesirable type of open-ended problems. Therefore, a clear definition is important. Open-ended problems are defined as those questions asked in a problem situation to which the correct answers oriented are indefinite in number (more than one). So, in finding the solutions, the learners should use diverging thinking pattern.

In the definition, the phrase "in a problem situation" puts learners in the central role of problem-solving. The phrase "the correct answers oriented" refers to exploring the targets set by learners in their process of solving the problems. Hence, it can be:

a) the condition of the Mathematics statement in the Open-ended problems;

b) the conclusion of the Mathematics statement;

c) the strategies and methods employed in the problem-solving process.

Major types of Open-ended problems

According to the classification of Open-ended problem by Dai (1996), it is believed that once the condition or the conclusion of the statement is unknown, the strategy and methods employed in solving the problem will be open-ended. Related to this, there are three types of Open-ended problem. These are:

open-condition type, open-conclusion type, and open-condition and conclusion type

The level of performance and ability targets set in using the Open-ended problems

The main aim of using open-ended problems of SSMSEE, however, is not to test the basic knowledge, but to test more of the learners' thinking process, abilities and potentials so as to reveal the difference in their respective level of performance and ability. In primary schools, cultivating and training Mathematics abilities is an important goal of Mathematics education, so testing Mathematics level of performance and ability is one of the main objectives in the whole test paper formulation process. Obviously, the scope and the specific abilities to be tested need to be defined clearly in using open-ended problems. After analyzing the history of ability requirements in China Mathematics Teaching Program since 1949(referred from Compilation for 20-century national Mathematical curriculum criterion and syllabus of primary school and middle school, 2001), and also analyzing Shanghai Multiple Mathematical Curriculum Criterions of Primary and Middle School(2004), we notice that China has always been paying attention to the cultivation of abilities in thinking, calculating, spatial perceiving, and practical problems solving. However, in all the past teaching programs or textbook requirements, there was no clear definition of ability targets. The priority then is to set up a testing and evaluating system, including corresponding performance or behavior targets on which the formulation of Open-ended problem can based on.

Besides, there is a need to understand the psychological features of cognition of junior middle school students, course standards that emphasize on students' experiences in exploring the quantitative relationships and rules in solving the problems.

Based on the above induction and thinking, a non-systematical framework in which open-ended problems in test paper are oriented to test students' capabilities of observing, logical thinking, mathematical concept about space should be set up. The following are some of the tangible benefits of this proposal.

(i) Capability of Observation in mathematics

It mainly refers to expressing the relationship with letters, Mathematical symbols and number, observing intentionally and promptly propositions, questions and the structure characteristics of charts, images or geometric forms.

For example (Quanzhou District, Fujian Province, 2002)

Refer to figure 2, a rectangle $ABCD$ consists of a square with $AB=a$, and two rectangles in which length is equal to a and width is equal to b. Write down different equations with a or b which be deduced from the diagram.

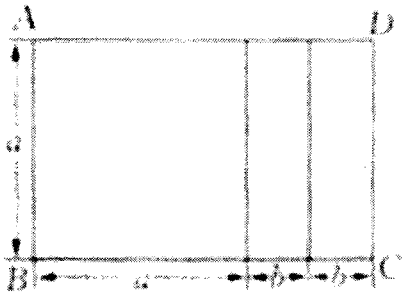

Figure 2. Quanzhou Exam

(ii) Logical thinking ability

The objective of logical thinking in Mathematics for the students in Junior High School lies in the understanding of the importance of Mathematics proof, mastering the basic rules and methods of deduction, expressing simply but clearly the process of deduction, analyzing reasonably the correctness of deduction. In SSMSEE, such ability manifests itself in

(a) correctly and gradually understanding the Mathematics concepts;

(b) deducting the result of new Mathematics materials by way of Mathematics logical deduction;

(c) changing, generalizing and reasoning the qualification of the given proposition.

For example (Liaoning Province, 2000)

(This item is similar to the exercise 3, page 39, the second term of grade 9 in Shanghai textbook)

Refer to figure 3, AB is diameter of circle O, D is the middle point of segment AC, and circle O passes through the point D, $DE \perp BC$ at E.

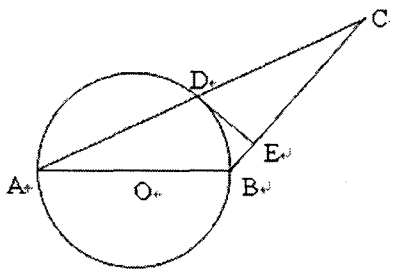

Figure 3. Liaoning Exam

(a) According to such condition, what conclusions can be drawn correctly? (Attention: No

signing other letter on the figure; No assistant line in your conclusions; No deducing; Only need four conclutions)

(b) If the condition that $\angle ABC$ is right angle is added up, what are the new conclusions that you can drawn and differed from those of (a)?

And hence draw the figure.

(iii) Mathematical Concept about Space

The aim of Mathematical concept about space for the students in Senior Middle School can be seen in imaging an geometric figure from a physical object, drawing a physical object in mind from an geometric figure, visualizing the motion and change of an geometric figure, telling the basic diagram from a rather complex one and analyzing the basic elements and the relations among them in it, and introducing the quality of a rather complex diagram from a basic simple one. In SSMSEE, concept about space will be examined in

(a) correctly analyzing the relations of measurement and location among the basic elements of a simple diagram;

(b) correctly judging the geometric figure and its location relation with the help of diagram and accurately parallel translation, circumvolving and folding the original diagram;

(c) correctly exchanging between letter or symbol language and diagram, that is, correctly drawing a correspondent diagram by reading the Mathematics language.

For example (Jingzhou District, Hubei Province, 2000)

Refer to figure 4, it is a steel plate. Now draw a line to separate it into two parts of equal area.

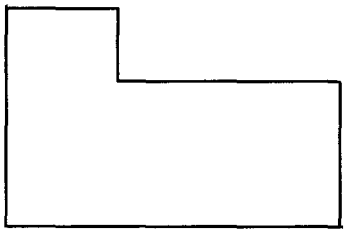

Figure 4. Jingzhou Exam

The Workout of the Entrance Examination for Senior Middle School

Due to the SSMSEE's educational importance and societal acceptance, especially in teaching, we mainly focus on adoption of original textbook and workbook. In the meanwhile, Dai, Zai-ping's on open-ended test questions are presented here. They are

(a) propositions tend to be diversified by omitting intentionally the old conclusion;

(b) the answers will be diversified by weakening the conditions of the old proposition;

(c) under the given condition and relation, Mathematics equation and geometric figure are left for the students to put forward;

(d) giving the conclusion, determine the various conditions that lead to the conclusion;

(e) comparing similarities and differences between some objectives;

(f) finding more solution and conclusion in real situations;

(g) setting up Mathematics problems with insufficient conditions and asking the students to find the necessary conditions that can result in different solutions to a problem.

DISCUSSION ABOUT THE DIFFICULTY ELEMENTS OF OPEN-ENDED PROBLEMS

The difficulty elements of open-ended problems will be discussed in the following three dimensions: the level of problems, the forward and converse directions of thinking, and the number of correct answers.

(i) The level of problems

A test-question might examine more than one type of capability. In this case, we cannot classify the level of problems simply from the angle of ability. At the same time, the standard shall be set in accordance to the cognitive level. Academically, open-ended problems are divided into low and high level by using the international standard (Stein, 2001).

(ii) The direction of thinking

The conclusions for a open-ended problem are not absolute and unique, and the solving process direction is uni-directional. Students may find it easier to solve. On the contrary, students in Junior Middle School may find it more difficult to solve propositions whose conditions are open and not absolute and therefore may get lower marks due to the need to solve them in converse direction. These open-ended problems are more difficult to be solved as they require bi-direction thinking.

(iii) The number of correct answers

With more correct answers, open-ended problems are more difficult to solve. Since a great majority of SSMSEE students will enter Senior Middle Schools, and even to key middle schools to study, the examination should be fair to the students. There is no deny that Open-ended problem is valuable in promoting teaching methods, students' learning power, and enhancing students' interest in their study. However, the open character of the problem makes the test targets vague and uncertain. Thus, it is difficult to mark and grade the papers. This problem poses challenge to the justice and fairness of the assessment.

CONCLUSION

The analyses and considerations on the content of the Open-ended problems in the SSMSEE are just preliminary discussions. However, they may be helpful to the future formulation of Open-ended problems on testing, and setting the appropriate level of difficulty of the test papers, thereby, increasing the validity of assessment. Nevertheless, these considerations still need further research.

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