유체에 잠긴 다공 원통형 쉘의 자유진동해석

Free Vibration Analysis of Perforated Shell Submerged in Fluid

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요 지

물에 잠긴 다공 원통형 쉘의 경우 물에 잠긴 상태로 유한요소해석을 하기에는 거의 불가능하므로 등가물성치를 사용하여야 한다. 다공 평판의 경우 이에 대한 등가물성치를 ASME 코드에서 제시하고 있지만, 다공 원통형 쉘의 등가물성치에 대한 연구는 아직까지 수행된 적이 없다. 따라서 본 연구에서는 유한요소해석을 이용하여 다공 원통형 쉘의 동적 해석에 이용할 수 있는 등가물성치를 제안하였고 그 타당성을 검증하였다.

핵심용어 : 다공 원통형 쉘, 유체-구조물 상호작용, 등기물성치, 고유진동수

Abstract

For the perforated cylindrical shell submerged in fluid, it is almost impossible to develop a finite element model due to the necessity of the fine meshing of the shell and the fluid at the same time. This necessitates the use of solid shell with equivalent material properties. Unfortunately the effective elastic constants are not found in any references even though the ASME code is suggesting those for perforated plate. Therefore in this study the equivalent material properties of perforated shell are suggested by performing several finite element analyses with respect to the ligament efficiencies.

Keywords : *perforated cylindrical shell, fluid structure interaction, equivalent material property, ligament efficiency, natural frequeney*

1. Introduction

The analysis of a plate or shell perforated with a large number of holes, by finite element method for instance, was a very costly and time-consuming technique which solves only one particular problem. But it is possible to model the perforated plate or shell and to analyze it and it is no more time-consuming theses days due to the rapid development of the computer and software. However, if a perforated plate or shell is submerged in fluid it is almost impossible to model and analyze it as is and the fluid at the same time, which is needed to investigate the effect of the fluid-structure interaction. The simplest way to avoid time-consuming and costly analysis of perforated plate or shell submerged in fluid is to replace the perforated plate or shell by an equivalent solid one considering weakening effect of holes.

Many authors have proposed experimental or theoretical method to solve this problem for the plate. Slot and O'Donnell(1971) determined the effective elastic constants for the thick perforated plates by equating strains in the equivalent solid material to the average strains in the perforated

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material. O'Donnell(1973) also presented those of thin perforated plates. These results are implemented in Article A-8000 of Appendix A to the ASME code Section III(ASME, 2004), which contains a method of analysis for flat perforated plates when subjected to directly applied loads or loadings resulting from structural interaction with adjacent members.

Unfortunately the effective elastic constants for the perforated shell are not found in any references. Therefore in this study the modal characteristics of the perforated shell are investigated and the equivalent material properties of perforated shell are suggested by performing several finite element analyses with respect to the ligament efficiencies. Two types of penetration patterns are considered and their modal characteristics are addressed.

Also this study deals with the free vibration characteristics of perforated cylindrical shell submerged in fluid, which is assumed to be incompressible, irrotational and frictionless. The natural frequencies of the fluid-coupled system are obtained by theoretical calculations and verified by three dimensional finite element analyses.

2. Analysis of Solid Shell

2.1 Theoretical Analysis

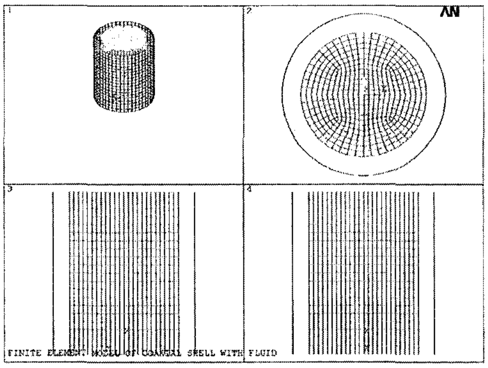
On the basis of the theory developed by Jhung *et al.*(2003, 1999), the frequency determinant is numerically solved using MathCAD in order to find the natural frequencies of the circular cylindrical shells with a bounded compressible fluid. In order to check the validity and accuracy of the results from the theoretical study, finite element analyses are also performed and frequency comparisons between them are carried out for the fluid-coupled system.

The inner and outer shells are coupled with a fluid-filled annular gap. The inner cylindrical shell has a mean radius of 100mm, a length of 300mm, and a wall thickness of 2mm. The outer cylindrical shell has a mean radius of 130mm with the same length and wall thickness. The physical properties of the shell material are as follows: Young's modulus=69.0GPa, Poisson's ratio=0.3, and mass density=2,700kg/m^3. Water is used as the contained fluid with a density of 1,000kg/m^3. The sound speed in water, 1483m/s, is equivalent to the bulk modulus of elasticity, 2.2GPa.

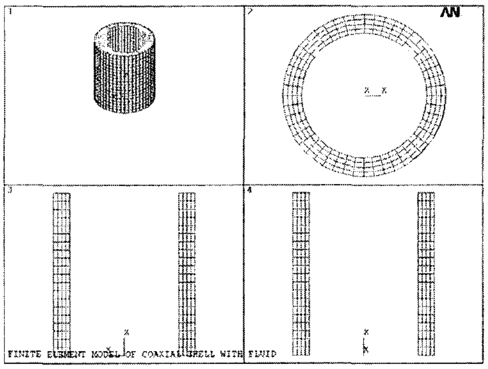
The frequency equations derived in the preceding sections involve an infinite series of algebraic terms. Before exploring the analytical method to obtain the natural frequencies of the fluid-coupled shell, it is necessary to conduct convergence studies and establish the number of terms required in the series expansions involved. In the numerical calculation, the Bessel-Fourier expansion term s is set to 200 and the expanding term m (or M) for the admissible function is set to 40, which gives an exact enough solution by convergence.

2.2 Finite Element Analysis

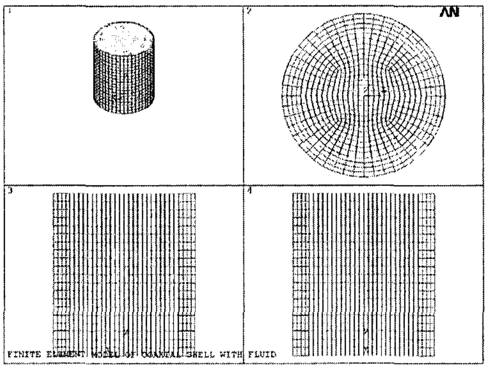
Finite element analyses using a commercial computer code ANSYS 9.0(ANSYS, 2005) are performed to verify the analytical results for the theoretical study. The results from finite element method are used as the baseline data. Four different three-dimensional models are considered for solid shells depending on the fluid as shown in Figure 1. The fluid region is divided into a number of identical 3-dimensional contained fluid elements(FLUID80) with eight nodes having three degrees of freedom at each node. The fluid element FLUID80 is particularly well suited for calculating hydrostatic pressures and fluid/solid interactions. The circular cylindrical shell is modeled as elastic shell elements(SHELL63) with four nodes.



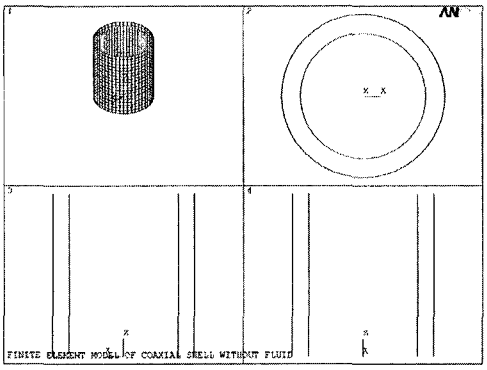
(a) Cylindrical shell with inner fluid



(b) Cylindrical shell with outer fluid



(c) Cylindrical shell with inner and outer fluid



(d) Cylindrical shell without fluid

Fig. 1 Finite element models

The fluid boundary conditions at the top and bottom of the shell are zero displacement and rotation. The nodes connected entirely by the fluid elements are free to move arbitrarily in three-dimensional space, with the exception of those, which are restricted to motion in the bottom and top surfaces of the fluid cavity. The radial velocities of the fluid nodes along the wetted shell surfaces coincide with the corresponding velocities of the shells. Clamped-clamped boundary conditions at both ends are considered for the shell.

Several cases of the finite element analyses are performed depending on the fluid existence. The Block Lanczos method is used for the eigenvalue and eigenvector extractions to calculate 300 frequencies including fluid modes(Grimes *et al.*, 1994). It uses the Lanczos algorithm where the Lanczos recursion is performed with a block of vectors. This method is as accurate as the subspace method, but faster. The Block Lanczos method is especially powerful when searching for eigenfrequencies in a given part of the eigenvalue spectrum of a given system. The convergence rate of the eigenfrequencies will be about the same when extracting modes in the midrange and higher end of the spectrum as when extracting the lowest modes.

3. Analysis of Perforated Shell

3.1 Triangular Penetration Pattern

Considering a cylindrical perforated shell with a triangular penetration pattern as shown in Figure 2, where the mean radius of the shell and pitch of the hole are 99.2mm and 36mm, respectively, the modal characteristics and the equivalent elastic constants are investigated as a typical case.

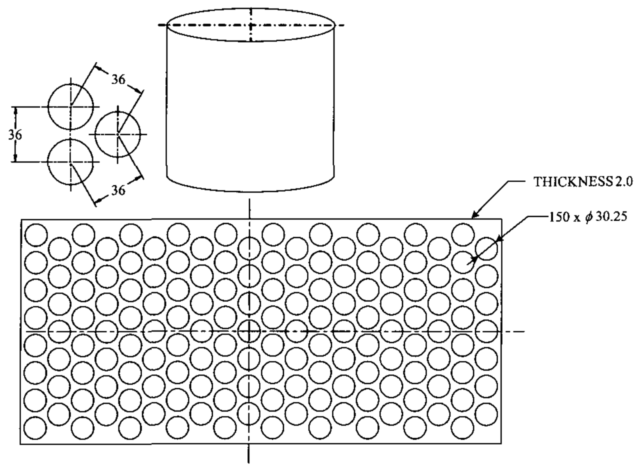


Fig. 2 Perforated shell with triangular penetration pattern

Frequencies of solid shell and perforated shell of ligament efficiency $\eta=0.05$ with triangular penetration pattern and their normalized values are shown in Figures 3 and 4, respectively. The inclusion of holes decreases the frequency significantly and this is more significant as the modal number decreases. Typical mode shapes are shown in Figure 5 for the perforated shell.

As mentioned earlier, the equivalent elastic constants of perforated shell are not found anywhere. Therefore it is necessary to define the equivalent elastic constants such that the modal characteristics of the perforated shell with original properties be the same with those of the solid shell with modified equivalent properties. The best way to find the equivalent constant is as follows:

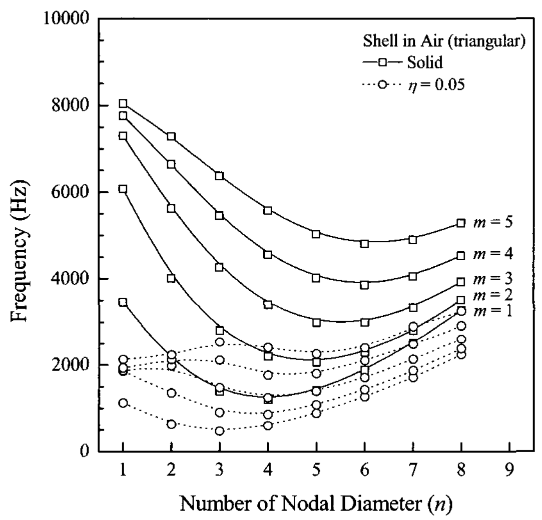


Fig. 3 Frequency comparisons between solid and perforated shells in air

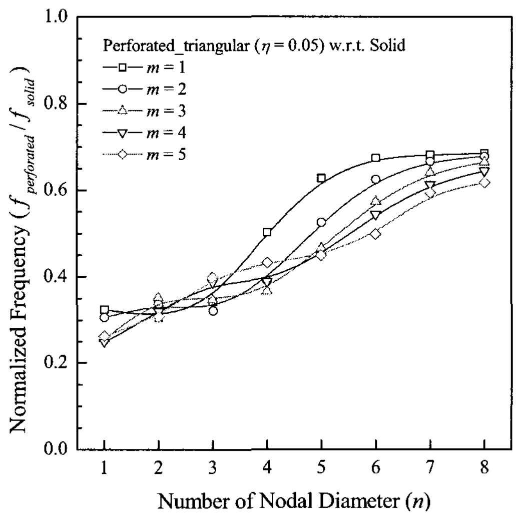


Fig. 4 Normalized frequencies of perforated she w.r.t. solid shell in air

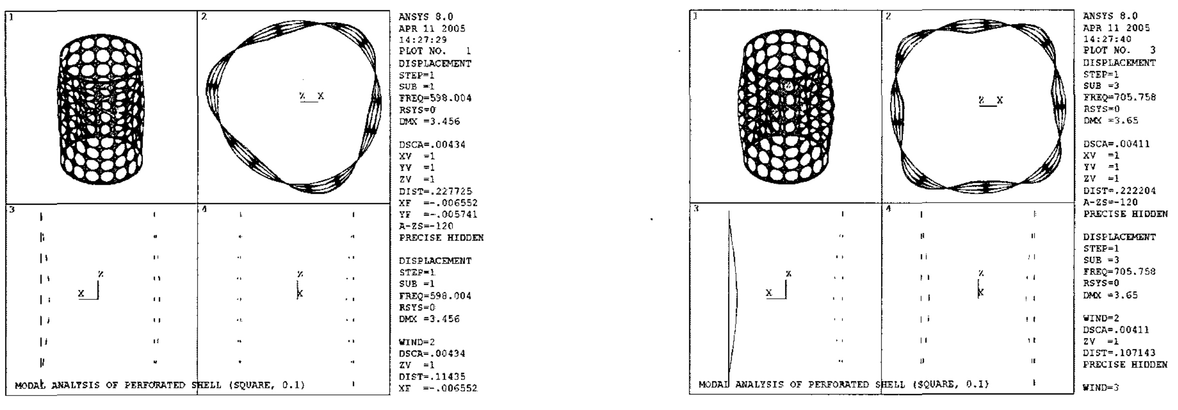


Fig. 5 Typical mode shapes of perforated shell

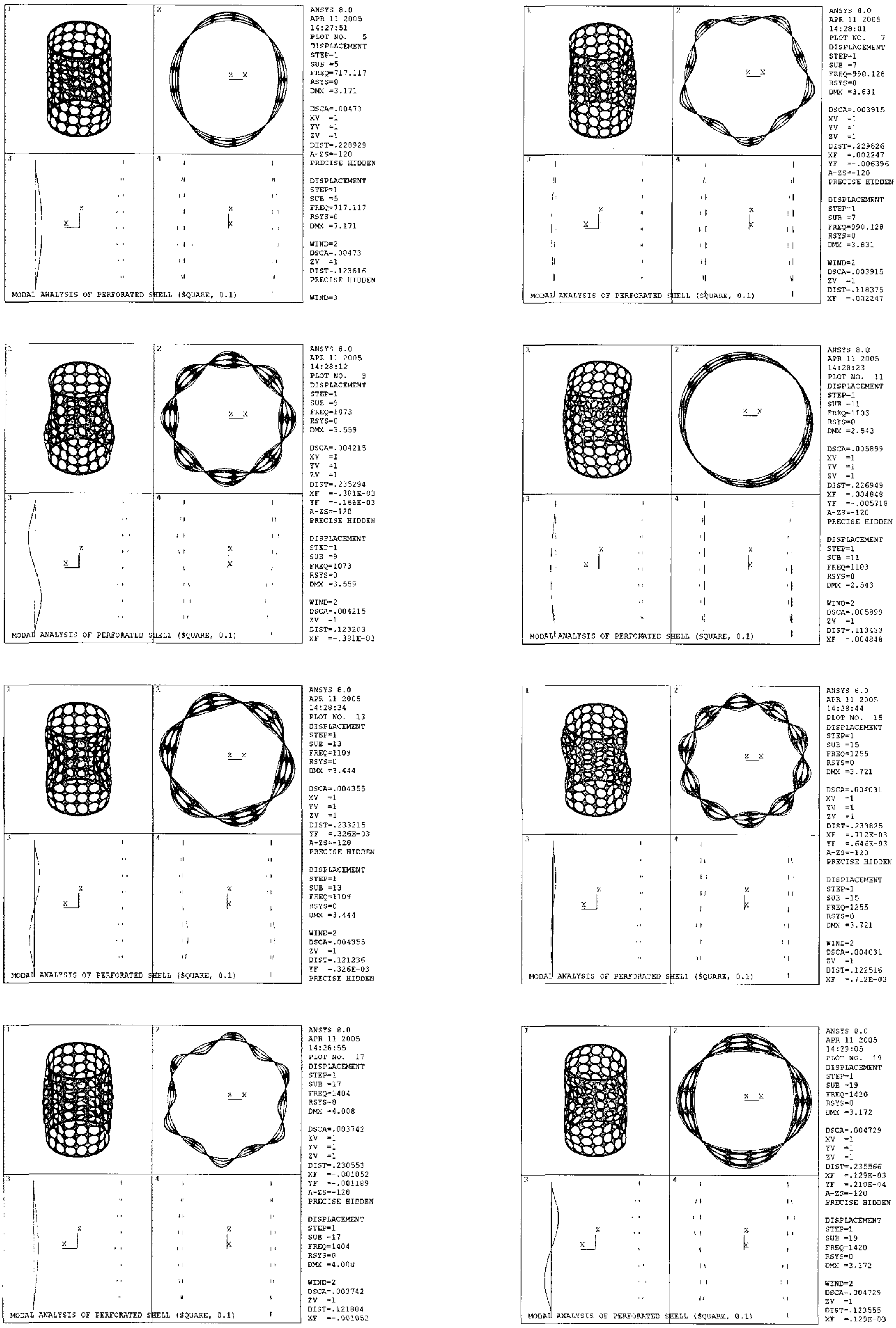


Fig. 5 Typical mode shapes of perforated shell (Cont'd)

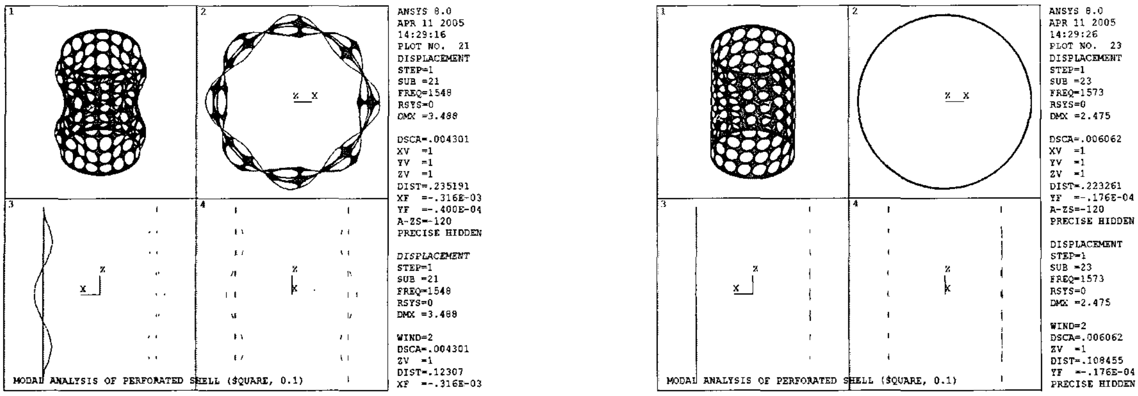


Fig. 5 Typical mode shapes of perforated shell (Cont'd)

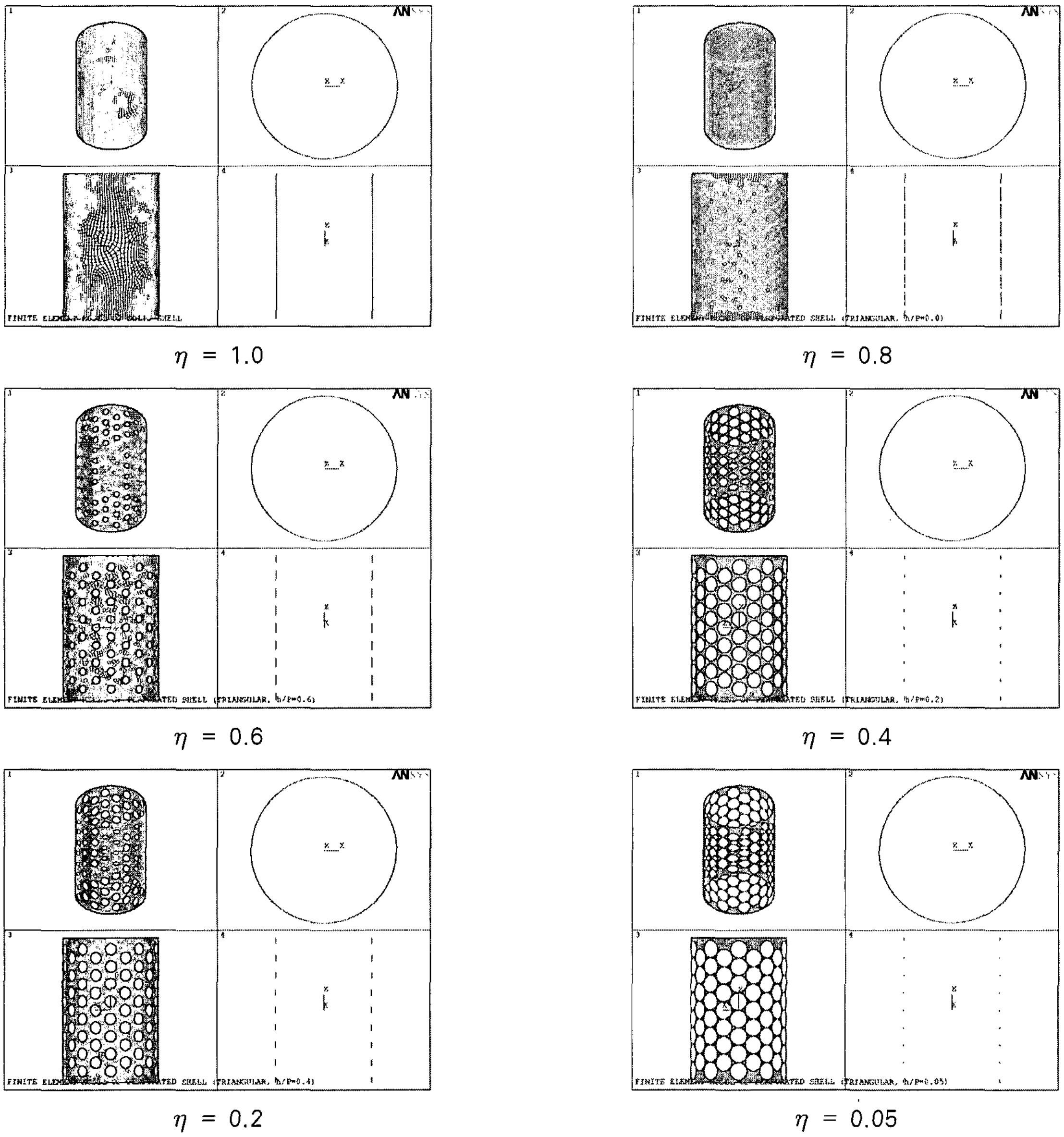


Fig. 6 Finite element models of solid and perforated shells with triangular penetration pattern

(1) Develop finite element model of solid shell and perforated shells with various ligament efficiencies(Figure 6).
(2) Perform the modal analyses of perforated and solid shell with original properties.
(3) Compare the frequencies and find the ratio of frequencies of perforated shell to those of solid shell.
(4) Find the multipliers of Young's modulus of solid shell to match frequencies of perforated shell with original properties using the relations between frequency and Young's modulus.
(5) Effective *elastic constant for each* ligament efficiency is averaged for all modes.

The natural frequencies are summarized in Figure 7 and their normalized values of perforated shell with respect to the solid shell are calculated. Because the elastic constant is proportional to the square of the frequency, the normalized frequencies are squared and these are the effective elastic constants with which the same modal frequencies as the perforated shell are obtained for the solid shell. Because the effective values are dependent on the mode numbers, their average values are used in Figure 8, which is the final effective elastic constants proposed for modal characteristics of the perforated shell with a triangular penetration pattern as;

$$\frac{E^*}{E} = 0.1610 + 1.7421\eta - 2.0365\eta^2 + 2.2733\eta^3 - 1.1471\eta^4 \tag{1}$$

Because the effective elastic constants are much changed depending on the mode numbers it is not proper to define one effective value for each ligament efficiency. For example, the effective elastic constant varies from the minimum 0.062 to the maximum 0.468 with the average of 0.242 for η=0.05. Therefore the frequency errors are expected for the case of using average equivalent elastic constants defined in Eq. (1). Therefore the equivalent elastic constant defined in Eq. (1) should be limited only for $0.5 \leq \eta \leq 0.8$ for the frequency errors of the perforated shell to be within 10%.

3.2 Square Penetration Pattern

Considering a cylindrical perforated shell with a square penetration pattern as shown in Figure 9, where the radius of the shell and pitch of the hole are 97.4mm and 36mm, respectively, the modal characteristics and the equivalent elastic constants are investigated as a typical case.

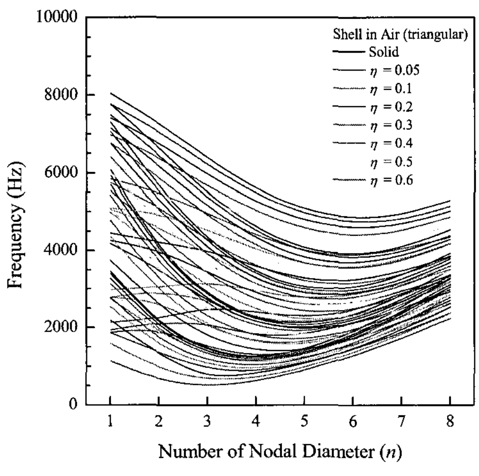


Fig. 7 Natural frequencies of shell in air for solid shell and perforated shells with various *ligament efficiencies*

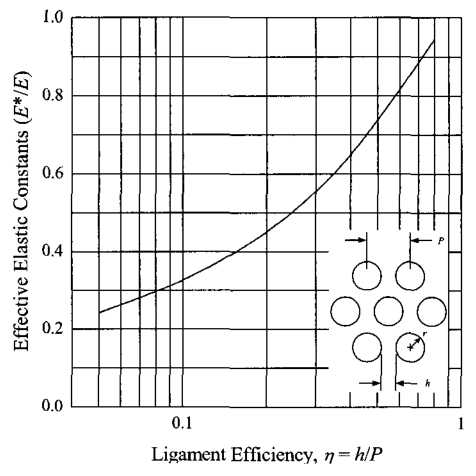


Fig. 8 Effective elastic constants of perforated shell with triangular penetration pattern

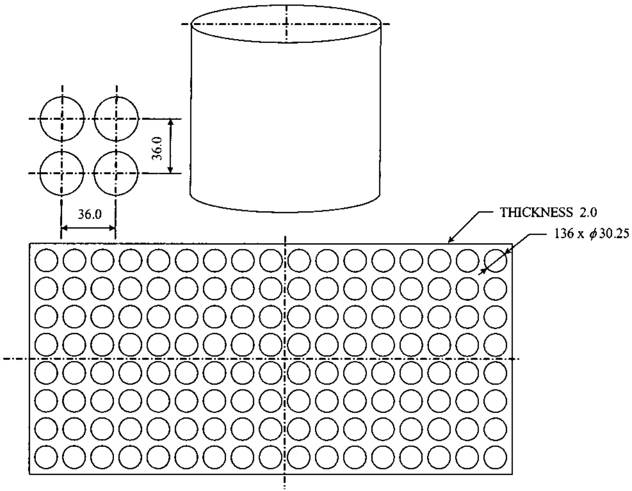


Fig. 9 Perforated shell with square penetration pattern

Finite element models of perforated shell with various ligament efficiencies are developed(Figure 10) and modal analyses for original properties are performed. The natural frequencies are summarized in Figure 11 and their normalized values of perforated shell with respect to the solid shell are calculated. Because the elastic constant is proportional to the square of the frequency, the normalized frequencies are squared and these are the effective elastic constants with which the same modal frequencies as the perforated shell are obtained for the solid shell. Because the effective values are dependent on the mode numbers, their average values are used.

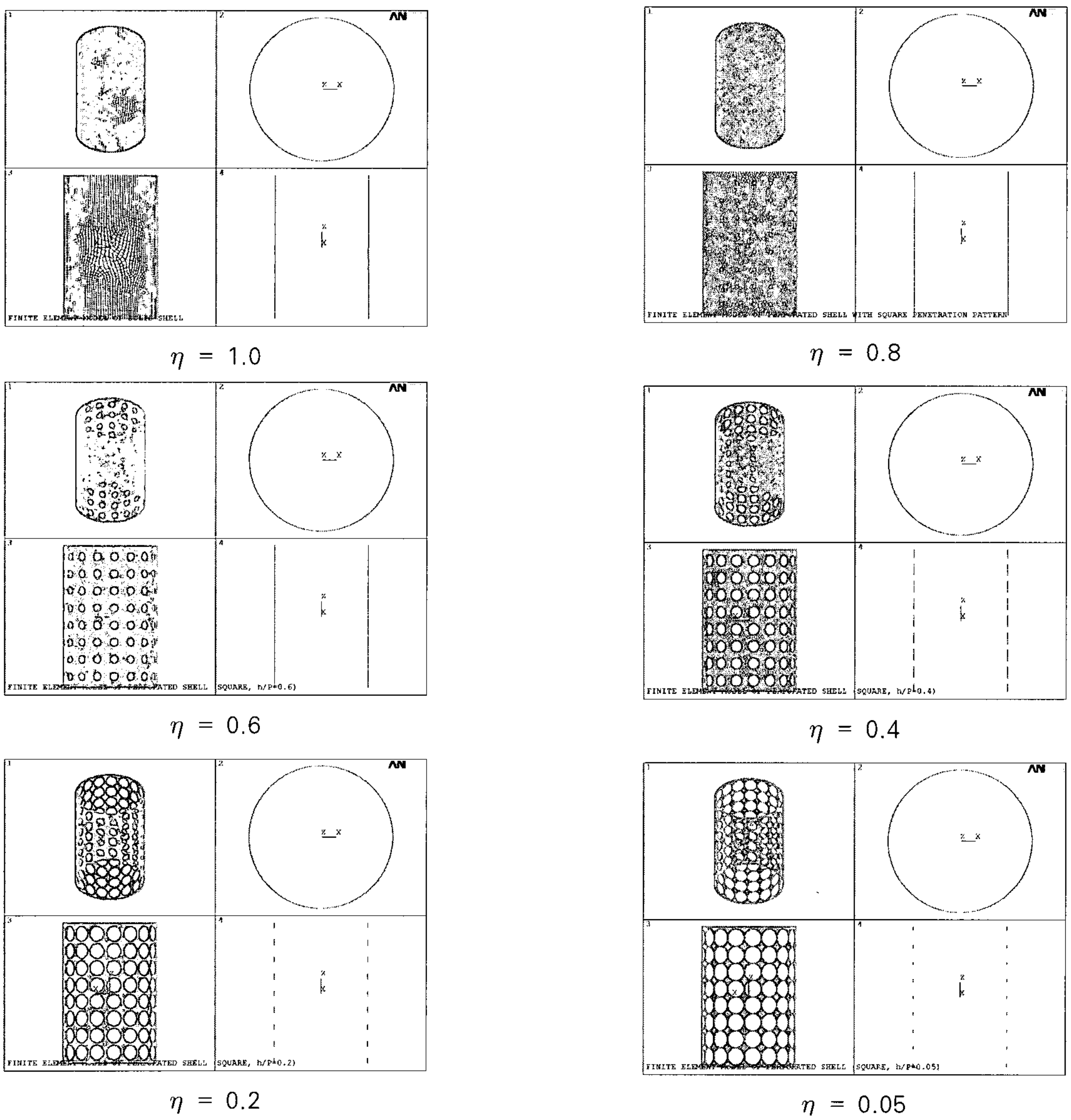


Fig. 10 Finite element models of solid and perforated shells with square penetration pattern

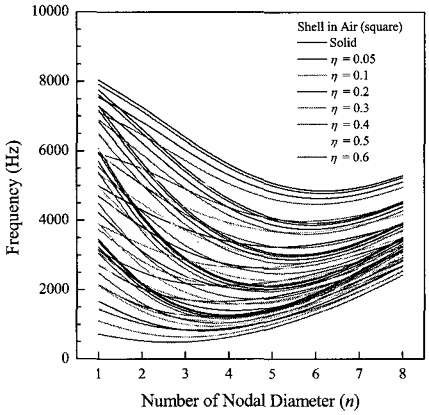


Fig. 11 Natural frequencies of shell in air for solid and perforated shells with square penetration pattern for various ligament efficiencies

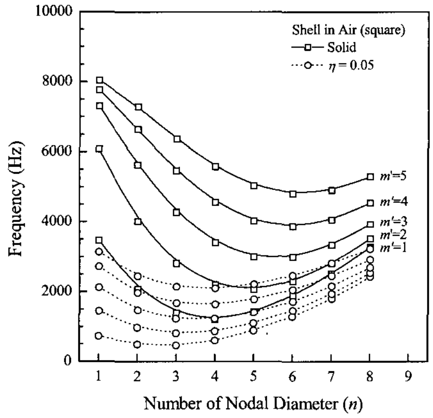


Fig. 12 Frequency comparisons between solid and perforated shells in air

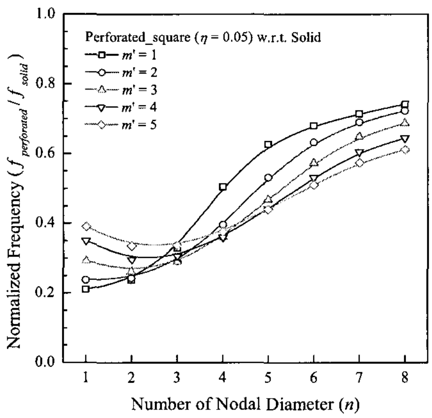


Fig. 13 Normalized frequencies of perforated shell w.r.t. solid shell in air

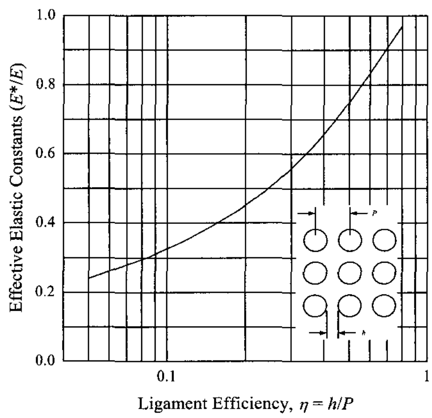


Fig. 14 Effective elastic constants of perforated shell with square penetration pattern

Frequency comparisons between solid and perforated shell and their normalized values for η = 0.05 are shown in Figures 12 and 13, respectively. Figure 14 is the final effective elastic constants proposed for modal characteristics of the perforated shell with a square penetration pattern, which can be represented for $0.5 \le \eta \le 0.8$ as;

$$\frac{E^*}{E} = 0.1593 + 1.7415\eta - 2.0387\eta^2 + 2.4894\eta^3 - 1.3499\eta^4 \qquad (2)$$

4. Results and Discussion

Mode shapes of the fluid-coupled shells are obtained by the finite element method and the frequency comparisons between analytical solution and finite element method are shown in Figure 15 for solid shell with fluid filled annulus. The symbol m' represents the number of axial mode of the wet mode and the symbol n means the number of circumferential mode. The discrepancy is defined as

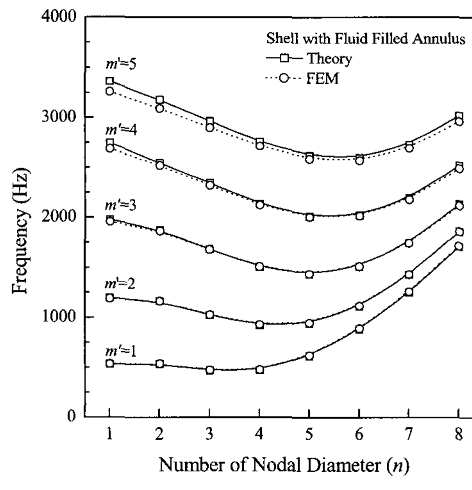


Fig. 15 Frequency comparisons of solid shell with fluid filled annulus between FEM and theory

$$\text{Discrepancy}(\%) = \frac{\text{frequency by FEM - theoretical frequency}}{\text{frequency by FEM}} \times 100 \quad (3)$$

The largest discrepancy between the theoretical and FEM results is 3.0% for $m'=5$ and $n=1$. Discrepancies defined by Eq. (3) are always less than 3%, therefore the theoretical results agree well with FEM results, verifying the validity of the analytical method. As the axial mode number increases, the discrepancy becomes large, which can be reduced by using the sufficient number of node in the axial direction in the finite element modelling. Also, the compressibility of the fluid was found to reduce the natural frequency of the lower wet modes in the case of a fluid-filled cylindrical shell(Jeong and Kim, 1998). Therefore, discrepancies may, also, be caused by the assumption that the water is incompressible in the theory.

Frequency comparisons of solid shell with fluid are shown in Figure 16, where it is found that with increasing axial modes frequencies of shell with fluid filled annulus become almost the same as those of shell with containing fluid inside. Shell submerged in fluid has lower frequencies as expected due to the added mass effect.

The frequency comparisons between shell in air and shell submerged in fluid are shown in Figure 17. The effect of fluid on the frequencies of shell wetted with fluid can be assessed using the normalized frequency defined as the natural frequency of a structure in contact with a fluid divided by the corresponding natural frequency in air. The normalized natural frequencies have values between one and zero due to the added mass effect of fluid. Figure 18 shows the normalized natural frequencies for the shell modes submerged in fluid. As the number of nodal diameters increases, the normalized natural frequencies increase by the gradual reduction of the relative added mass effect. Therefore, an increase of nodal diameters causes an increase in the normalized natural frequencies for all cases of modes.

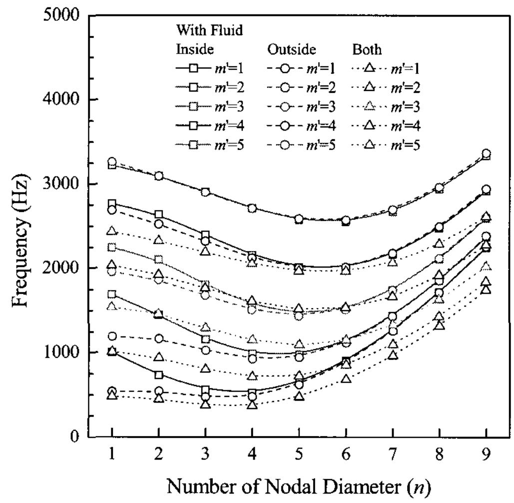


Fig. 16 Frequency comparisons of solid shell with fluid

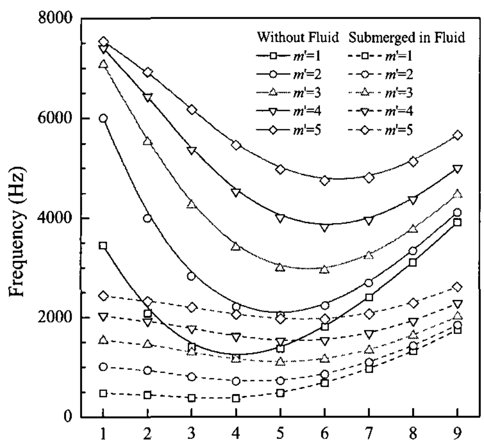


Fig. 17 Frequency comparisons of solid shell between with- and without-fluid

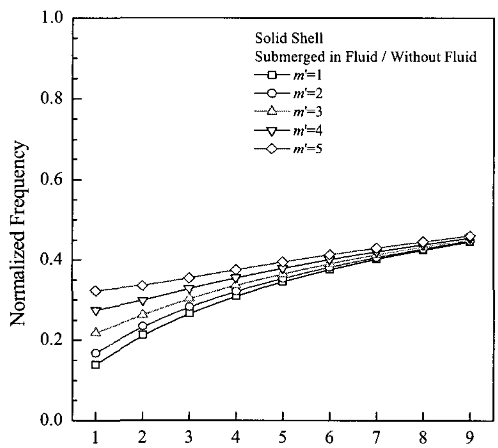


Fig. 18 Normalized frequency of solid shell submerged in fluid with respect to shell without fluid

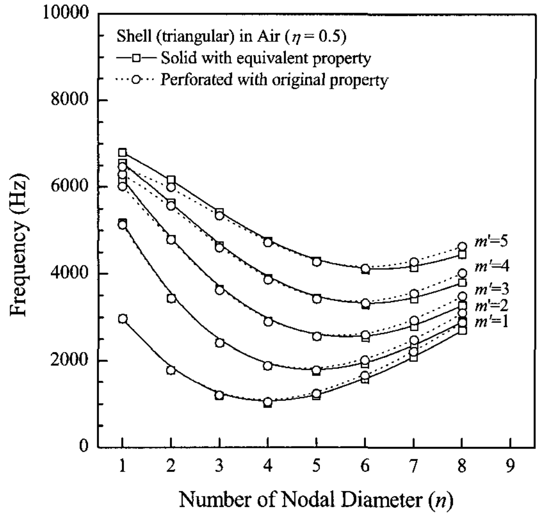


Fig. 19 Comparison of frequencies between perforated shell with original property and solid shell with equivalent property

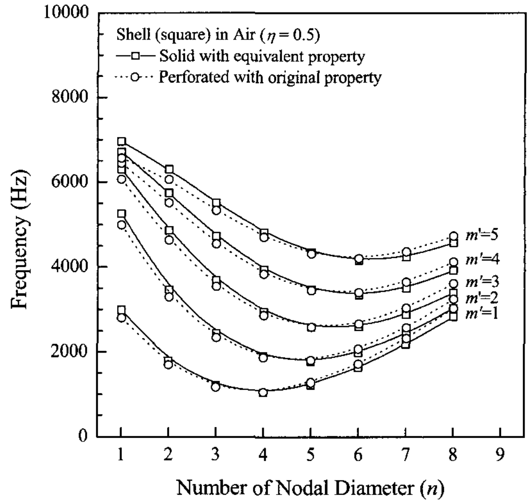


Fig. 20 Comparison of frequencies between perforated shell with original property and solid shell with equivalent property

Figure 19 shows the frequency comparisons between perforated shell($\eta=0.5$) with a triangular penetration pattern with original properties and solid shell with effective elastic constant determined in this study and it is found to be in good agreement between them with the difference of less than 6.27% verifying the validity of the method developed here to calculate the effective elastic constants for the modal analysis of the perforated shell with a triangular penetration pattern.

Figure 20 shows the frequency comparisons between perforated shell with a square penetration pattern with original properties and solid shell with effective elastic constant determined in this study and it is found to be in good agreement between them with the difference of less than 6.6% for $\eta=0.5$ verifying the validity of the method developed here to calculate the effective elastic constants for the modal analysis of the perforated shell with a square penetration pattern.

Comparisons of effective elastic constant between triangular and square penetration patterns show that for the ligament efficiency of more than $\eta=0.5$, the effective elastic constants of a square penetration pattern are higher than those of a triangular penetration pattern. But the differences are less than 10% and are not so significant.

Assuming that there is no flow across the hole the perforated shell submerged in fluid can be modeled as a solid one with equivalent material properties defined in Eqs. (1) and (2) for the triangular and square penetration pattern, respectively. In this case the frequencies is lowered tremendously as expected due to the hole effect as well as the added mass effect of the fluid. For example, the normalized frequencies of perforated shell with triangular penetration pattern for $\eta=0.5$ submerged in fluid are between 0.10 and 0.39 as shown in Figure 21.

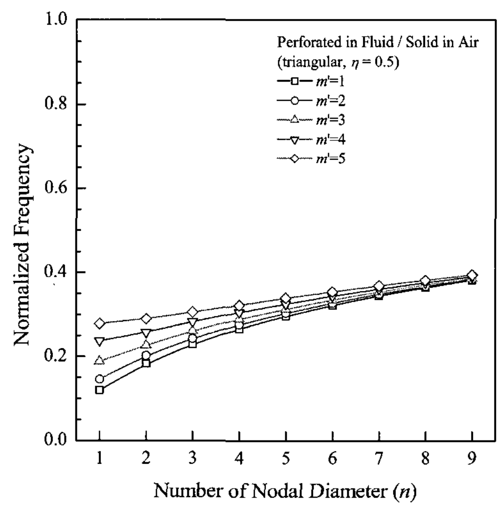


Fig. 21 Normalized frequencies of perforated shell in fluid w.r.t. solid shell in air

5. Conclusions

For the perforated shell submerged in fluid, it is almost impossible to develop a finite element model due to the necessity of the fine meshing of the shell and the fluid at the same time. This necessitates the use of solid shell with equivalent material properties. Unfortunately the effective elastic constants are not found anywhere for the perforated shell analysis. Therefore in this study the equivalent material properties of perforated shell are suggested by performing several finite element analyses with respect to the ligament efficiencies.

The Young's moduli proposed for the modal analysis of the perforated shells with triangular and square penetration patterns are defined in Eqs. (1) and (2). Using the equivalent Young's modulus defined in this study, the modal analysis of the perforated shell submerged in fluid is performed. The normalized frequencies of perforated shell submerged in fluid with respect to the perforated shell in air are calculated and they are compared with those of solid shell. It is found that the frequency decreases significantly due to the hole effect as well as the added mass effect of the fluid.

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